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Applications of Graphs in Real-Life

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Abstract: Graphs are becoming increasingly significant as it is applied to other areas of mathematics, science and technology. This paper gives an overview of the applications of graph theory in heterogeneous fields. It is being actively used in fields as varied as biochemistry (genomics), electrical engineering (communication networks and coding theory), computer science (algorithms and computation) and operations research (scheduling). The powerful combinatorial methods found in graph theory have also been used to prove fundamental results in other areas of pure mathematics. This paper, besides giving a general outlook of these facts, includes new graph theoretical proofs of Fermat's Little Theorem and the Nielson-Schreier Theorem. New applications to DNA sequencing (the SNP assembly problem) and computer network security using minimum vertex covers in graphs are discussed. We also show how to apply edge coloring and matching in graphs for scheduling (the timetabling problem) and vertex coloring in graphs for map coloring and the assignment of frequencies in GSM mobile phone networks.

Keywords: DNA, SNP, GSM:Groups Social Mobile, Voronoi

I. INTRODUCTION

Graph theory is rapidly moving into the mainstream of mathematics mainly because of its applications in diverse fields which include biochemistry (genomics), electrical engineering (communications networks and coding theory), computer science (algorithms and computations) and operations research (scheduling). For example, a data structure can be designed in the form of tree which in turn utilized vertices and edges. Similarly modeling of network topologies can be done using graph concepts. In the same way the most important concept of graph coloring is utilized in resource allocation, scheduling. Also, paths, walks and circuits in graph theory are used in tremendous applications say traveling salesman problem, database design concepts, resource networking. This leads to the development of new algorithms [1] and new theorems that can be used in tremendous applications [2]. In this paper, we present a few selected applications of graphs to other parts of mathematics and to various other fields in general.

II. APPLICATIONS

A. Graphs in Chemistry:

Drug discovery is a time consuming and extremely expensive undertaking. Graphs are natural representation for chemical compounds. In chemical graphs [3], nodes represent atoms and edges represent bond between atom. Graphs are used in the field of chemistry to model chemical compounds. In computational biochemistry some sequences of cell samples have to be excluded to resolve the conflicts between two sequences. This is modelled in the form of graph where the vertices represent the sequences in the sample. An edge will be drawn between two vertices if and only if there is a conflict between the corresponding sequences.

B. Graphs in Biology and The SNP Assembly Problem:

Biology graphs are usually on higher level where nodes represent amino acids and edges represent connections or contacts among amino acids. In computational biochemistry there are many situations where we wish to resolve conflicts between sequences in a sample by excluding some of the sequences. Of course, exactly what constitutes a conflict must be precisely defined in the biochemical context. We define a conflict graph where the vertices represent the sequences in the sample and there is an edge between two vertices if and only if there is a conflict between the corresponding sequences. The aim is to remove the fewest possible sequences that will eliminate all conflicts. Recall that given a simple graph *G*, a vertex cover *C* is a subset of the vertices such that every edge has at least one end in *C*. Thus, the aim is to find a minimum vertex cover in the conflict graph *G* (in general, this is known to be a NP-complete problem). We look at a specific example of the SNP assembly problem given in [4] and show how to solve this problem, using the vertex cover algorithm. A Single Nucleotide Polymorphism (SNP, pronounced "snip") is a single base mutation in DNA. It is known that SNPs are the most common source of genetic polymorphism in the human genome (about 90% of all human DNA polymorphisms).

