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# Maximum Entropy Functions of Discrete Random Fuzzy Variables and Genetic Algorithm

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***Abstract:*** Due to deficiency of information, the membership functions and probability distribution of a random fuzzy variable cannot be obtained explicitly. It is a challenging work to find an appropriate membership function and an appropriate probability distribution when certain partial information about a random fuzzy variable is given, such as expected value or moments. This paper solves such problems for the maximum entropy of discrete random fuzzy variables with certain constraints. A genetic algorithm is designed to solve the general maximum entropy model for discrete random fuzzy variables, which is illustrated by some numerical experiments.

***Keywords****:* Random fuzzy variables; Chance measure; Entropy; Genetic algorithm

## Ⅰ. INTRODUCTION

We usually meet many uncertain phenomena because "uncertainty is absolute and certainty is relative" in the real world. In these uncertain events, fuzziness and randomness are two basic types of uncertainty. Probability theory is a branch of mathematics for studying the behavior of random phenomena. The study of probability theory was started by Pascal and Fermat (1654), and an axiomatic foundation of probability theory was given by Kolmogoroff (1933) in his *Foundations of Probability Theory.* Credibility theory is a branch of mathematics for studying the behavior of fuzzy phenomena. The study of credibility theory was started by Liu and Liu (2002), and an axiomatic foundation of credibility theory was given by Liu (2004) in his *Uncertainty Theory*. Sometimes, fuzziness and randomness simultaneously appear in a system. In order to describe this phenomena, a random fuzzy variable was proposed by Liu [12] as a fuzzy element taking "random variable" values. In addition, a hybrid variable was introduced by Liu [13] as a tool to describe the quantities with fuzziness and randomness. Fuzzy random variable and random fuzzy variable are instances of hybrid variable. In order to measure hybrid events, a concept of chance measure was introduced by Li and Liu [14].

Entropy is use to provide a quantitative measurement of the degree of uncertainty, which has widely been applied in transportation [19]&[20], risk analysis [22], signal processing [21] and economics [25]. Since the Shannon entropy of random variables was proposed by Shannon [6], Jaynes [15] provided the maximum entropy principle of random variables when some constraints were given. Fuzzy entropy was first initialized by Zadeh [7] to quantify the fuzziness, who defined the entropy of a fuzzy event as a weighted Shannon entropy. Up to now, fuzzy entropy has been studied by many researchers such as De Luca and Termini [4], Kaufmann [5], Yager [17], Kosko [16], Pal and Pal [10], Bhandari and Pal [1], Pal and Bezdek [11]. However, those definitions of entropy characterize the uncertainty resulting primarily from the linguistic vagueness rather than resulting from information deficiency, and vanishes when the fuzzy variable is an equipossible one. In order to measure the uncertainty of fuzzy variables, Liu [9] suggested that an entropy of fuzzy variables should meet at least three basic requirements: (i) minimum; (ii) maximum; (iii) universality. In order to meet those requirements, Li and Liu [8] provided a new definition of fuzzy entropy to characterize the uncertainty resulting from information deficiency which is caused by the impossibility to predict the specified value that a fuzzy variable takes and provided the maximum entropy principle of fuzzy variables. In order to measure the uncertainty of hybrid variables, Li X, and Liu B [14] provided the concept of hybrid entropy. However, given some constraints, for example, expected value and variance, there are usually multiple compatible membership functions and probability distributions. Which membership functions and probability distributions shall we take? Because fuzziness and randomness simultaneously appear in a system, we can not get the maximum entropy of hybrid variables through Euler-Lagrange equation. For fuzzy variables, Li and Liu [2] gave an analytical method to find the maximum entropy membership function of continuous fuzzy variables and Gao and You [3] gave an analytical method to find the maximum entropy membership function of discrete fuzzy variables. On the basis of their work，we promote their ideas to solve the problem for maximum entropy functions of discrete random fuzzy variables in this paper.

