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MULTIVALUED POSITIVE BOOLEAN DEPENDENCIES BY GROUPS IN THE DATABASE MODEL OF BLOCK FORM

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Abstract -- The article proposed the concept of multivalue positive Boolean dependencies by groups in the database model of block form, proved the equivalence of three types derived: m-deduction by logic, m-deduction in groups by block, m-deduction in groups by block not exceeding p elements,, necessary and sufficient conditions for a block to be a m-tight representation by groups of a set of multivalue positive Boolean dependencies by groups on the block ... In addition, some properties multivalue positive Boolean dependencies by groups have been stated and proved here.

Keywords -- Multivalued positive Boolean dependencies by groups, block, block schemes.

I. INTRODUCTION

In recent years, research to expand the relational data model has been interested by many scientists around the world.. Following this research direction, there are some proposed database models such as: Multidimensional data model [1],[2],[3], data block [4],[5], data warehouse [6],[7],… the database model of block form [8].

In a database model of block form, the concepts: blocks, block diagrams, slices, relational algebra over blocks, functional dependencies, closures of index attribute set ... have been studied [8]. However, the study of extended logical dependencies in this data model is limited, many types of dependencies have not been studied. This article wants to propose and study the properties of a new type of logical dependency in a database model of block form: that is multivalued positive Boolean dependency by groups

II. THE DATABASE MODEL OF BLOCK FORM

II.1 The block, slice of the block

Definition II.1 [8]

Let $R = (id; A_1, A_2,..., A_n)$ is a finite set of elements, where id is non-empty finite index set, A_i (i=1..n) are attributes. Each attribute A_i (i=1..n) there is a corresponding value domain $dom(A_i)$. A block r on R, denoted r(R) consists of a finite number of elements that each element is a family of mappings
from the index set id to the value domain of the attributes A_i (i=1..n). In other words:

$$t\in r(R) \Leftrightarrow t = \{ t^i : id \to dom(A_i)\}_{i=1..n}.$$

Then, block is denoted r(R) or $r(id; A_1, A_2,..., A_n)$, if without fear of confusion we simply denoted r.

Definition II.2 [8]

Let $R = (id; A_1, A_2,..., A_n)$, r(R) is a block over R. For each $x\in id$ we denoted $r(R_x)$ is a block with $R_x = (\{x\}; A_1, A_2,..., A_n)$ such that:

$$t_x\in r(R_x) \Leftrightarrow t_x = \{t^i_x = t^i\big|_x\}_{i=1..n}, \text{ where } t\in r(R),$$

$$t = \{ t^i: id \to dom(A_i)\}_{i=1..n}.$$

Then $r(R_x)$ is called a slice of the block r(R) at point x.

II.2 Functional dependencies

Here, for simplicity we use the notation:

$x^{(i)} = (x; A_i)$; $id^{(i)} = \{x^{(i)} \mid x \in id\}$,

and $x^{(i)}$ ($x \in id$, $i = 1..n$) is called an index attribute of block scheme $R = (id; A_1,A_2,...,A_n)$.

Definition II.3 [8]

Let $R = (id; A_1,A_2,...,A_n)$, r(R) is a block over R and $X, Y \subseteq \bigcup_{i=1}^{n} id^{(i)}$, $X\to Y$ is a notation of functional dependency. A block r satisfies $X \to Y$ if $\forall\, t_1, t_2 \in r$ such that $t_1(X) = t_2(X)$ then $t_1(Y) = t_2(Y)$.

Definition II.4 [9]

Let block scheme $\alpha = (R,F)$, $R = (id; A_1, A_2,..., A_n)$, F is the set of functional dependencies over R. Then, the closure of F denoted F^+ is defined as follows:

$$F^+ = \{X \to Y \mid F \Rightarrow X \to Y\}.$$

If $X = \{x^{(m)}\} \subseteq id^{(m)}$, $Y= \{y^{(k)}\} \subseteq id^{(k)}$ then we denoted functional dependency $X \to Y$ is simply $x^{(m)} \to y^{(k)}$.

The block r satisfies $x^{(m)}\to y^{(k)}$ if $\forall\, t_1, t_2 \in r$ such that $t_1(x^{(m)}) = t_2(x^{(m)})$ then $t_1(y^{(k)}) = t_2(y^{(k)})$.

where: $t_1(x^{(m)}) = t_1(x; A_m)$, $t_2(x^{(m)}) = t_2(x; A_m)$,

$t_1(y^{(k)}) = t_1(y; A_k)$, $t_2(y^{(k)}) = t_2(y; A_k)$.

Henceforth, for convenience, we used notation for subsets of functional dependencies on R:

$F_h =\{ X\to Y \mid X = \bigcup_{i\in A} x^{(i)}, Y = \bigcup_{j\in B} x^{(j)}$, $A, B \subseteq \{1,2,...,n\}$, $x\in id \}$,

$$F_{hx} = F_h \Big|_{\bigcup_{i=1}^{n} x^{(i)}} = \{X \to Y\in F_h \mid X, Y \subseteq \bigcup_{i=1}^{n} x^{(i)}\}.$$

Definition II.5 [9]

Let block scheme $\alpha=(R,F_h)$, $R=(id; A_1, A_2,..., A_n)$, then F_h is called the complete set of functional depen-dencies if: $F_{hx} = F_h \left| \bigcup_{i=1}^{n} x^{(i)} \right.$ is the same with every $x \in$ id.

A more specific way:

F_{hx} is the same with every $x \in$ id mean:

$\forall$ x, y$\in$ id: $M \rightarrow N \in F_{hx} \Leftrightarrow M' \rightarrow N' \in F_{hy}$, with M', N' respectively, formed from M, N by replacing x by y.