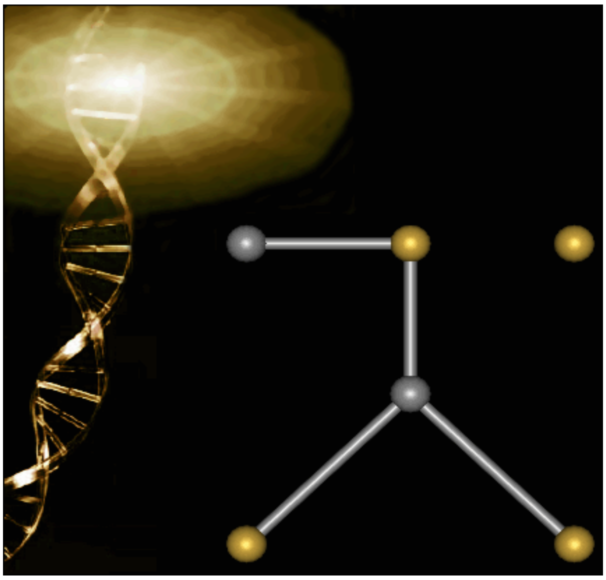

Figure.1The DNA double helix and SNP assembly problem

The SNP Assembly Problem is defined as follows. A SNP assembly is a triple (S, F, R) where $S = \{s_1, ..., s_n\}$ is a set of n SNPs, $F = \{f_1, ..., f_m\}$ is a set of m fragments and R is a relation R: $S \times F \rightarrow \{0, A, B\}$ indicating whether a SNP $s_i \in S$ does not occur on a fragment $f_j \in F$ (marked by 0) or if occurring, the non-zero value of s_i (A or B). Two SNPs s_i and s_j are defined to be in conflict when there exist two fragments f_k and f_l such that exactly three of $R(s_i, f_k)$, $R(s_i, f_l)$, $R(s_j, f_k)$, $R(s_j, f_l)$ have the same non-zero value and exactly one has the opposing non-zero value. The problem is to remove the fewest possible SNPs that will eliminate all conflicts. The following example from [3] is shown in the table below. Note that the relation R is only defined for a subset of $S \times F$ obtained from experimental values. Note, for instance, that s_1 and s_5 are in conflict because $R(s_1, f_2) = B$, $R(s_1, f_5) = B$, $R(s_5, f_2) = B$, $R(s_5, f_5) = A$. Again, s_4 and s_6 are in conflict because $R(s_4, f_1) = A$, $R(s_4, f_3) = A$, $R(s_6, f_1) = B$, $R(s_6, f_3) = A$. Similarly, all pairs of conflicting SNPs are easily determined from the table. The conflict graph G corresponding to this SNP assembly problem is shown below in figure 2.

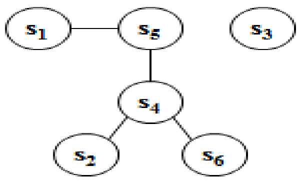

Figure.2 The conflict graph G

We now use the vertex cover algorithm to find the minimal vertex covers in the conflict graph G. The input is the number of vertices 6, followed by the adjacency matrix of G shown below in figure 3. The entry in row i and column j of the adjacency matrix is 1 if the vertices s_i and s_j have an edge in the conflict graph and 0 otherwise.

0	0	0	0	1	0
0	0	0	1	0	0
0	0	0	0	0	0
0	1	0	0	1	1
1	0	0	1	0	0
0	0	0	1	0	0

Figure.3 The input for the vertex cover algorithm

The vertex cover program finds two distinct minimum vertex covers:-

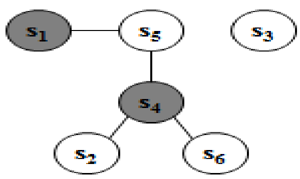

Figure.4 Minimum Vertex Cover: s_1, s_4

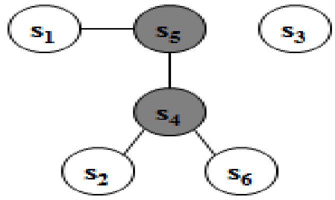

Figure.5 Minimum Vertex Cover: s_4, s_5

Thus, either removing s_1, s_4 or removing s_4, s_5 solves the given SNP assembly problem.