The organization of our work is as follows: In section 2, some basic concepts and results on random fuzzy variables are reviewed. In section 3, we introduce some constraints. In sections 4 and 5, an effective genetic algorithm is introduced to solve general maximum entropy models for discrete random fuzzy variables and some computational experiments are given in illustration of it. Finally, the conclusion is given in the last section.

## II. PRELIMINARIES

Let $\xi$ be a fuzzy variable with the membership function $\mu(x)$ which satisfies the normalization condition, i.e., $\sup_x \mu(x)=1$. In the setting of credibility theory, the credibility measure for fuzzy event $\{\xi\in B\}$ deduced from $\mu(x)$ is given by

$$\mathrm{Cr}\{\xi\in B\}=\frac{1}{2}\left(\sup_{x\in B}\mu(x)+1-\sup_{x\in B^c}\mu(x)\right)$$

(2.1)

Where $B$ is any subset of the real numbers $R$, and $B^c$ is the complement of set $B$. Conversely, for a fuzzy variable $\xi$, its membership function can be derived from the credibility measure by

$$\mu(x)=\left(2\mathrm{Cr}\{\xi=x\}\right)\wedge 1,\qquad x\in R$$

(2.2)

**Definition 2.1** (X.Li, B.Liu [23]) A random fuzzy variable is a function from a credibility space $(\Theta,P,\mathrm{Cr})$ to the set of random variables defined on a probability space $(\Omega,A,\mathrm{Pr})$.

In the following, we give some examples of random fuzzy variables.

**Example 2.1** (Uniformly distributed random fuzzy variable) A random fuzzy variable $\xi$ is said to be uniform if for each $\theta$, $\xi(\theta)$ is a uniformly distributed random variables, i.e., $\xi(\theta)\sim U\left[X(\theta),Y(\theta)\right]$, with $X$ and $Y$ are fuzzy variables defined on the space $\Theta$ such that $X\le Y$.

**Example 2.2** (Normally distributed random fuzzy variable) A random fuzzy variable $\xi$ is said to be normal if for each $\theta$, $\xi(\theta)$ is a normally distributed random variable, i.e., $\xi(\theta)\in N\left(X(\theta),Y(\theta)\right)$, with $X$ and $Y$ are fuzzy variables defined on the space $\Theta$ such that $Y>0$. A normally distributed random fuzzy variable is usually denoted as $\xi\in N(X,Y)$, and the fuzziness of random fuzzy variable $\xi$ is said to be characterized by fuzzy vector $(X,Y)$.

**Example 2.3** (Exponentially distributed random fuzzy variable) A random fuzzy variable $\xi$ is said to be exponential if for each $\theta$, $\xi(\theta)$ is an exponentially distributed random variable whose density function is defined as

$$f_{\xi(\theta)}(t)=\begin{cases}0, & t<0\\ X(\theta)\exp\left(-X(\theta)t\right), & t\ge 0\end{cases}$$

(2.3)

with $X$ is a positive fuzzy variable defined on the space

 

$\Theta$. An exponentially distributed random fuzzy variables is often denoted by $\xi \sim \exp(X)$, and the fuzziness of random fuzzy variable $\xi$ is said to be characterized by fuzzy variable $X$.

Generally, let $\xi$ be a random fuzzy variable. The fuzziness of $\xi$ is said to be characterized by fuzzy variable $X$, if for each $\theta \in \Theta$, the distribution of random variable $\xi(\theta)$ is determined by the parameter $X(\theta)$.