II.3 Closure of the index attributes sets

Definition II.6 [10]

Let block scheme $\alpha=(R,F)$, $R=(id; A_1, A_2,..., A_n)$, F is the set of functional dependencies on R. With each $X \subseteq \bigcup_{i=1}^{n} id^{(i)}$, we define closure of X for F denoted X^+ as follows:

$$X^+ = \{ x^{(i)}, x \in id, i = 1..n \mid X \rightarrow x^{(i)} \in F^+\}.$$

III. MULTIVALUED BOOLEAN FORMULARS

III.1 The operations and multivalued logical function

Definition III.1 [11]

For the set of Boolean values $B = \{b_1,b_2, ...,b_k\}$ including k values in [0;1], $k \geq 2$ are in ascending order and satisfy the following conditions:

(i) $0 \in B$.

(ii) $\forall b \in B \Rightarrow 1- b \in B$.

We choose the operations and basic multivalued logical function:

$\forall a, b \in B$

- $a \wedge b = \min(a, b)$,
- $a \vee b = \max(a, b)$,
- $\neg a = 1-a$
- $\forall b \in B$ we define the function I_b:

$\forall x \in B$: $I_b(x) = 1$ if $x = b$ and $I_b(x) = 0$ if $x \neq b$.

The functions I_b, $b \in B$ called generalized negative functions.

Definition III.2 [11]

Let $P = \{x_1, x_2, ..., x_n\}$ is a finite set of Boolean variables, B is the set of Boolean values. Then the multivalued boolean formulas (CTBĐT) also known as multivalued logic formulas are constructed as follows:

(i) Each value in B is a CTBĐT.

(ii) Each variable in P is a CTBĐT.

(iii) Each function I_b, $b\in B$ is a CTBĐT.

(iv) If a is a multivalued Boolean formula then (a) is a CTBĐT.

(v) If a and b are CTBĐT then $a\wedge b$, $a\vee b$ and $\neg a$ are CTBĐT.

(vi) Only formulas created by rules from (i) – (v) are CTBĐT.

We denoted MVL(P) as a set of CTBĐT building on the set of variables $P = \{x_1, x_2, ..., x_n\}$ and set of values $B = \{b_1, b_2,..., b_k\}$ including k values in [0;1], $k \geq 2$.

Definition III.3 [11]

We define $a\rightarrow b$ equivalent to CTBĐT $(\neg a)\vee b$ and then: $a\rightarrow b = \max(1- a, b)$.

Definition III.4 [11]

Each vector of elements $v = \{v_1, v_2, ..., v_n\}$ in space $B^n = B \times B \times ... \times B$ is called a value assignment. Thus, with each CTBĐT $f \in MVL(P)$ we have $f(v) = f(v_1, v_2, ..., v_n)$ is the value of formula f for v value assignments.

We understand the symbol $X \subseteq P$ at the same time performing for the following subjects:

- An attribute set in P.
- A set of logical variables in P.
- A multivalued Boolean formula is the logical union of variables in X.

On the other hand, if $X = \{B_1, B_2, ..., B_n\} \subseteq P$, we denoted:

$\wedge X = B_1\wedge B_2\wedge ... \wedge B_n$ called the associational form.

$\vee X = B_1\vee B_2\vee ... \vee B_n$ called the recruitmental form.

For each finite set CTBĐT $F = \{f_1, f_2, ..., f_m\}$ in MVL(P), we consider F as a formatted formula $F = f_1 \wedge f_2 \wedge ... \wedge f_m$. Then we have:

$$F(v) = f_1(v) \wedge f_2(v)\wedge ... \wedge f_m(v).$$

III.2 Table of values and truth tables

With each formula f on P, table of values for f, denote that V_f contains n+1 columns, with the first n columns containing the values of the variables in U, and the last column contains the value of f for each values signment of the corresponding row. Thus, the value table contains k^n row, n is the element number of P, k is the element number of B.

Definition III.5 [11]

Let $m \in [0;1]$, truth table with m threshold of f or the m-truth table of f, denoted $T_{f,m}$ is the set of assignments v such that f(v) receive value not less than m: $T_{f,m} = \{v \in B^n \mid f(v) \geq m\}$

Then, the m-truth table $T_{F,m}$ of finite sets of formulas F on P, is the intersection of the m-truth tables of each member formula in F.

$$T_{F,m} = \bigcap_{f\in F} T_{f,m}.$$

We have: $v\in T_{F,m}$ necessary and sufficient are $\forall f\in F$: $f(v) \geq m$.

III.3 Logical deduction

Definition III.6 [11]

Let f, g are two CTBĐT and value $m\in B$. We say formula f derives formula g from threshold m and denoted $f \models_m g$ if $T_{f,m} \subseteq T_{g,m}$. We say f and g are two m-equivalent formulas, denoted $f \equiv_m g$ if $T_{f,m} = T_{g,m}$.

With F, G in MVL(P) and value $m\in[0;1]$, we say F derives G from threshold m, denoted $F \models_m G$, if $T_{F,m} \subseteq T_{G,m}$.

Moreover, we say F and G are m-equivalents, denoted $F \equiv_m G$ if $T_{F,m} = T_{G,m}$.

III.4 Multivalued positive Boolean formula

Definition III.7 [11]

Formula $f\in MVL(P)$ is called a multivalued positive Boolean formula (CTBDĐT) if f(e) = 1 with e is the unit value assignment: e = (1, 1, ..., 1), we denoted MVP(P) is the set of all multivalued positive Boolean formulas on P.