C. Map Coloring and GSM Mobile Phone Networks:

Groups Special Mobile (GSM) is a mobile phone network where the geographical area of this network is divided into hexagonal regions or cells. Each cell has a communication tower which connects with mobile phones within the cell. All mobile phones connect to the GSM network by searching for cells in the neighbours. Since GSM operate only in four different frequency ranges, it is clear by the concept of graph theory that only four colors can be used to color the cellular regions. These four different colors are used for proper coloring of the regions.

Therefore, the vertex coloring algorithm [5] may be used to assign at most four different frequencies for any GSM mobile phone network. Given a map drawn on the plane or on the surface of a sphere, the four color theorem assets that it is always possible to color the regions of a map properly using at most four distinct colors such that no two adjacent regions are assigned the same color. Now, a dual graph is constructed by putting a vertex inside each region of the map and connect two distinct vertices by an edge if their respective regions share a whole segment of their boundaries in common. Then proper coloring of the dual graph gives proper coloring of the original map. Since, coloring the regions of a planar graph G is equivalent to coloring the vertices of its dual graph and vice versa. By coloring the map regions using four color theorem [6], the four frequencies can be assigned to the regions accordingly.GSM networks operate in only four different frequency ranges.

D. Computer Network Security:

A team of computer scientists led by Eric Filiol [7] at the Virology and Cryptology Lab, ESAT, and the French Navy, ESCANSIC, have recently used the vertex cover algorithm to simulate the propagation of stealth worms on large computer networks and design optimal strategies for protecting the network against such virus attacks in real-time.

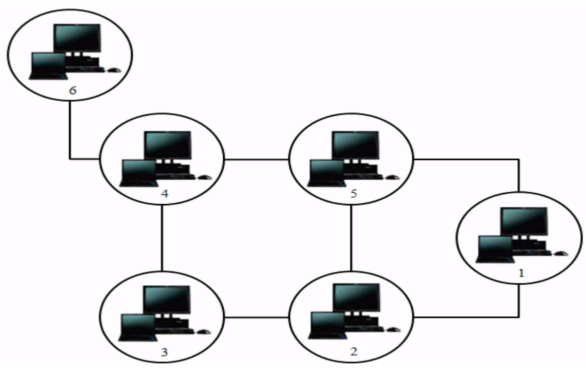

Figure.6 The set {2, 4, 5} is a minimum vertex cover in this computer network

The simulation was carried out on a large internet-like virtual network and showed that that the combinatorial topology of routing may have a huge impact on the worm propagation and thus some servers play a more essential and significant role than others. The real-time capability to identify them is essential to greatly hinder worm propagation. The idea is to find a minimum vertex cover in the graph whose vertices are the routing servers and whose edges are the (possibly dynamic) connections between routing servers. This is an optimal solution for worm propagation and an optimal solution for designing the network defense strategy.

E. Airline route maps:

Vertices represent airports, and there is an edge from vertex A to vertex B if there is a direct flight from the airport represented by A to the airport represented by B. Airlines use minimum spanning trees to work out their basic route system. Assuming that there are k aircrafts and they have to be assigned n flights. The ith flight should be during the time interval (ai, bi). If two flights overlap, then the same aircraft cannot be assigned to both the flights. This problem is modeled as a graph as follows. The vertices of the graph correspond to the flights. Two vertices will be connected, if the corresponding time intervals overlap. Therefore, the graph is an interval graph that can be colored optimally in polynomial time.

F. T ime table scheduling:

Allocation of classes and subjects to the professors is one of the major issues if the constraints are complex. Graph theory [8] plays an important role in this problem. For m professors with n subjects the available number of p periods timetable has to be prepared. This is done as follows. A bipartite graph (or bigraph is a graph whose vertices can be divided into two disjoint sets *U* and *V* such that every edge connects a vertex in *U* to one in *V*; that is, *U* and *V* are independent sets [9]) G where the vertices are the number of professors say m1, m2, m3, m4, ..mk and n number of subjects say n1, n2, n3, n4,.. nm such that the vertices are connected by pi edges.

