DOI: http://dx.doi.org/10.26483/ijarcs.v13i6.6919

Volume 13, No. 6, November-December 2022

ISSN No. 0976-5697

International Journal of Advanced Research in Computer Science

RESEARCH PAPER

Available Online at www.ijarcs.info

STOCK MARKET FORECASTING USING CONTINUOUS WAVELET TRANSFORM AND LONG SHORT-TERM MEMORY NEURAL NETWORKS

José Alfredo Acuna-García
Computer Science Faculty
Autonomous University of Queretaro
Querétaro, Querétaro

Sandra Luz Canchola-Magdaleno
Computer Science Faculty
Autonomous University of Queretaro
Querétaro, Querétaro

Carlos Alberto Olmos Trejo
Computer Science Faculty
Autonomous University of Queretaro
Querétaro, Querétaro

Abstract: The analysis and exploitation of complex and large-volume data requires new approaches, and modeling it in time series is a very successful technique. A characteristic time series is the one that defines the dynamic financial market and its asset prices. This research presents a novel forecasting methodology, which uses the Continuous Wavelet Transform for the definition of representative elements that define a time series, and a recurrent neural network architecture for the forecast of prices of financial stocks related by the item of income in the short and medium time term. The proposed model, inspired by the Continuous Wavelet Transform and Neural Networks of the "Long short-term memory" type, uses the most representative coefficients of the Wavelet transform based on the time series in the time domain, for the prediction of future prices of stocks in short prospective periods. The results show a very successful projection using this methodology. Future research will analyze the interrelationship presented by the price time series of the same stock market section, in the domain of Wavelets, and how it affects the stock market forecast.

Keywords: Data science, Wavelet, Continuous Wavelet Transform, Forecast, Long short-term memory.

I. INTRODUCTION

The price of financial assets has been considered non-predictable given the dynamism that the financial market presents. The forecast is the proposal of future values based on forward-looking methods that analyze price behavior using past times. The set of values that represent a phenomenon that changes over time is known as a time series, and the prices of financial stocks are represented as values in a time series. In equation 1 the mathematical form of values of a time series is showed.

$$x(t) = x_1, x_2, x_3, \dots x_n \qquad (1)$$

Where x_n are the values that the series takes at time n, and $x(t)$ is the function that groups them, a value $x(t+h)$ where $h \geq 1$ and that defines the next state of the function based in the previous known states. In addition to being able to be applied to the prices of stock assets, the time series and their projection into the future, they can be used in applications such as the control of industrial processes, signal processing, meteorology, economics, among others. Forecast analysis is a research area that is still open to the proposal of solutions today [1].

The use of Wavelets in time series analysis is an area that is becoming increasingly important in the analysis of non-obvious information. One method to analyze signals in time series is to transform them to a domain that provides valuable information for analysis, such as the frequency domain. The Continuous Wavelet Transform (CWT) allows decomposing a data set from an original domain to a domain that represents frequency elements of the data being transformed. CWT is applicable to data organized in unidimensional vectors [2,3,4].

There are proposed methods for predicting price behavior considering only a market asset and change points, such as the Quandt-Andrews method proposed in [5]. Several analyzes of the behavior of the change points in time series, in the representation of stock market prices, using probabilities are the investigations documented in [6,7,8].

Neural networks are popular for discovering behavior patterns, allowing mathematical models to be defined for temporal processes and consequently for time series analysis. Neural Networks (NNs), allow analyzing the functional relationships that exist between data of a phenomenon and have been used to predict future values in series of events over time [9].

One type of neural networks is known as the Recurrent Neural Network (RNN), which have been very suitable for modeling non-stationary dynamic systems in industrial applications. This type of network models has non-linear partial autocorrelation structure of time series and directly captures effects such as seasonality and trends. The RNNs are used to forecast electricity load, weather data, and stock prices. The RNNs have results in applications such as speech processing, using techniques of Convolutional Neural Networks of Long Short-Term Memory (CNN-LSTM). In [10] the application in time series for the quantification of uncertainty and using Bayesian estimation for better forecasting and improved coverage is shown.

Dixon [11] shows how exponentially smoothed RNNs are suitable for modeling dynamical systems arising in big-data applications. The application of exponentially smoothed RNNs taking Bitcoin prices as a sample, allows us to see the effectiveness of exponential smoothing for time series forecasting. The algorithms used by Alpha-RNN are considered to be complex "black box" architectures. The use of deep neural networks (Deep NN, DNN) that are powerful types of artificial neural networks, has been used in speech transcription and image recognition for their predictive properties. In [12] the application of DNN to predict the directions of financial market movements is described, using a set-and-train approach to demonstrate its application in a simple trading strategy on 43 future average commodity prices and different currencies in intervals of 5 minutes.