IV. RESEARCH RESULTS

IV. The multivalued truth block by groups of the data block

Definition IV.1

Let R = (id; A_1, A_2,..., A_n), r(R) is a block over R, we convention that each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, x∈id), 1≤ i ≤ n, contains at least p (p≥2) elements. Then, with each value domain d_i, we consider the mapping $\beta_i:(d_i)^p \rightarrow$ B, satisfies the following properties:

(i) Reflectivity: $\forall a \in (d_i)^p$: $\beta_i(a)$= 1, if in a contains at least two identical components.

(ii) Commutation: $\forall a \in (d_i)^p$: $\beta_i(a) = \beta_i(a')$, where a' is permutation of a.

(iii) Sufficiency: $\forall m \in$ B, $\exists a \in (d_i)^p$: $\beta_i(a) = m$.

Thus, we see the mapping β_i is an evaluation on a group containing p (p≥2) values of d_i satisfying reflection and commutative properties. Equality relation is a separate case of this relation.

Example IV.1

Let R = ({1, 2}, A_1, A_2); then the index attribute of R are U = {$1^{(1)}$, $1^{(2)}$, $2^{(1)}$, $2^{(2)}$}, with:

A_1: Weight of the ball (C: high, K: quite high, M: average, S: low),

A_2: Color of the ball (Đ: red, V: yellow, X: blue, N: brown).

r is a block over R, includes 4 elements: t_1, t_2, t_3, t_4 as follows:

$t_1.1^{(1)}$ = C, $t_1.1^{(2)}$ = Đ, $t_1.2^{(1)}$ = C, $t_1.2^{(2)}$ = Đ.
$t_2.1^{(1)}$ = M, $t_2.1^{(2)}$ = V, $t_2.2^{(1)}$ = M, $t_2.2^{(2)}$ = V.
$t_3.1^{(1)}$ = S, $t_3.1^{(2)}$ = X, $t_3.2^{(1)}$ = S, $t_3.2^{(2)}$ = X.
$t_4.1^{(1)}$ = K, $t_4.1^{(2)}$ = N, $t_4.2^{(1)}$ = K, $t_4.2^{(2)}$ = N.

With p = 3, corresponding to each group has 3 balls, then:

We consider the mapping β_i: $(d_i)^3 \rightarrow \{0, 0.5, 1\}$, d_i: is the value domain of the attribute A_i , i=1..2;

$\forall a \in (d_1)^3$, we assign $\beta_1(a)$=1 if in a we have at least 2 balls of the same weight, $\beta_1(a)$ = 0.5 if in a we have 3 balls with different weights for each pair and 1 ball with high weight, the remaining cases we have $\beta_1(a)$ = 0.

$\forall a \in (d_2)^3$, we assign $\beta_2(a)$=1 if in a we have at least 2 balls of the same color, $\beta_2(a)$ = 0.5 if in a we have 3 balls with different colors for each pair and 1 ball with red color, the remaining cases we have $\beta_2(a)$ = 0.

Then we have:

- With a = ($t_1.1^{(1)}$, $t_2.1^{(1)}$, $t_3.1^{(1)}$), then $\beta_1(a)$= β_1(C, M, S) =0.5;
- With a = ($t_2.1^{(2)}$, $t_3.1^{(2)}$, $t_4.1^{(2)}$), then $\beta_2(a)$= β_2(V, X, N) = 0;
- With a = ($t_1.2^{(2)}$, $t_1.2^{(2)}$, $t_1.2^{(2)}$), then $\beta_2(a)$= β_2(Đ, Đ, Đ)= 1;
- With a = ($t_1.2^{(1)}$, $t_2.2^{(1)}$, $t_4.2^{(1)}$), then $\beta_1(a)$= β_1(C, M, K)= 0.5; ....

Definition IV.2

Let R = (id; A_1,A_2,...,A_n), r(R) is a block over R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, x∈id,1≤ i ≤ n), contains at least p elements, β_i is an evaluation on groups containing p (p≥2) values of $x^{(i)}$, x∈id, 1≤ i ≤ n. For each group of p elements: u_1, u_2, ..., u_p arbitrary (not necessarily distinguish) on the block, we call $\beta(u_1, u_2, ..., u_p)$ is the value assignment:

$\beta(u_1, u_2, ..., u_p) = (t_{x1}, t_{x2}, ..., t_{xn})$ with $t_{xi} = \beta_i(u_1.x^{(i)}, u_2.x^{(i)}, ..., u_p.x^{(i)})$, x∈id, 1≤ i ≤n. Then, for each block r we denote the multivalued truth block by groups of block r as T^p_r:

$$T^p_r = \{ \beta(u_1, u_2, ..., u_p) \mid u_j \in r, 1\le j \le p \}.$$

Example IV.2:

With the given block in the example IV.1, r is a block of 4 elements: t_1, t_2, t_3, t_4, as follows:

$t_1.1^{(1)}$ = C, $t_1.1^{(2)}$ = Đ, $t_1.2^{(1)}$ = C, $t_1.2^{(2)}$ = Đ.
$t_2.1^{(1)}$ = M, $t_2.1^{(2)}$ = V, $t_2.2^{(1)}$ = M, $t_2.2^{(2)}$ = V.
$t_3.1^{(1)}$ = S, $t_3.1^{(2)}$ = X, $t_3.2^{(1)}$ = S, $t_3.2^{(2)}$ = X.
$t_4.1^{(1)}$ = K, $t_4.1^{(2)}$ = N, $t_4.2^{(1)}$ = K, $t_4.2^{(2)}$ = N.

with defined functions β_i: $(d_i)^3 \rightarrow \{0, 0.5, 1\}$, i=1..2. Then we have the elements a_1, a_2, a_3, a_4, a_5,... of the truth block T^p_r as follows:

- With a_1 = (t_1, t_2, t_3), then: $a_1.1^{(1)} = \beta_1(t_1.1^{(1)}, t_2.1^{(1)}, t_3.1^{(1)})$ = β_1(C, M, S) =0.5;
$a_1.1^{(2)} = \beta_2(t_1.1^{(2)}, t_2.1^{(2)}, t_3.1^{(2)})$ = β_1(Đ, V, X) =0.5;
$a_1.2^{(1)} = \beta_1(t_1.2^{(1)}, t_2.2^{(1)}, t_3.2^{(1)})$ = β_1(C, M, S) =0.5;
$a_1.2^{(2)} = \beta_2(t_1.2^{(2)}, t_2.2^{(2)}, t_3.2^{(2)})$ = β_1(Đ, V, X) =0.5;
$a_1 = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$.

- With a_2 = (t_1, t_2, t_4), then: $a_2.1^{(1)} = \beta_1(t_1.1^{(1)}, t_2.1^{(1)}, t_4.1^{(1)})$ = β_1(C, M, K) =0,5;
$a_2.1^{(2)} = \beta_2(t_1.1^{(2)}, t_2.1^{(2)}, t_4.1^{(2)})$ = β_1(Đ, V, N) =0,5;
$a_2.2^{(1)} = \beta_1(t_1.2^{(1)}, t_2.2^{(1)}, t_4.2^{(1)})$ = β_1(C, M, K) =0,5;
$a_2.2^{(2)} = \beta_2(t_1.2^{(2)}, t_2.2^{(2)}, t_4.2^{(2)})$ = β_1(Đ, V, N) =0,5;
$a_2 = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$.

- With a_3 = (t_1, t_3, t_4), then: $a_3.1^{(1)} = \beta_1(t_1.1^{(1)}, t_3.1^{(1)}, t_4.1^{(1)})$ = β_1(C, S, K) =0,5;
$a_3.1^{(2)} = \beta_2(t_1.1^{(2)}, t_3.1^{(2)}, t_4.1^{(2)})$ = β_1(Đ, X, N) =0.5;
$a_3.2^{(1)} = \beta_1(t_1.2^{(1)}, t_3.2^{(1)}, t_4.2^{(1)})$ = β_1(C, S, C) =0,5;
$a_3.2^{(2)} = \beta_2(t_1.2^{(2)}, t_3.2^{(2)}, t_4.2^{(2)})$ = β_1(Đ, X, N) =0.5;
$a_3 = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$.

- With a_4 = (t_2, t_3, t_4), then: $a_4.1^{(1)} = \beta_1(t_2.1^{(1)}, t_3.1^{(1)}, t_4.1^{(1)})$ = β_1(M, S, K) =0;
$a_4.1^{(2)} = \beta_2(t_2.1^{(2)}, t_3.1^{(2)}, t_4.1^{(2)})$ = β_1(V, X, N) =0;
$a_4.2^{(1)} = \beta_1(t_2.2^{(1)}, t_3.2^{(1)}, t_4.2^{(1)})$ = β_1(M, S, K) =0;
$a_4.2^{(2)} = \beta_2(t_2.2^{(2)}, t_3.2^{(2)}, t_4.2^{(2)})$ = β_1(V, X, N) =0;
$a_4 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

- With a_5 = (t_1, t_1, t_1), then: $a_5.1^{(1)} = \beta_1(t_1.1^{(1)}, t_1.1^{(1)}, t_1.1^{(1)})$ = β_1(C, C, C) =1;
$a_5.1^{(2)} = \beta_2(t_1.1^{(2)}, t_1.1^{(2)}, t_1.1^{(2)})$ = β_1(Đ, Đ, Đ) =1;
$a_5.2^{(1)} = \beta_1(t_1.2^{(1)}, t_1.2^{(1)}, t_1.2^{(1)})$ = β_1(C, C, C) =1;
$a_5.2^{(2)} = \beta_2(t_1.2^{(2)}, t_1.2^{(2)}, t_1.2^{(2)})$ = β_1(Đ, Đ, Đ) =1;
$a_5 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ ....

In the case id ={x}, then the block degenerates into a relation and the concept of the multivalued truth block by groups of the block becomes the concept of multivalued truth table by groups of relation in the relational data model. In other words, the multivalued truth block by groups of a block is to expand the concept of the multivalued truth table by groups of relation in the relational data model.

IV.2 The multivalued positive Boolean dependencies by groups of a data block

Definition IV.3

Let R = (id; A_1,A_2,...,A_n), r(R) is a block over R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, x∈id,1≤ i ≤ n), contains at least p (p≥2) elements, β_i is an evaluation on groups containing p (p≥2) values of d_i. With evaluations β_i on the value domain of the index attribute $x^{(i)}$, x∈id, 1≤ i ≤n, then a multivalued positive Boolean dependency by groups is a multivalued positive Boolean formula in

MVP(U) with $U = \bigcup_{i=1}^{n} id^{(i)}$.

Let value $m \in B$, we say block r is m-satisfying by groups the multivalued positive Boolean dependency by groups (PTBDĐTTNB) f and denoted $r^p(f,m)$ if $T^p_r \subseteq T_{f,m}$.

The block r is m-satisfying by groups set PTBDĐTTNB F and denoted $r^p(F,m)$ if r is m-satisfying by groups all f in F:

$$r^p(F,m) \Leftrightarrow \forall f \in F: r^p(f,m) \Leftrightarrow T^p_r \subseteq T_{F,m}.$$

If $r^p(F,m)$ then we say set PTBDĐTTNB F is m-right by groups in the block r.