**Definition 2.2** (Li and Liu [18]) Let $(\Theta, P, \mathrm{Cr}) \times (\Omega, A, \Pr)$ be a chance space. Then a chance measure of an event $\Lambda$ is defined as

$$\mathrm{Ch}\{\Lambda\} = \begin{cases} \sup\limits_{\theta\in\Theta}\left(\mathrm{Cr}\{\theta\} \wedge \Pr\{\Lambda(\theta)\}\right), \\ \qquad \text{if} \quad \sup\limits_{\theta\in\Theta}\left(\mathrm{Cr}\{\theta\} \wedge \Pr\{\Lambda(\theta)\}\right) < 0.5 \\ 1-\sup\limits_{\theta\in\Theta}\left(\mathrm{Cr}\{\theta\} \wedge \Pr\{\Lambda^c(\theta)\}\right), \\ \qquad \text{if} \quad \sup\limits_{\theta\in\Theta}\left(\mathrm{Cr}\{\theta\} \wedge \Pr\{\Lambda(\theta)\}\right) \ge 0.5 \end{cases} \tag{2.4}$$

In fact, chance measure may be defined in different ways. For example, we may employ the following chance measure,

$$Ch\{\Lambda\} = \frac{1}{2}\left(\sup_{\theta\in\Theta}\left(\mu(\theta)\times\Pr\{\Lambda(\theta)\}\right) + 1 - \sup_{\theta\in\Theta}\left(\mu(\theta)\times\Pr\{\Lambda^c(\theta)\}\right)\right) \tag{2.5}$$

Where $\mu(\theta) = (2Cr\{\theta\}) \wedge 1$.

**Theorem 2.1** (Li X, Liu B [24]) Let $(\Theta, P, \mathrm{Cr}) \times (\Omega, A, \Pr)$ be a chance space and $\mathrm{Ch}$ a chance measure. Then for any event $\Lambda$, we have

$$\sup_{\theta\in\Theta}\left(\mathrm{Cr}\{\theta\} \wedge \Pr\{\Lambda(\theta)\}\right) \vee \sup_{\theta\in\Theta}\left(\mathrm{Cr}\{\theta\} \wedge \Pr\{\Lambda^c(\theta)\}\right) \ge 0.5 \tag{2.6}$$

$$\sup_{\theta\in\Theta}\left(\mathrm{Cr}\{\theta\} \wedge \Pr\{\Lambda(\theta)\}\right) + \sup_{\theta\in\Theta}\left(\mathrm{Cr}\{\theta\} \wedge \Pr\{\Lambda^c(\theta)\}\right) \le 1 \tag{2.7}$$

**Proof:** It follows from the basic properties of probability and credibility that

$$\begin{aligned} &\sup_{\theta\in\Theta}\left(\mathrm{Cr}\{\theta\} \wedge \Pr\{\Lambda(\theta)\}\right) \vee \sup_{\theta\in\Theta}\left(\mathrm{Cr}\{\theta\} \wedge \Pr\{\Lambda^c(\theta)\}\right) \\ &\ge \sup_{\theta\in\Theta}\left(\mathrm{Cr}\{\theta\} \wedge \left(\Pr\{\Lambda(\theta)\} \vee \Pr\{\Lambda^c(\theta)\}\right)\right) \\ &\ge \sup_{\theta\in\Theta}\mathrm{Cr}\{\theta\} \wedge 0.5 = 0.5 \end{aligned}$$

and

$$\begin{aligned} &\sup_{\theta\in\Theta}\left(\mathrm{Cr}\{\theta\} \wedge \Pr\{\Lambda(\theta)\}\right) + \sup_{\theta\in\Theta}\left(\mathrm{Cr}\{\theta\} \wedge \Pr\{\Lambda^c(\theta)\}\right) \\ &= \sup_{\theta_1,\theta_2\in\Theta}\left(\mathrm{Cr}\{\theta_1\} \wedge \Pr\{\Lambda(\theta_1)\} + \mathrm{Cr}\{\theta_2\} \wedge \Pr\{\Lambda^c(\theta_2)\}\right) \\ &\le \sup_{\theta_1\neq\theta_2}\left(\mathrm{Cr}\{\theta_1\} + \mathrm{Cr}\{\theta_2\}\right) \vee \sup_{\theta\in\Theta}\left(\Pr\{\Lambda(\theta)\} + \Pr\{\Lambda^c(\theta)\}\right) \\ &\le 1 \vee 1 = 1. \end{aligned}$$