In the research [13] a learning algorithm based on particle swarm optimization and evolutionary algorithms (PSO-EA) is proposed that uses RNNs for the prediction of time series data. The implemented algorithm trains an Elman network to predict time series and test with the CATS Benchmark an artificial time series containing 5000 values. CATS contains 100 missing values spread across 5 sections. The forecast of the missing values is carried out using 5 networks; each network predicts 20 values. The prediction of all missing values with respect to the true values has a mean square error (MSE) of 351.

A so-called chaotic neural network was implemented in [14] based on a biological model called the KIII set. This network, capable of oscillating in a complex non-periodic way, is trained using a Hebbian learning rule. The training set is composed of 45 input samples, each containing 10 values from the time series. The predictions has a Mean Square Error (MSE) of 73. The model used has important prediction results to take into account in the definition of new methodologies. In another investigation, a hybrid complex neural network (HCNN) is proposed, which is a long-term recurrent neural model for time series prediction proposed by Gómez-Gil [15]. HCNN uses Fourier analysis to gain insights into time series behavior to train various harmonic generators. The generator is a fully connected neural network with the ability to autonomously generate sine series. The overtone generator is connected to other recurrent neurons. The results indicate an MSE value of 2.5^{e-3}.

The use of the wavelet transform in the attenuation of undesirable noise in a series of time, is treated in multiple investigations with efficient results, for the treatment of signals and the elimination of possible risks in the interpretation of contaminated signals in the geophysical area. Its methodology uses a comparative analysis of signals obtained from the same source and uses the Wavelet transform in the process of attenuation of high frequencies by identifying the corresponding coefficients [16]. Similarly, the Wavelet transform is used for electrical noise reduction in industrial signal processing. The use of the discrete transform is very useful for processing electrical signals, using it to transform the signal to the time-frequency domain, and after filtering the determined coefficients, perform the inverse transformation and return the signal to a time series [17].

In the processing of speech signals for the recognition of spoken natural language, the signals may contain a large amount of ambient noise. The Discrete Wavelet Transform (DWT) is presented as a very useful tool in the treatment of the signal with the purpose of attenuating the noise present [18]. In the area of electrical signals that are contaminated by static generated by electrical storms or by discharges from electrical power equipment, DWT is used for time series signal analysis, weighting coefficients of the wavelet transform to determine the amount of noise on the signal [19]. In the industrial area, the wavelet transform is used for the analysis of possible risks for security systems. The analysis is based on determining a signal distribution index in its representation in the frequency-time domain. Their results analyze the correlation of the spectra obtained with frequency indicators in the signal time spaces [20].

The present investigation establishes a novel forecasting methodology, which uses the CWT for the definition of representative elements that define a time series, and an LSTM neural network architecture for the price forecast of related financial assets because they belong to the same financial category, in the short and medium time term. The proposed model uses the CWT and CNN-LSTM on the most representative coefficients of the time series based on the contribution to the time domain, it shows a well-defined behavior in the forecast of time series in short prospective periods.

The importance of this research lies in the area of time series forecasting, being possible to apply it to any phenomenon that can be represented by means of a time series, allowing the identification of events that allow anticipating measures that take advantage of or attenuate effects that are prospected with this methodology.

This work deals with the investigation in 5 main sections, the first being the introduction and theoretical framework of the investigation. The second section proposes the methodology of the present investigation. The results section shows charts and images applying the defined methodology followed by a discussion section, and finally conclusions and future projects.

*A. **Theory***

A signal that changes over time, also known as a time series, represented by a series of discrete data is

$$f(t) = \sum_{k=1}^{\infty} \mu_k \emptyset_k(t) \qquad (2)$$

mathematically represented in Equation 2.

where $f(t)$ is a signal defined by a time series, $\mu_k = \langle f(t)|\emptyset_k(t)\rangle\ y\ \emptyset_k(t)$ they are basic functions composed of various wavelet signals. Consequently, the coefficients of the integrated wavelet signals in the functions are given by different values of k and can be generalized as indicated by equation 3.

$$\mu_k = \langle f(t)|\emptyset_k(t)\rangle = \int f(t)\,\emptyset_k(t)dt \qquad (3)$$

A Wavelet is a function characterized by different waves that depend on their integration on their location along an independent variable on the time axis *(t)*. A Wavelet that has the property of being normalized must meet the condition that its mean value must be zero as expressed in equation 4 [21,3].

$$\int_{-\infty}^{\infty} \varphi_{i(t-i)dt=0} \qquad (4)$$

where $\varphi_{i(t-i)}$ represents the Wavelet, i is a value that varies on the x axis, and whose value integration is zero on average.