Proposition IV.1

Let $R = (id; A_1,A_2,...,A_n)$, r(R) is a block on R, $U = \bigcup_{i=1}^{n} id^{(i)}$. Then:

i) If r is m-satisfying by groups the multivalued positive Boolean dependency by groups f: $r^p(f,m)$ then $r^p_x(f_x,m)$, $\forall x \in id$.

ii) If r is m-satisfying by groups set of multivalued positive Boolean dependency by groups F: $r^p(F,m)$ then $r^p_x(F_x,m)$, $\forall x \in id$.

Proof

i) Under the assumption we have $r^p(f,m) \Rightarrow T^p_r \subseteq T_{f,m} \Rightarrow T^p_{rx} = (T^p_r)_x \subseteq (T_{f,m})_x = T_{fx,m}$, $\forall x \in id$

So we have $T^p_{rx} \subseteq T_{fx,m}$, $\forall x \in id \Rightarrow r^p_x(f_x,m)$, $\forall x \in id$.

ii) Under the assumption $r^p(F,m) \Rightarrow T^p_r \subseteq T_{F,m} \Rightarrow T^p_{rx} = (T^p_r)_x \subseteq (T_{F,m})_x = T_{Fx,m}$, $\forall x \in id$

Therefore: $T^p_{rx} \subseteq T_{Fx,m}, \forall x \in id \Rightarrow r^p_x(F_x,m)$, $\forall x \in id$.

Proposition IV.2

Let $R = (id; A_1,A_2,...,A_n)$, r(R) is a block on R, $U = \bigcup_{i=1}^{n} id^{(i)}$, $f = \bigcup_{x \in id} f_x$. Then:

i) If $r^p_x(f_x,m), \forall x \in id$ then r is m-satisfying by groups the multivalued positive Boolean dependency by groups f: $r^p(f,m)$.

ii) If $r^p_x(F_x,m), \forall x \in id$ then r m-satisfying by groups set of multivalued positive Boolean dependency by groups F: $r^p(F,m)$.

Proof

i) Under the assumption we have: $r^p_x(f_x,m)$, $\forall x \in id \Rightarrow T^p_{rx} \subseteq T_{fx,m}$, $\forall x \in id \Rightarrow (T^p_r)_x \subseteq (T_{f,m})_x$, $\forall x \in id$.

So we have: $T^p_r \subseteq T_{f,m} \Rightarrow r^p(f,m)$.

$\Rightarrow$ r is m-satisfying by groups the multivalued positive Boolean dependency by groups f.

ii) Under the assumption $r^p_x(F_x,m)$, $\forall x \in id \Rightarrow T^p_{rx} \subseteq T_{Fx,m}$, $\forall x \in id \Rightarrow (T^p_r)_x \subseteq (T_{F,m})_x$, $\forall x \in id$

So we have: $T^p_r \subseteq T_{F,m} \Rightarrow r^p(F,m)$.

$\Rightarrow$ r m-satisfying by groups set of multivalued positive Boolean dependency by groups F.

From the proposition IV.1 and IV.2 we have the following necessary and sufficient theorem:

Theorem IV.1

Let $R = (id; A_1,A_2,...,A_n)$, r(R) is a block on R, $U = \bigcup_{i=1}^{n} id^{(i)}$, $f = \bigcup_{x \in id} f_x$. Khi đó:

i) $r^p_x(f_x,m), \forall x \in id \Leftrightarrow$ r is m-satisfying by groups the multivalued positive Boolean dependency by groups f: $r^p(f,m)$.

ii) $r^p_x(F_x,m), \forall x \in id \Leftrightarrow$ r m-satisfying by groups set of multivalued positive Boolean dependency by groups F: $r^p(F,m)$.

Let set PTBDĐTTNB F and PTBDĐTTNB f:

- We have F is m-deduced f by block with groups and denoted $F |\!-^p_m f$ if: $\forall r: r^p(F,m) \Rightarrow r^p(f,m)$.

- We have F is m-deduced f by block with groups, block contains no more than p elements and denoted $F |\!-^p_{p,m} f$ if $\forall r_p: r^p_p(F,m) \Rightarrow r^p_p(f,m)$.

We have the following equivalent theorem:

Theorem IV.2

Let $R = (id; A_1,A_2,...,A_n)$, r(R) is a block over R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, $x \in id, 1 \le i \le n$), contains at least p ($p \ge 2$) elements, β_i are evaluations on groups containing p value of the index attribute $x^{(i)}$, $x \in id$, $1 \le i \le n$, set PTBDĐTTNB F and PTBDĐTTNB f. Then the following three propositions are equivalent:

(i). $F \models_m f$ (m-deduction by logic),

(ii). $F \vdash^p_m f$ (m-deduction in groups by block),

(iii). $F \vdash^p_{p,m} f$ (m-deduction in groups by block has no more than p elements).

Proof

(i) $\Rightarrow$ (ii): We need proof: $F \models_m f \Rightarrow F \vdash^p_m f$.

Indeed, under the assumption we have $F \models_m f \Rightarrow T_{F,m} \subseteq T_{f,m}$. (1)

Let r be an arbitrary block m-satisfying by groups F: $r^p(F,m)$, then by definition: $T^p_r \subseteq T_{F,m}$. (2)

From (1) and (2) we infer: $T^p_r \subseteq T_{f,m} \Rightarrow r^p(f,m)$.

So that: $r^p(F,m) \Rightarrow r^p(f,m)$ mean: $F \vdash^p_m f$.

Since then we have: $F \models_m f \Rightarrow F \vdash^p_m f$.

(ii) $\Rightarrow$ (iii): We need proof: $F \vdash^p_m f \Rightarrow F \vdash^p_{p,m} f$.