**Example 2.4**: Let $\eta_1, \eta_2, \cdots, \eta_m$ be random variables, and let $u_1, u_2, \cdots, u_m$ be nonnegative numbers with $u_1 \vee u_2 \vee \cdots \vee u_m = 1$. Then

$$\xi = \begin{cases} \eta_1 \text{ with membership degree } u_1 \\ \eta_2 \text{ with membership degree } u_2 \\ \quad \cdots \\ \eta_m \text{ with membership degree } u_m \end{cases} \tag{2.8}$$

is clearly a random fuzzy variable. If $\eta_1, \eta_2, \cdots, \eta_m$ have probability density functions $\phi_1, \phi_2, \cdots, \phi_m$, respectively, then for any Borel set $B$ of real numbers,

$$\mathrm{Ch}\{\xi \in B\} = \begin{cases} \max\limits_{1\le i\le m}\left(\dfrac{u_i}{2} \wedge \int_B \phi_i(x)\,dx\right), & \text{if } \max\limits_{1\le i\le m}\left(\dfrac{u_i}{2} \wedge \int_B \phi_i(x)\,dx\right) < 0.5 \\ 1-\max\limits_{1\le i\le m}\left(\dfrac{u_i}{2} \wedge \int_{B^c} \phi_i(x)\,dx\right), & \text{if } \max\limits_{1\le i\le m}\left(\dfrac{u_i}{2} \wedge \int_B \phi_i(x)\,dx\right) \ge 0.5 \end{cases} \tag{2.9}$$

**Definition 2.3.** (Li and Liu [22]) Let $\xi$ be a random fuzzy variable. Then the expected value of $\xi$ is defined by

$$E[\xi]=\int_0^{+\infty}\mathrm{Ch}\{\xi\ge r\}dr-\int_{-\infty}^{0}\mathrm{Ch}\{\xi\le r\}dr$$

(2.10)

provided that at least one of the two integrals is finite.

In fact, the expected value $E[\xi]$ of $\xi$ may be defined by

$$E[\xi]=\int_0^{+\infty}Cr\{\theta\in\Theta \mid E[\xi(\theta)]\ge r\}dr-\int_{-\infty}^{0}Cr\{\theta\in\Theta \mid E[\xi(\theta)]\le r\}dr$$

(2.11)

provided that at least one of the two integrals is finite, where $E[\xi(\theta)]$ is the expected value of random variable $\xi(\theta)$.

According to the Li and Liu [23] we get a new definition of entropy of random fuzzy variable $\xi$, denoted by $H[\xi]$.

**Definition 2.4** Suppose that $\xi$ is a discrete random fuzzy variable taking values in $\{x_1,x_2,\cdots\}$. Then its entropy is defined by

$$H[\xi]=\sum_{i=1}^{\infty}S(\mathrm{Ch}\{\xi=x_i\}) .$$

(2.12)

where $S(t)=-t\ln t-(1-t)\ln(1-t)$. If there exists some index $k$ such that $\mathrm{Ch}\{\xi=x_k\}=1$, and 0 otherwise, then its entropy $H[\xi]=0$. Suppose that $\xi$ is a simple random fuzzy variable taking values in $\{x_1,x_2,\cdots,x_n\}$ . If $\mathrm{Ch}\{\xi=x_i\}=0.5$ for all $i=1,2,\cdots,n$, then its entropy $H[\xi]=n\ln 2$. Suppose that $\xi$ is a discrete random fuzzy variable taking values in $\{x_1,x_2,\cdots\}$. Then $H[\xi]\ge 0$ and equality holds if and only if $\xi$ is essentially a deterministic / crisp number.