CWT is based on the continuous integration of two functions along their axis of independent variable (the time axis for a time series). The one that is the subject of analysis, which is a signal of indefinite duration, and the Wavelet has a duration defined in a period of time. The method applies the Wavelet to the analysis signal by integrating it, over the length of the Wavelet, and shifting it over the time axis to continue covering the analysis signal. This wavelet with which the original signal is integrated has an established distribution and is called the Kernel Wavelet.

The process of discretely integrating the signals is called mathematical convolution. In CWT processing, the variation of the Kernel frequency varies depending on the number of Wavelet Frequency Bands (WFBs) with which it is required to decompose the original signal. The graphical representation of all the frequencies into which an original signal is decomposed is known as the CWT spectrum. Coefficients expressed in equations 5 and 6 are obtained from this process, for a multi-resolution analysis of two resolutions.

$$A_i = \int_{-\infty}^{\infty} 2^{\frac{1}{2}} \emptyset_0 (2^{-1}t - i)\, f(t)dt \tag{5}$$

$$D_i = \int_{-\infty}^{\infty} 2^{\frac{1}{2}} \varphi_0 (2^{-1}t - i)\, f(t)dt \tag{6}$$

where $\emptyset(t)$ and $\varphi_0(t)$ are the convolution Wavelets in the time domain, which define coefficients of high and low frequency bands, A_i represents the higher frequency coefficients of the Wavelet transformation and D_i the lower frequency ones [21,3]. This process can be seen schematized in figure 1.

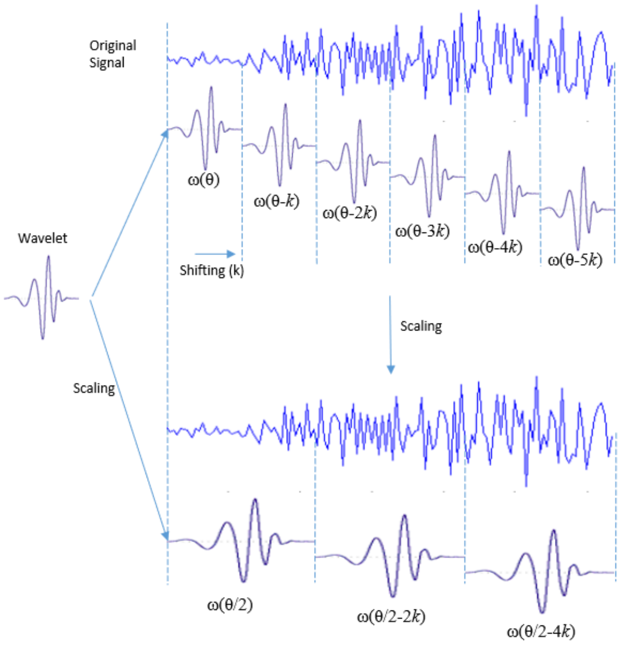


Figure 1. Schematic of the Wavelet Transformation Continues exemplifying two levels of resolution in 1D. Source: self-made.

In the area of machine learning, Deep Learning has proven to be a widely used tool in a large number of different fields, such as object recognition, image classification, language recognition, in the financial area in the fraud detection, pricing, portfolio creation, risk management, and more. Data represented in time series is used as input to this type of network for training and testing, and plays an important role in these problems, which demonstrates the ability of neural networks to process this type of information.

LSTM neural networks are a special type of recurrent convolutional networks, and they are very useful in learning non-periodic sequences, which makes them very attractive for the present investigation. This type of network requires training with a large amount of input data, to improve its performance in determining a next step in the series, a step that is suggested when using it for forecasting a time series. A CNN-LSMT is defined in equation 7.

$$y_j(t+1) = \varphi\Big(\sum_{i=1}^{m+n} \omega_{ij} \emptyset_i\,(t)\Big) \tag{7}$$

$$\emptyset_i(t) = \begin{cases} y_j(t+1), & i \le n \\ u_{i-n}, & i > n \end{cases}$$

where $y_j(t+1)$ is the output of the $j-th$ neuron and ω_{ij} the connection of the $i-th$ y $j-th$ neuron. The m es el is the number of inputs, n is the number of output and hidden neurons, and $\varphi()$ is an arbitrary differential function (it can be a sigmod, proportional, etc.). The external inputs u_i and the recursive inputs are y_i are represented by $\emptyset_i$. [10,11].