Obviously, because inference by the block has no more than p elements is the special case of inference by block.

(iii) $\Rightarrow$ (i): We need proof: $F \vdash^p_{p,m} f \Rightarrow F \models_m f$.

Indeed, under the assumption $F \vdash^p_{p,m} f$, then every block there is no more than p elements we have: $r^p_p(F,m) \Rightarrow r^p_p(f,m)$, We need proof $F \models_m f$ mean $T_{F,m} \subseteq T_{f,m}$.

Suppose $t = (t_{x1}, t_{x2}, ..., t_{xn})_{x \in id}$, $t \in T_{F,m}$, we proof $t \in T_{f,m}$.

If t = e then we have $t \in T_{f,m}$ because as we know f is a multivalued positive Boolean formula.

If $t \neq e$, we build the block r including p elements as follows:

From the properties of the mapping $\beta_i: (d_i)^p \to B$ with each index attribute $x^{(i)}$, $x \in id$, $1 \le i \le n$ we have:

$\exists a_{xi} \in (d_i)^p$: $a_{xi} = (a_{xi1}, a_{xi2}, ..., a_{xip})$ such that the $\beta_i(a_{xi}) = t_{xi}$.

Then, with each index attribute $x^{(i)}$ in $U = \bigcup_{i=1}^{n} id^{(i)}$,

We fill in the column of this index attribute of block r values $a_{xi1}, a_{xi2}, ..., a_{xip}$.

According to the way of building block r, we have: $T^p_r = \{e, t\} \subseteq T_{F,m}$ with e is the unit value assignment. Thus r is a block with p elements and m-satisfying by groups set PTBDĐTTNB F.

Under the assumption if r is m-satisfying by groups F then r will m-satisfy by groups f, this means: $T^p_r = \{e, t\} \subseteq T_{f,m}$, infer: $t \in T_{f,m}$.

Consequence IV.1

Let $R = (id; A_1,A_2,...,A_n)$, r(R) is a block over R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$,

$x\in id, 1\le i\le n$), contains at least p ($p\ge 2$) elements, β_i are evaluations on groups containing p value of the index attribute $x^{(i)}$, $x\in id$, $1\le i\le n$, set PTBDĐTTNB F and PTBDĐTTNB f. Then on r_x the following three propositions are equivalent:

(i) $F_x \models_m f_x$ (m-deduction by logic),

(ii) $F_x \vdash^p_m f_x$ (m-deduction in groups by slice r_x),

(iii) $F_x \vdash^p_{p,m} f_x$ (m-deduction in groups by slice r_{px} have no more than p elements).

In the case of index set id = {x}, then the block r degenerates into a relation and the above equivalence theorem becomes the equivalent theorem in the relational data model. Specifically, we have the following consequences:

Consequence IV.2

Let R = (id; $A_1,A_2,...,A_n$), r(R) is a block over R, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, $x\in id, 1\le i\le n$), contains at least p ($p\ge 2$) elements, β_i are evaluations on groups containing p value of the index attribute $x^{(i)}$, $x\in id$, $1\le i\le n$, set PTBDĐTTNB F and PTBDĐTTNB f. Then, if id = {x} then the block r degenerates into a relation and in the relational data model the following three propositions are equivalent:

(i) $F \models_m f$ (m-deduction by logic),

(ii) $F \vdash^p_m f$ (m-deduction in groups by relation),

(iii) $F \vdash^p_{p,m} f$ (m-deduction in groups by relation has no more than p elements).

Definition IV.4

Cho R = (id; $A_1,A_2,...,A_n$), r(R) is a block over R, U = $\bigcup_{i=1}^{n} id^{(i)}$, m $\in$ B, , each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, $x\in id, 1\le i\le n$), contains at least p ($p\ge 2$) elements, β_i are evaluations on groups containing p value of the index attribute $x^{(i)}$, $x\in id$, $1\le i\le n$. With Σ is the subset PTBDĐTTNB on U, we denote $(\Sigma,m)^+$ is the set of all PTBDĐTTNB m-deduced from Σ, in other words:

$(\Sigma,m)^+ = \{ f \mid f\in MVP(U), \Sigma \models_m f \} = \{ f \mid f\in MVP(U), T_{\Sigma,m} \subseteq T_{f,m} \}$.

Definition IV.5

Let R = (id; $A_1,A_2,...,A_n$), r(R) is a block over R, U = $\bigcup_{i=1}^{n} id^{(i)}$, m $\in$ B, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, $x\in id, 1\le i\le n$), contains at least p ($p\ge 2$) elements, β_i are evaluations on groups containing p value of the index attribute $x^{(i)}$, $x\in id$, $1\le i\le n$. Then, we denoted NMBD(r,m) is the set of all PTBDĐTTNB m-right by groups in block r, means:

$$NMBD(r,m) = \{f \mid f\in MVP(U), r^p(f,m)\}.$$

Theorem IV.3

Let R = (id; $A_1,A_2,...,A_n$), r(R) is a block over R, U= $\bigcup_{i=1}^{n} id^{(i)}$, m $\in$ B, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, $x\in id, 1\le i\le n$), contains at least p ($p\ge 2$) elements, β_i are evaluations on groups containing p value of the index attribute $x^{(i)}$, $x\in id$, $1\le i\le n$. Then we have:

$$(NMBD(r,m),m)^+ = NMBD(r,m).$$

Proof

By definition, we have:

$(NMBD(r,m),m)^+ = \{ f \mid f\in MVP(P),\ NMBD(r,m,) \models_m f \} = \{ f \mid f\in MVP(U), T_{(NMBD(r,m),m)} \subseteq T_{f,m} \}$.