## III. MOMENT CONSTRAINTS

In this section, we consider discrete random fuzzy variables. Let $\xi$ be a discrete random fuzzy variable taking values in $\{x_1,x_2,\cdots,x_n\}$ (in this paper we always assume that $x_1<x_2<\cdots<x_n$) with membership degrees $\{u_1,u_2,\cdots,u_n\}$ and probability $\{p_1,p_2,\cdots,p_n\}$, respectively, where $u_1\vee u_2\vee\cdots\vee u_n=1$. Then the expected value of $\xi$ can be written as (without loss of generality, suppose $x_{k-1}<0\le x_k$).

$$E[\xi]=\int_0^{x_k}\mathrm{Ch}\{\xi\ge r\}dr+\sum_{i=k+1}^{n}\int_{x_{i-1}}^{x_i}\mathrm{Ch}\{\xi\ge r\}dr-\sum_{i=2}^{k-1}\int_{x_{i-1}}^{x_i}\mathrm{Ch}\{\xi\le r\}dr-\int_{x_{k-1}}^{0}\mathrm{Ch}\{\xi\le r\}dr$$

$$=\sum_{i=1}^{k-1}\left(\mathrm{Ch}\{\xi\le x_i\}-\mathrm{Ch}\{\xi<x_i\}\right)\cdot x_i+\sum_{i=k}^{n}\left(\mathrm{Ch}\{\xi\ge x_i\}-\mathrm{Ch}\{\xi>x_i\}\right)\cdot x_i$$

$$=\begin{cases}\sum_{i=1}^{n}\frac{1}{2}\left(\max\limits_{1\le j\le i}\left(\frac{u_j}{2}\wedge p_j\right)-\max\limits_{1\le j<i}\left(\frac{u_j}{2}\wedge p_j\right)+\max\limits_{i\le j\le n}\left(\frac{u_j}{2}\wedge p_j\right)-\max\limits_{i<j\le n}\left(\frac{u_j}{2}\wedge p_j\right)\right)\cdot x_i \\ \qquad if\quad \max\limits_{1\le j\le i}\left(\frac{u_j}{2}\wedge p_j\right)<0.5\,\&\max\limits_{i\le j\le n}\left(\frac{u_j}{2}\wedge p_j\right)<0.5 \\ \sum_{i=1}^{n}\frac{1}{2}\left(1-\max\limits_{i<j\le n}\left(\frac{u_j}{2}\wedge p_j\right)-\max\limits_{1\le j<i}\left(\frac{u_j}{2}\wedge p_j\right)+1-\max\limits_{1\le j<i}\left(\frac{u_j}{2}\wedge p_j\right)-\max\limits_{i<j\le n}\left(\frac{u_j}{2}\wedge p_j\right)\right)\cdot x_i \\ \qquad if\left(\frac{u_i}{2}\wedge pi\right)\ge 0.5\,\&\max\limits_{1\le j<i}\left(\frac{u_j}{2}\wedge p_j\right)<0.5\,\&\max\limits_{i<j\le n}\left(\frac{u_j}{2}\wedge p_j\right)<0.5 \\ \text{Similarly, five forms can be introduced}\end{cases}$$

$$=\sum_{i=1}^{n}\omega_i x_i$$

(3.1)

It is easy to verify that all $\omega_i\ge 0$ and $\sum_{i=1}^{n}\omega_i\le 1$. If

$$\max_{1\le j\le i}\left(\frac{u_j}{2}\wedge p_j\right)\vee\max_{i<j\le n}\left(\frac{u_j}{2}\wedge p_j\right)\ge 0.5, i\in\{1,2,\cdots,n\}$$

, then $\sum_{i=1}^{n}\omega_i=1$.