Signal comparison allows knowing the similarity or difference between two or more groups of values that represent a signal. There are several methods to perform signal comparisons, being one of the most commonly used, the index based on the Euclidean distance between the elements of two signals to be compared. If a signal is represented by means of a time series, the Euclidean distance represents the differentiation between the two series. This method is also known as the MSE and a variation is the Square Root of the MSE (RMSE), which provides information directly related to the original dimensions of the information that is analyzed, and whose percentage. In relation to the maximum representation value of an element of the time series, it represents the similarity between the signals it compares. The greatest similarity of this index is presented when the error is 0.00. The MSE is defined expressly in equation 8.

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(x_i - y_i)^2 \tag{8}$$

That for two signals x and y their values are defined by each one of the elements of the time series of the coordinates i and N s the dimension of the one-dimensional vector of elements.

Another comparison index is the Structural Similarity Index (SSIM), which is considered a benchmark when it comes to comparing elements. It can be explained firstly that this index shows contrast and visibility characteristics in the signal comparison. Furthermore, it reflects a structural covariance between the signals compared [22].

The SSIM index, unlike the MSE index, shows the characteristics of the structure of the signals to be compared, and not just the difference between point elements. One of the main advantages it has over MSE is that it better reflects a similarity correlation as it is perceived by human eyes. Images with a similar MSE can be visually very different. The SSIM formula is showed in equation 9 [23].

$$SSIM(x,y) = 1 - \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \tag{9}$$

where μ_x and μ_y are the mean values of the signals x and y respectively. σ_x^2 and σ_y^2 are the variances of the corresponding signals. The expression σ_{xy} is the covariance of x and y, the constants C_1 and C_2 are two variables used to stabilize the division with a weak denominator obtained from the dynamic range of the values of the pixels of the images multiplied by two constants defined in the index. The greatest similarity between two signals by means of this index, occurs when its value is closer to 1.00.

II. Method

New approaches have emerged for forecasting based on neural networks and transformation of time series in non-temporal domains. In the present investigation, an innovative methodology is proposed that takes as a source of information a series of data on the results of the financial stock prices of the companies of a corporate. The financial stock analyzed belong to companies that have the common factor of being part of a corporation and of the same financial category, whose purposes are similar and/or complementary. The information belongs to the company and is extracted from its information banks to process and standardize it in a format that facilitates its processing. The standardized information presents 6,650 readings spaced at a time of 3 hours, in a time period of 831.25 historical days, which represents 2.27 years, beginning in January 2019. 9 stocks are analyzed and information is presented on the 3 most representative.

The series of financial stock prices are analyzed in the wavelet domain using the CWT, weighting the coefficients that present periodicities representative of the series, determining their contribution to the time series in the time domain based on their average absolute value, and discriminating the coefficients that have a smaller contribution to the CWT coefficients. The weighting is performed when the contribution of the coefficients determines 83% of the total contribution of the specific values in the time series. 83% of representation was considered to give good results in the forecast made, after analyzing different contribution percentages.

Coefficients are standardized and tokenized to determine the number of input neurons of the neural network. The coefficients considered with greater representation in the Wavelet domain are introduced to a CNN-SLTM type neural network for training, considering as input the selected neurons that correspond to the integer values of the coefficients. The output neurons are standardized and tokenized based on the stock price values in the stock price series presented in the time series.

Figure 2 shows the general diagram of the CNN-LSTM used. A sigmoid activation function is used on the output.

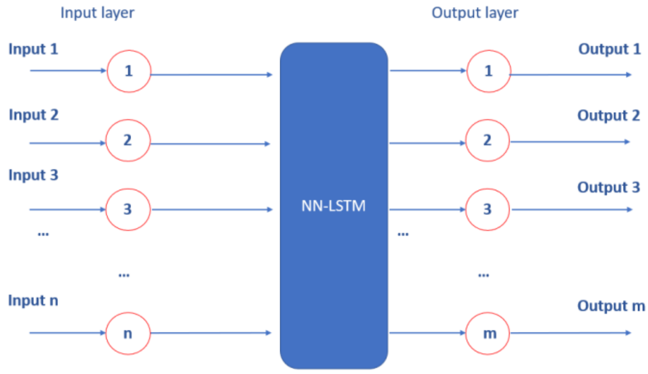


Figure 2. LSTM Neural Network used. Source: self-made.

Based on the standardization of the input coefficients, the number "n" of input neurons is defined (for example, for the first signal 7,800 were defined), and "m" output neurons (for the first signal, 12,000 were obtained) with an average layer of 300 neurons. For each stock market share, the number of input and output neurons is defined according to the variation in prices it presents. In the LSTM neural network, the process is performed recursively with the values of each point of the time series, feeding back the neural network for 6,650 periods at 3-hour intervals each. This process is carried out for the training of the neural network, to proceed in a following phase with the application of the trained Neural Network, and applied to the forecast of the time series of stock market values.