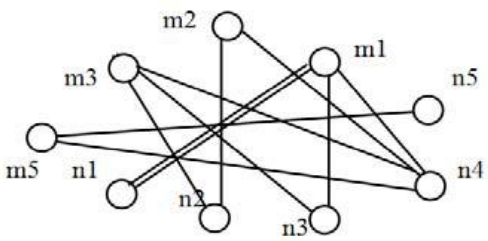

Figure.7

It is presumed that at any one period each professor can teach at most one subject and that each subject can be taught by maximum one professor.

p	n1	n2	n3	n4	n5
m1	2	0	1	1	0
m2	0	1	0	1	0
m3	0	1	1	1	0
m4	0	0	0	1	1

Figure.8

Consider the first period. The timetable for this single period corresponds to a matching in the graph and conversely, each matching corresponds to a possible assignment of professors to subjects taught during that period. So, the solution for the timetabling problem will be obtained by partitioning the edges of graph G into minimum number of matching. Also the edges have to be colored with minimum number of colors. This problem can also be solved by vertex coloring algorithm [5]. The line graph L(G) of G has equal number of vertices and edges of G and two vertices in L(G) are connected by an edge iff the corresponding edges of G have a vertex in common. The line graph L(G) is a simple graph and a proper vertex coloring of L(G) gives a proper edge coloring of G by the same number of colors. So, the problem can be solved by finding minimum proper vertex coloring of L(G). For example, Consider there are 4 professors namely m1, m2, m3, m4, and 5 subjects say n1, n2, n3, n4, n5 to be taught. The teaching requirement matrix p = [pij] shown above.

G. Modeling sensor networks as graph:

The sensor networks have got variety of applications. Tracking of mobile objects, collection of environmental data, defense applications, health care etc.

In this, a message pruning tree with minimum cost is converted to track the moving objects in wireless sensor networks. The sensor network is modeled as a graph to analyse the communication efficiency. Voronoi graph is taken to model the sensor network. (A Voronoi diagram [10] is a special kind of decomposition of a metric space determined by distances to a specified discrete set of objects in the space. Because voronoi graph is constructed in a plane in the form of polygons with nodes as the sensors and the polygon boundaries can be considered as the sensing range of each sensor. Consider, the plane as the sensing field and S be the sensors. The sensing field is partitioned into a voronoi graph as shown. Any object located in a polygon of the voronoi graph is closest to the sensor in the corresponding polygon. The polygon can be considered as the sensing range of these sensors. Among these sensors one sensor will be the cluster head for reporting function. Two sensors are considered as neighbours if their sensing range share a common boundary in the voronoi graph. In the diagram a, b are neighbours. Similarly, e,f; e,d; e,I; are also neighbours. When the objects cross the boundary of one sensor i.e. the sensing range of one sensor, and enter into the sensing range of another sensor it should be reported to the neighbouring sensor properly by the previous sensor.

The event rate between two neighbouring sensors implies the strength of the detection. Since, it is assumed that the sensor's transmission range is large enough such that any two neighbours can directly communicate with each other, the network is represented as an undirected weighted graph G(VG, EG, WG) where v belongs VG, edge (u,v) belongs EG. V implies the sensors, u,v implies the neighbours. WG(u,v) the weighted edge of (u,v) belongs EG. The authors have used the concept of coverings. (K-cover [11], is defined as a set of sensors M such that each point in the sensor network is "covered" by at least K different sensors in M, and the communication graph induced by M is connected.

H. Electrical Circuits:

Vertices represent diodes, transistors, capacitors, switches, etc., and edges represent wires connecting them.