Furthermore, $E\left[(\xi-e)^2\right]$ is called the variance of $\xi$ and $E\left[\xi^n\right]$ the nth moment of $\xi$. If the random fuzzy variable $\xi$ reduces to a fuzzy variable, i.e., for any $i\in\{1,2,\cdots,n\}$, $p_i\equiv 1$, then the expected value reduces to the following form

$$E[\xi]=\sum_{i=1}^{n}\frac{1}{2}\left(\max_{1\le j\le i}u_j-\max_{1\le j<i}u_j+\max_{i\le j\le n}u_j-\max_{i<j\le n}u_j\right)\cdot x_i$$

(3.2)

Which is just the expected value of discrete fuzzy variable $\xi$. Thus, the expected value of discrete random fuzzy variable is a natural extension of discrete fuzzy variable. Let $\xi$ be a nonnegative discrete random fuzzy variable taking values in $\{x_1,x_2,\cdots,x_n\}$ with membership degrees $\{u_1,u_2,\cdots,u_n\}$ and probability $\{p_1,p_2,\cdots,p_n\}$, respectively., where , and $k$ a positive number. Then the k-th moment

$$E\left[\xi^k\right]=k\int_0^{+\infty}r^{k-1}Ch\{\xi\ge r\}dr$$

$$=k\sum_{i=1}^{n}Ch\{\xi\ge x_i\}.x_i^k$$

$$=\begin{cases}k\sum_{i=1}^{n}\left(\max_{i\le j\le n}\left(\frac{u_j}{2}\wedge p_j\right)\right)\cdot x_i^k & \text{if } \max_{i\le j\le n}\left(\frac{u_j}{2}\wedge p_j\right)<0.5\\ k\sum_{i=1}^{n}\left(1-\max_{1\le j<i}\left(\frac{u_j}{2}\wedge p_j\right)\right)\cdot x_i^k & \text{if } \max_{i\le j\le n}\left(\frac{u_j}{2}\wedge p_j\right)\ge 0.5\end{cases}$$

(3.3)

If the random fuzzy variable $\xi$ reduces to a fuzzy variable, i.e., for any $i\in\{1,2,\cdots,n\}$, $p_i\equiv 1$, then the k-th moment reduces to the following form

$$E\left[\xi^k\right]=k\int_0^{+\infty}r^{k-1}Cr\{\xi\ge r\}dr$$

$$=k\sum_{i=1}^{n}Cr\{\xi\ge x_i\}.x_i^k$$

$$=k\sum_{i=1}^{n}\frac{1}{2}\left(\max_{i\le j\le n}u_j+1-\max_{1\le j<i}u_j\right)\cdot x_i^k$$

(3.4)

Which is just the k-th moment of discrete fuzzy variable $\xi$. If $k=1$, which is just the expected value of discrete random fuzzy variable $\xi$.

## IV. GENETIC ALGORITHM FOR GENERAL MAXIMUM ENTROPY MODEL

Genetic algorithm is a stochastic search method for global optimization problems based on the mechanics of natural selection and natural genetics. Genetic algorithm has demonstrated enormous success in providing good solutions to many complex optimization problems. In this section, we will design an effective genetic algorithms integrated with random fuzzy simulation for solving the maximum entropy model for discrete random fuzzy variables.

Let $\xi$ be a discrete random fuzzy variable taking values in $\{x_1,x_2,\cdots,x_n\}$ with membership degrees $\{u_1,u_2,\cdots,u_n\}$ and probability $\{p_1,p_2,\cdots,p_n\}$, respectively. We have the natural relation $0\le u_i\le 1$, $0\le p_i\le 1$ and $\max_{1\le i\le n}u_i=1$. By using maximum entropy principle, we have the following maximum entropy model:

$$\begin{cases} \max \sum_{i=1}^{n} S\left(Ch\{\xi = x_i\}\right) \\ \text{subject to} \\ \qquad \sum_{i=1}^{n} \omega_i x_i \le e, \\ \qquad 0 \le u_i \le 1, i = 1, 2, \cdots, n. \\ \qquad 0 \le p_i \le 1, i = 1, 2, \cdots, n \\ \qquad u_i = 1, i \in \{1, 2, \cdots, n\} \\ \qquad \sum_{i=1}^{n} \omega_i = 1 \end{cases}$$