The mathematical processes of the CWT transform was programmed in Python Ver. 3.8. The CWT transformation was performed using the Mallat algorithm (Chui, 1992), and was processed using numpy and scipy libraries for Python on the Google Colab platform. The CNN-LSTM neural network process was processed using Python Ver. 3.8 and TensorFlow 1.9.0-cp37-cp37m-amd64 libraries, on the Google Colab platform.

In the final part of the process, the results obtained by the predicted processing was analyzed, contrasting it against the real results of the stocks, showing the predicted values, the actual values of the stocks, and discussing the results of the methodology. The results allow analyzing the relevance of the length of the prediction period based on the deviations detected. This research presents qualitative results.

Figure 3 presents the general scheme of the methodology used.

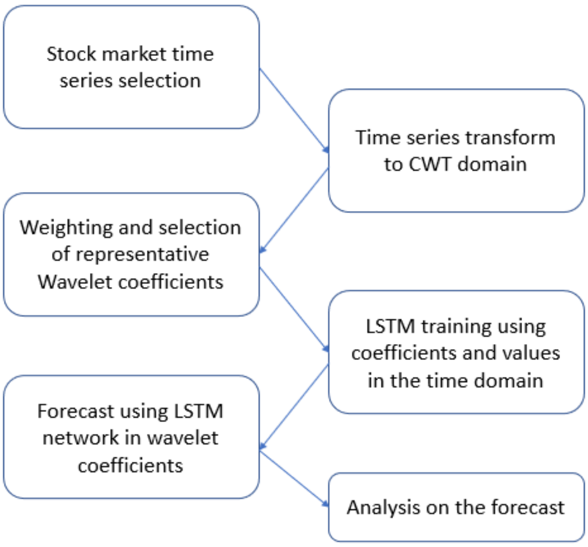


Figure 3. Scheme of the methodology. Source: self-made.

III. RESULTS AND DISCUSSION

The results of applying the methodology are presented in the following figures, with three of the most representative time series of the 9 that were extracted from the companies' data. Figure 4 shows the graphs that represent the time series of the stock market shares, the spectrum of the coefficients of the CWT transformation and the graph of a short section of values of the price time series.

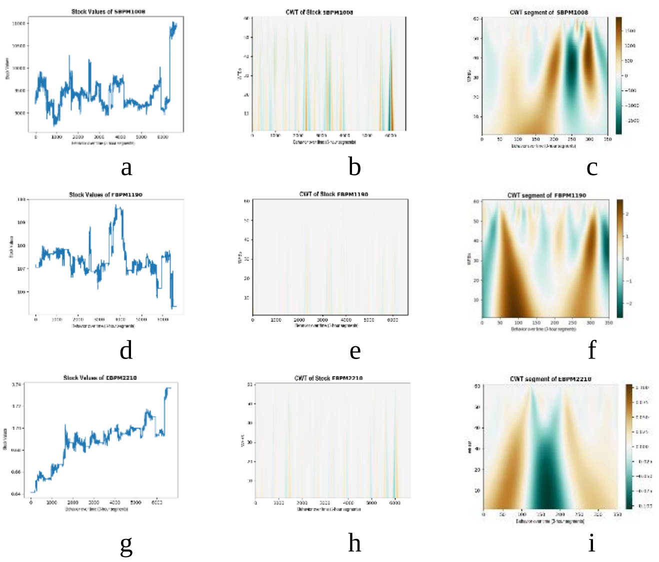


Figure 4. Time series plots and CWT spectra. Source: self-made.

Figure 4 (a), (d) and (g) shows the graphs of the time series of the market stocks selected as representative of the behavior of all the stocks processed. In addition, Figure 4 presents the spectrum of the coefficients of the CWT transformation and the graph of a short section of values of the price series. Applying the CWT to the time series, the graph of the coefficient spectrum of the complete CWT transformation is obtained, which is shown in Figure 4 (b), (e) and (h). In the graphs of figure 4(c), (f) and (i), a section of the spectrum is shown with the scale of color values used in the spectra, where we can notice the variation within the range from -1,900 to 1,900 for the first time series shown in the same plots, as an example.

Figure 5 presents the graphs that show the spectra data and the Wavelet coefficients, weighted selected for the best representation of the time series in the time domain.