We infer: $(NMBD(r,m),m)^+ \supseteq NMBD(r,m)$ (3)

On the other hand, suppose we have: $g \in (NMBD(r,m),m)^+$, We need proof $g \in NMBD(r,m)$.

Indeed, the hypothesis:

$g \in (NMBD(r,m),m)^+ = \{ f \mid f\in MVP(U), T_{(NMBD(r,m),m)} \subseteq T_{f,m} \} \Rightarrow g\in MVP(U), T_{(NMBD(r,m),m)} \subseteq T_{g,m}$.

Which by definition of $NMBD_{(r,m)}$ we have:

$T^p_r \subseteq T_{(NMBD(r,m),m)} \Rightarrow T^p_r \subseteq T_{g,m} \Rightarrow$ block r is m-satisfying by groups PTBDĐTTNB g.

From there we have: $g \in NMBD(r,m)$.

$\Rightarrow (NMBD(r,m),m)^+ \subseteq NMBD(r,m)$ (4)

From (3) and (4) we have:

$$(NMBD(r,m),m)^+ = NMBD(r,m).$$

Consequence IV.3

Let R = (id; $A_1,A_2,...,A_n$), r(R) is a block over R, U = $\bigcup_{i=1}^{n} id^{(i)}$, m$\in$ B, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, $x\in id, 1\le i\le n$), contains at least p ($p\ge 2$) elements, β_i are evaluations on groups containing p value of the index attribute $x^{(i)}$, $x\in id, 1\le i\le n$. Then on r_x we have:

$$(NMBD(r_x,m),m)^+ = NMBD(r_x,m).$$

Consequence IV.4

Let R = (id; $A_1,A_2,...,A_n$), r(R) is a block over R, U = $\bigcup_{i=1}^{n} id^{(i)}$, m$\in$ B, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, $x\in id, 1\le i\le n$), contains at least p ($p\ge 2$) elements, β_i are evaluations on groups containing p value of the index attribute $x^{(i)}$, $x\in id, 1\le i\le n$. Then we have: if id = {x} then block r degenerates into relation and in the relational data model: $(NMBD(r,m),m)^+ = NMBD(r,m)$.

Theorem IV.4

Let R = (id; $A_1,A_2,...,A_n$), r(R) is a block over R, U = $\bigcup_{i=1}^{n} id^{(i)}$, m$\in$ B, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, $x\in id, 1\le i\le n$), contains at least p($p\ge 2$) elements, β_i are evaluations on groups containing p value of the index attribute $x^{(i)}$, $x\in id$, $1\le i\le n$. Then we have:

$$T^p_r = T_{(NMBD(r,m),m)}.$$

Proof

According to the definition of the set PTBDĐTTNB NMBD(r,m) we have: if $f \in NMBD(r,m) \Rightarrow$ block r is m-satisfying by groups PTBDĐTTNB f $\Rightarrow T^p r \subseteq T_{f,m}$.

From the properties of the relationship between Boolean formulas and truth blocks, with truth block T^p_r we have found a multivalued Boolean formula h so that: $T_{h,m} = T^p_r$.

On the other hand, because $e \in T^p_r = T_{h,m}$ so h is a multivalued positive Boolean formula.

From the equality: $T^p_r = T_{h,m}$ We deduce that block r is m-satisfying by groups PTBDĐTTNB h, means:

$h \in NMBD(r,m)$.

So infer: $NMBD(r,m) \models_m h$. Hence we have:

$T_{(NMBD(r,m),m)} \subseteq T_{h,m} = T^p_r \Rightarrow T_{(NMBD(r,m),m)} \subseteq T^p_r$ (5)

From the definition of NMBD(r,m) we have:

$T^p_r \subseteq T_{(NMBD(r,m),m)}$ (6)

From (5) and (6) we infer: $T^p_r = T_{(NMBD(r,m),m)}$.

Consequence IV.5

Let R = (id; $A_1,A_2,...,A_n$), r(R) is a block over R, U= $\bigcup_{i=1}^{n} id^{(i)}$, m$\in$ B, each value domain d_i of attribute A_i

(is also of index attribute $x^{(i)}$, $x\in$ id,$1\le i\le n$), contains at least p ($p\ge 2$) elements, β_i β_i are evaluations on groups containing p value of the index attribute $x^{(i)}$, $x\in id$, $1\le i\le n$. Then we have: if id = {x} then block r degenerates into relation and in the relational data model:

$$T^p_r = T_{(NMBD(r,m),m)}.$$

Definition IV.6

Let R = (id; $A_1,A_2,...,A_n$), r(R) is a block over R, U $=\bigcup_{i=1}^{n} id^{(i)}$, $m\in B$, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, $x\in id,1\le i\le n$), contains at least p ($p\ge 2$) elements, β_i are evaluations on groups containing p value of the index attribute $x^{(i)}$, $x\in id$, $1\le i\le n$. We say block r is m-representation by groups set PTBDĐTTNB Σ nếu NMBD(r,m) $\supseteq (\Sigma,m)^+$ and block r is m-tight representation by groups set PTBDĐTTNB Σ if NMBD(r,m) = $(\Sigma,m)^+$.

If r is m-tight representation by groups set PTBDĐTTNB Σ then we say r is the block m-Armstrong by groups of set PTBDĐTTNB Σ.

Theorem IV.5

Let R = (id; $A_1,A_2,...,A_n$), r(R) is a block over R, U$=\bigcup_{i=1}^{n} id^{(i)}$, $m\in B$, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, $x\in id,1\le i\le n$), contains at least p ($p\ge 2$) elements, β_i are evaluations on groups containing p value of the index attribute $x^{(i)}$, $x\in id$, $1\le i\le n$. Then r is m-tight representation by groups set PTBDĐTTNB Σ if and only if $T^p_r = T_{\Sigma,m}$.