I. Graph theory relevant to ad-hoc networks:

In Adhoc networks [12], issues such as connectivity, scalability, routing, modeling the network and simulation are to be considered. Since a network can be modeled as a graph, the model can be used to analyze these issues. Graphs can be algebraically represented as matrices. Also, networks can be automated by means of algorithms. The issues such as node density, mobility among the nodes, link formation between the nodes and packet routing have to be simulated. To simulate these concepts random graph theory is sued. The connectivity issues are analyzed by using graph spanners, (A geometric spanner or a *k*-spanner graph or a *k*-spanner was initially introduced as a weighted graph over a set of points as its vertices and every pair of vertices has a path between them of weight at most *k* times the spatial distance between these points, for a fixed *k* proximity graphs,(A proximity graph is simply a graph in which two vertices are connected by an edge if and only if the vertices satisfy particular geometric requirements), sparsification and spectral graph theory. Various algorithms are also available to analyze the congestion in MANET's where these networks are modeled based on graph theoretical ideas.

III. CONCLUSION AND FUTURE RESEARCH

The main aim of this paper is to present the importance of graph theoretical ideas in various areas of Real life. An overview is presented especially to project the idea of graph. Researches may get some information related to graph theory and its applications in computer field and can get some ideas related to their field of research. Graph arise in the context of number of applications such as social networking, in which the communications between large group of users are captured in the form of a graph. Such applications are very challenging since the data cannot be localized on a disk for the purpose of structural analysis. Therefore new techniques are required to summarize the structural behaviour of graph and use them for a variety of analytical scenarios. Graphs combine the advantage of intuitive understanding with mathematical feasibility. Rule-basedness is considered to be a powerful and manageable method for specifying the dynamic behaviour of systems. Putting graphs and rules together yields graph grammars. We expect that graph transformation will be shown to be a suitable base for the development of integrated specification environments, fractal pattern processing, design of expert systems. The interest in the applicability of graph will grow with the quality of interactive tools on a large scale.

IV. REFERENCES

[1] "Planarizing Graphs", Journal of Graph Algorithms and Applications, http://www.cs.brown.edu/publications/jgaa/vol. 5, no. 1, 2001.

[2] Shariefuddin Pirzada and Ashay Dharwadker, "Applications of Graph Theory", Journal of the Korean Society for Industrial and applied Mathematics, Volume 11, No.4,2007.

[3] Alexandru T. Balaban, Journal of Chemical Information and Computer Science, 1985, 25 (3), pp 334–343 DOI: 10.1021/ci00047a033.

[4] G. Lancia, V. Afna, S. Istrail, L. Lippert, and R. Schwartz,SNPsProblems, Complexity and Algorithms, ESA 2002, LNCS 2161, pp. 182-193, 2001. Springer-Verlag 2001.

[5] Ashay Dharwadker, The Vertex Coloring Algorithm, 2006, http://www.dharwadker.org/vertex_coloring

[6] Ashay Dharwadker, " a new proof of The four colour theorem", Institute of Mathematics, Haryana, India. http://www.dharwadker.org/

[7] Eric Filiol, Edouard Franc, Alessandro Gubbioli, Benoit Moquet and Guillaume Roblot, Combinatorial Optimisation of Worm Propagation on an Unknown Network, Proc. World Acad. Science, Engineering andTechnology, Vol 23, August 2007.

[8] Daniel Marx, "Graph Coloring problems and their applications in scheduling".

[9] Narasingh Deo, "Graph theory with applications to engineering and computer science", Prentice Hall of India, 1990.

[10] M. Abellanas, F. Hurtado, C. Icking, R. Klein, E. Langetepe, L. Ma, B. Palop, and V. Sacristan. "The farthest color Voronoi diagram and related problems". In Abstracts 17th European Workshop Comput. Geom., pages 113–116. Freie University¨at Berlin, 2001.

[11] Zongheng Zhou, Samir R. Das, Himanshu Gupta, "Connected K-Coverage Problem in Sensor Networks", Proc. of the 13th In*t*. C*onf*. on Computer Communicati*on* and Networks *(*IC3N*)*, Chicago, IL, Oct 2004.

[12] M. A. Rajan, M. Girish Chandra, Lokanatha C. Reddy, Prakash Hiremath "Concepts of graph theory relevant to Adhoc Networks", Int. Journal of Computers, Communications & Control, ISSN 1841-9836, E-ISSN 1841-9844 Vol. III 2008.