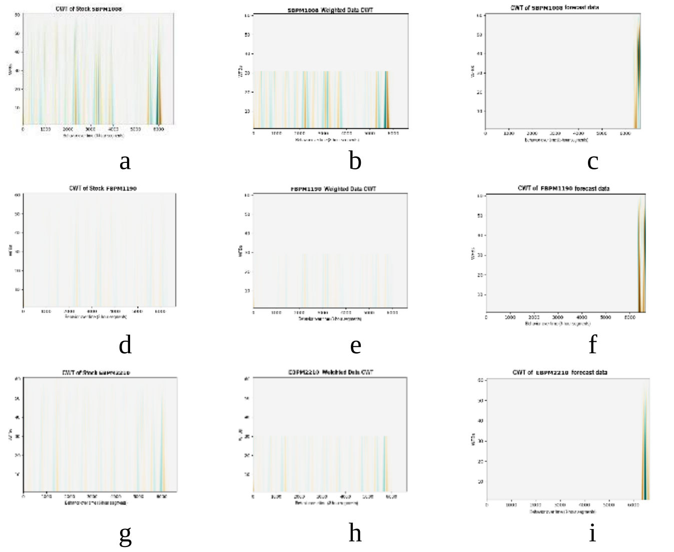
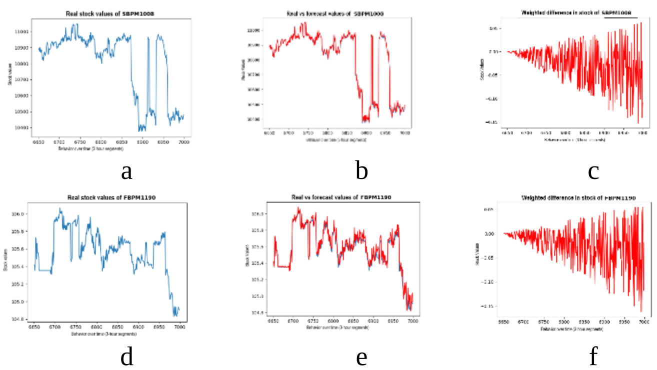
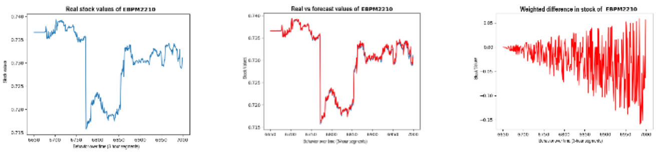


Figure 5. Coefficient weighting and segmentation plots Source: self-made.

The graphs in Figure 5 (b), (e) and (h) show the weighting of the most representative coefficients that allow a reconstruction of the time series avoiding noise in the signal and considering the wavelet coefficients in their WFBs, which better represent the cyclical elements of the time series. These coefficients are standardized and tokenized to introduce them to the CNN-LSTM into the training process in the Google Colab platform. After training, the test or application process is started, and the forecasted prices are obtained, transformed back to the CWT domain and re-introduced to the trained model to obtain new forecasts. The spectrum of the forecasted prices is presented in the graphs of figure (c), (f) and (i), which allow obtaining the new forecasted stock prices. The values of these forecast prices increase as each new prospective value of the corresponding stock is forecast. The new forecast prices are taken as facts, and on the basis of them the new security prices are projected.

Figure 6 shows the graphs of real prices taken from the company, the graphs of compared forecast prices and the graphs of deviations.





g h i

Figure 6. Forecast Price and Difference Charts. Source: self-made.

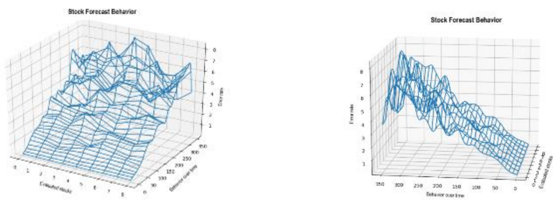
The graphs in figure 6 (a), (d) and (g) show the actual values of 350 periods of stock prices recorded after the training period, which are compared with the forecast prices, and whose comparison is shown in the graphs of figure 6 (b), (e) and (h) where the blue line are the actual values and the red line are those predicted with the methodology. As a result of the previous graph, the effective differences graph is presented in the graphs of figure (c), (f) and (i). In these last graphs it is observed how the differences between the stock prices increase as the forecast times are extended, making the real time series less similar against the forecast time series. Table 1 presents the values of the MSE of the prices of the 9 shares registered without standardizing and their SSIM.

Table 1: Difference indices between actual and forecast prices. Source: self-made.