Proof

Use the results of the theorem IV.3 and IV.4 for PTBDĐTTNB we have:

$$(NMBD(r,m),m)^+ = NMBD(r,m)$$

and $T^p_r = T_{(NMBD(r,m),m)}$. Then:

Block r is m-tight representation by groups set PTBDĐTTNB Σ if and only if: NMBD(r,m) = $(\Sigma,m)^+ \Leftrightarrow$ NMBD(r,m) $\equiv_m \Sigma \Leftrightarrow T_{(NMBD(r,m),m)} = T_{\Sigma,m} \Leftrightarrow T^p_r = T_{\Sigma,m}$.

So that, block r is m-tight representation by groups set PTBDĐTTNB $\Sigma \Leftrightarrow T^p_r = T_{\Sigma,m}$.

Consequence IV.6

Let R = (id; $A_1,A_2,...,A_n$), r(R) is a block over R, U$=\bigcup_{i=1}^{n} id^{(i)}$, $m\in B$, each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, $x\in id,1\le i\le n$), contains at least p ($p\ge 2$) elements, β_i are evaluations on groups containing p value of the index attribute $x^{(i)}$, $x\in id$, $1\le i\le n$. Then we have, if id = {x} then block r degenerates into relation and in the relational data model: r is m-tight representation by groups set PTBDĐTTNB Σ if and only if $T^p_r = T_{\Sigma,m}$.

Here we denoted: $\Sigma_x = \Sigma \Big|_{\bigcup_{i=1}^{n} x^{(i)}}$.

Consequence IV.7

Let R = (id; $A_1,A_2,...,A_n$), r(R) is a block over R, U= $\bigcup_{i=1}^{n} id^{(i)}$, $m\in B$,Σ is set PTBDĐTTNB on U, $\Sigma = \bigcup_{x\in id}\Sigma_x$, $\Sigma_x \neq\emptyset$. Each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, $x\in id,1\le i\le n$), contains at least p ($p\ge 2$) elements, β_i are evaluations on groups containing p value of the index attribute $x^{(i)}$, $x\in id$, $1\le i\le n$. Then r_x is m-tight representation by groups set PTBDĐTTNB Σ_x if and only if $T^p_{rx} = T_{\Sigma x,m}, \forall x\in id$

Theorem IV.6

Let R = (id; $A_1,A_2,...,A_n$), r(R) is a block over R, U$=\bigcup_{i=1}^{n} id^{(i)}$, $m\in B$, Σ is set PTBDĐTTNB on U, $\Sigma = \bigcup_{x\in id}\Sigma_x$ $\Sigma_x \neq\emptyset$. Each value domain d_i of attribute A_i (is also of index attribute $x^{(i)}$, $x\in id,1\le i\le n$), contains at least p ($p\ge 2$) elements, β_i are evaluations on groups containing p value of the index attribute $x^{(i)}$, $x\in id$, $1\le i\le n$. Then, with every block r(R) is otherwise empty on R we have: r is m-tight representation by groups set PTBDĐTTNB Σ if and only if r_x is m-tight representation by groups set Σ_x, $\forall x\in id$.

Proof

$\Rightarrow$) Suppose r is m-tight representation by groups set PTBDĐTTNB Σ we need proof r_x is m-tight representation by groups set Σ_x, $\forall x\in id$.

Indeed, under the assumption we have: r is m-tight representation by groups set PTBDĐTTNB Σ, using the results of theorem IV.5 we have: $T^p_r = T_{\Sigma,m}$.

Thence inferred: $(T^p_r)_x = (T_{\Sigma,m})_x$, $\forall x\in id$.

Which we have: $T^p_{rx} = (T^p_r)_x = (T_{\Sigma,m})_x = T_{\Sigma x,m}$, $\forall x\in id$ $\Rightarrow T^p_{rx} = T_{\Sigma x,m} \Rightarrow r^p_x(\Sigma_x,m)$, $\forall x\in id$.

So r_x is m-tight representation by groups set Σ_x, $\forall x\in id$.

$\Leftarrow$) Suppose r_x is m-tight representation by groups set Σ_x,$\forall x \in id$ we need proof r is m-tight representation by groups set PTBDĐTTNB Σ.

Indeed, under the assumption r_x is m-tight representation by groups set Σ_x ,$\forall x\in id \Rightarrow T^p_{rx} = T_{\Sigma x,m}$, $\forall x\in id$.

Inferred: $(T^p_r)_x = T^p_{rx} = T_{\Sigma x,m} = (T_{\Sigma,m})_x$, $\forall x\in id$.

Which we have: $T^p_r = \bigcup_{x\in id} T^p_{rx}$, $T_{\Sigma,m} = \bigcup_{x\in id} T_{\Sigma x,m}$

$\Rightarrow T^p_r = T_{\Sigma,m}$.

So r is m-tight representation by groups set PTBDĐTTNB Σ.

V. CONCLUSIONS

From a proposed concept are functions that evaluate values on a group with p elements, The article gave the definition of the multivalued truth block by groups of data blocks. From there build a new type of dependency: it is a multivalued positive Boolean dependency by groups in the database model of block form. From the new concept of dependency is proposed, the authors have stated and proved the equivalent theorem for multivalued positive Boolean dependencies by groups on the block, the necessary and sufficient condition for a block r is m-tight representation set PTBDĐTTNB Σ… From these results we can further study the relationship between other types of extended logical dependencies on the data block.

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