Stock market	MSE	SSIM
Stock 0	175.8512	0.8535
Stock 1	26.9232	0.9674
Stock 2	1.0162	0.8536
Stock 3	9.0666	0.8364
Stock 4	0.0030	0.8357
Stock 5	2.7920	0.8579
Stock 6	1.2021	0.9999
Stock 7	0.0041	0.8608
Stock 8	0.0049	0.8648
Average		0.8811

In table 1 the behavior of the MSE and SSIM indicators can be observed, understanding that the MSE value depends on the value of the stock price, a good behavior can be seen, which is better visualized by the SSIM index that shows the structural similarity of the real time series against the forecast, being generally greater than 0.83.

Figure 7 shows the differences between the actual price readings against those forecast in standardized and absolute values.



a b

Figure 7. Standardized Absolute Difference Analysis. Source: self-made.

The graphs in Figure 7 (a) and (b) show the same data, but allow analyzing the behavior of the nine stocks evaluated by the forecast methodology, forecasting 350 readings in periods of 3 hours (43.75 days), showing the error percentage on the vertical axis.

A. Discussion

The results presented show a good performance of the proposed methodology, present in the SSIM index that shows the structural similarity of the time series of stock prices between the real values that occurred in times after the considered training period of the CNN-LSTM, against forecast prices. A behavior greater than 0.83 and an average of 0.8811 is presented in all stocks, considering that the greatest similarity would be 1.00.

In the graphs of Figure 7, it can be seen how, as the forecast is made over long periods of time, the error that occurs is greater, reaching 8% in some stocks. From the graph in figure 7(b) it can be seen how in short periods of time it has a good performance, being on average a difference of 2.25% in 150 readings (18.75 days).

In period 250 there is already an average difference between the stocks of 4.12%, and it reaches more than 8% at the end of the forecast period. Analyzing these results from the point of view of the company's analysts who allow us their data, it is adequate up to 150 readings or less. The forecast after about 150 readings can be considered not very accurate. After that period of time, the predicted prices are too far from the real ones, so the forecast loses value.

In general, all the results show a consistent behavior, considering that only some of the CWT coefficients were weighted as input to the neural network, it is possible to consider reducing the coefficients even more to analyze if this improves the results obtained by the methodology.

IV. Conclusions

The information resulting from the processing of the data that is presented in a time series contains hidden information that is not evident in the time domain and is not identified if it is only analyzed from its original domain. It is concluded that there is a periodicity in the signal represented in its time series, which is present in the coefficients of the Wavelet CWT transformation, and that it is not easily identified in the time series in the time domain. The periodicity can be clearly seen in the graphs of Figure 4 (b), (e) and (h), from a visual perspective, analyzing the periodicity of the values of the Wavelet coefficients present in the color tones. Since these coefficients are the input values for the analysis in the Neural Network, a better prospecting of future values is obtained.

In the particular case of the stock market time series, it is important for the company to be able to identify the trends that its stocks may have as support in the analysis in managerial decision making. This research allows us to conclude that the CWT applied in conjunction with CNN_LSTM is a powerful tool in time series processing, considering the WFBs that are generated as representative forecast elements of a time series. Advantages of the present implementation are defined by the use of the method to process the information by means of wavelet, and the way in which processing identifies the most present periodicities in time series analyzed.

In future works, it is proposed to carry out a study in relation to the interrelationship presented by the time series of prices of the same stock market section, in the domain of Wavelets, and how it affects the forecast of projected stock values.The present work has future applications in the identification of cases where the analysis of the time series, in the Wavelet domain, determines that there are events that affect other events such as industrial production, the behavior of maintenance events in machinery weather events, among others.

V. ACKNOWLEDGMENT

I am grateful to the Autonomous University of Querétaro, for its support in the current research, and to the professors who supported with their valuable contributions and reviews.

VI. REFERENCES

[1] Parmar K, Bhardwaj R. Trend Time Series and Wavelet Analysis of River Water Dynamics. Non-Linear Dynamics Lab, Department of Mathematics University School of Basic and Applied Sciences Guru Gobind Singh Indraprastha.2014;https:/doi.org/10.1007/s13762-012-0086.

[2] Zhang S, Cong X. The Application of Wavelet Analysis in Financial Multiple Change Points Time Series. 5th International Conference on Industrial Economics System and Industrial Security Engineering. 2018;https:/doi.org/10.1109/ieis.2018.8597980.

[3] Chui C. Wavelet analysis and its applications, USA. Academic Press. 1992; doi.org/10.1142/5251 | April 2003.

[4] Acuna-Garcia J, Canchola-Magdaleno S. Análisis de singularidades en líneas de producción utilizando transformada Wavelet en un sistema embebido. Revista Electrónica de Divulgación de la Investigación Vol. 18.2019.

[5] Andrews W, Ploberger W. Optimal tests when a nuisance parameter is present only under the alternative. Econometrica, vol. 62, pp. 1383-1414.1994.

[6] Zhang S, Bao G. Tian B, Li Y. Semiparametric test for multiple change-points based on empirical likelihood. Communication in Statistics-Theory and Methods, vol.46, pp. 3574– 3585. 2017;https:/doi.org/10.1080/03610926.2015.1066815.

[7] Zhang S, Tian B. Inference for random coefficient INAR(k) with the occasional level shift random noise based on empirical likelihood. Communication in Statistics-Theory and Methods, vol.46, pp. 6994–7006.2017.

[8] Zhang S, Tian B. Semiparametric method for identifying multiple change-points in financial market. Journal Communications in Statistics - Simulation and Computation, vol.4, pp. 2004–2519. 2017; https:/doi.org/10.1080/03610918.2017.1359285.

[9] Lendasse A, Oja E, Simula O, Verleysen M. Time series prediction competition: The CATS benchmark. Neurocomputing, vol. 70, no. 13-15, pp. 2325–2329. 2007; https:/doi.org/10.1016/j.neucom.2007.02.013.

[10] Dixon M. Industrial Forecasting with Exponentially Smoothed Recurrent Neural Networks. 2020;https:/doi.org/10.2139/ssrn.3572181.

[11] Dixon M, London J. Financial Forecasting with alpha-RNNs: A Time Series Modeling Approach. Frontiers in Applied Mathematics and Statistics 6. 2020; https:/doi.org/10.3389/fams.2020.551138.

[12] Dixon M, Klabjan D, Bang J. Classification-Based Financial Markets Prediction Using Deep Neural Networks. SSRN Electronic Journal. 2017; https:/doi.org/10.2139/ssrn.2756331.

[13] Cai X, Zhang N, Venayagamoorthy G. Time series prediction with recurrent neural networks traindes by hibrid pso-ea algorithm. Neurocomputing, Vol 70, no. 13-15, pp. 2342-2353. 2007;https:/doi.org/10.1016/j.neucom.2005.12.138.

[14] Beliaev I, Kozma R. Time series prediction using chaotic neural networks on the CATS Benchmark. Neurocomputing, Vol. 70, no.13-15, pp.2426-2439. 2007; https:/doi.org/10.1016/j.neucom.2006.09.013

[15] Gómez P, Ramirez M. Expriments with a hibrid-complex neural networks for long term prediction of electrocardiogram. Proceedings of the IEEE 2006 international world congress of computational intelligence. Vancouver Canada. 2006.

[16] Yu Z, Abma R, Etgen J, Sullivan C. Attenuation of noise and simultaneous source interference using wavelet denoising. Society of Exploration Geophysicists. 2017; https:/doi.org/10.1190/geo2016-0240.1.

[17] Cedeño A, Trujillo R. Esquema basado en Wavelet para la reducción de ruido online en señales industriales. Revista Cubana de Ciencias Informáticas, Vol. 8, No. 3.2014; https:/doi.org/10.13140/RG.2.2.35865.42085.

[18] Yaseen A, Pavlov A, Hramov A. Speech signal denoising with wavelet-transforms and the mean opinion score characterizing the filtering quality. Proceedings Volume 9707, Dynamics and Fluctuations in Biomedical Photonics XIII. 2016; https:/doi.org/10.1117/12.2211384.

[19] Santamaría F, Cortés C, Román F. Uso de la Transformada de Ondeletas (Wavelet Transform) en la Reducción de Ruidos en las Señales de Campo Eléctrico producidas por Rayos. Información Tecnológica Vol. 23 Nº 1. 2012; https:/doi.org/10.4067/S0718-07642012000100008.

[20] Amosov O, Amosova S, Muller N. Identification of Potential Risks to System Security Using Wavelet Analysis, the Time-and-Frequency Distribution Indicator of the Time Series and the Correlation Analysis of Wavelet-Spectra. International Multi-Conference on Industrial Engineering and Modern Technologies. 2018; https:/doi.org/10.1109/FarEastCon.2018.8602675.

[21] Lone M. Performance Analysis of DWT Families. Conference: 3rd International Conference on Intelligent Sustainable Systems (ICISS) At: Thoothukudi, India, India. 2021;https:/doi.org/10.1109/ICISS49785.2020.9315960.

[22] Larkin K. Structural Similarity Index SSIMplified. Occasional Texts in the Pursuit of Clarity and Simplicity in Research. Series 1, Number 1.2015.

[23] Gandhi S, Kulkarni C. MSE Vs SSIM. International Journal of Scientific & Engineering Research, Volume 4, Issue 7, July.2013.