(4.1)

Where $\omega_i$ from (3.1),

$$S(t) = -t \ln t - (1-t)\ln(1-t),$$

$$Ch\{\xi = x_i\} = \begin{cases} \frac{u_i}{2} \wedge p_i & \left(\frac{u_i}{2} \wedge p_i\right) < 0.5 \\ 1 - \max\limits_{\substack{1 \le j \le n \\ j \ne i}} \left(\frac{u_j}{2} \wedge p_j\right) & \left(\frac{u_i}{2} \wedge p_i\right) \ge 0.5 \end{cases}.$$

In general, the expected value constraint can be replaced by other moment constraints. For the search spaces of the maximum entropy model (4.1) are particularly irregular, genetic algorithm has succeeded in providing good solutions to complex moment conditions.

As an illustration, the following steps show how the genetic algorithm works.

Step 1: Initialize pop-size feasible chromosomes $U_t = \{u_1^t, u_2^t, \cdots, u_n^t\}$ and $P_t = \{p_1^t, p_2^t, \cdots, p_n^t\}$ for $t = 1, 2, \cdots,$ pop-size from $(0,1) \times (0,1) \times \cdots \times (0,1)$, in which the maximum $u_i^t, i = 1, 2, \cdots, n$ for each $t$ is set to be 1.

Step 2: Calculate the expected values for all chromosomes $U_t$ and $P_t$, $t = 1, 2, \cdots,$ pop-size, respectively. If the expected values do not satisfy the constraints, we regenerate a chromosome to replace the original one until it is feasible.

Step 3: Calculate the entropy of each random fuzzy variable which is represented by each chromosome. The entropy denoted by $H(U_t \wedge P_t)$, is to assign a probability of reproduction to each chromosome $U_t$ and $P_t$ so that its likelihood of being selected is proportional to its entropy relative to the other chromosomes in the population. That is, the chromosomes with larger entropy will have more chance to produce offspring by using roulette wheel selection.

Step 4: Select the chromosomes for a new population by spinning the roulette wheel according to the value of the entropy of all chromosomes.

Step 5: Renew the chromosomes by crossover operations with a predetermined parameters $P_c$, which is called the probability of crossover. In order to determine the parents for crossover operation, let us do the following process repeatedly from $t = 1$ to pop-size: generating a random number $r$ from the interval $[0,1]$, the chromosome $U_t$ and $P_t$ is selected as a parent if $r < P_c$. We denote the selected parents by $U_1{}^t, U_2{}^t, U_3{}^t, \cdots$ and $P_1^t, P_2^t, P_3^t, \cdots$ and divide them into the following pairs:

$$\left(U_1{}^t, U_2{}^t\right), \left(U_3{}^t, U_4{}^t\right), \left(U_5{}^t, U_6{}^t\right), \cdots$$

$$\left(P_1^t, P_2^t\right), \left(P_3^t, P_4^t\right), \left(P_5^t, P_6^t\right), \cdots$$

Let us illustrate the crossover operator on each pair by $\left(U_1{}^t, U_2{}^t\right)$ and $\left(P_1^t, P_2^t\right)$. At first, we generate a random number $c$ from the open interval $(0,1)$. Then the crossover operator on $U_1{}^t$ and $U_2{}^t$, $P_1^t$ and $P_2^t$ will produce two children $X$ and $Y$, $X'$ and $Y'$ as follows:

$$X = c \cdot U_1{}^t + (1-c) \cdot U_2{}^t, Y = (1-c) \cdot U_1{}^t + c \cdot U_2{}^t$$

$$X' = c \cdot P_1^t + (1-c) \cdot P_2^t, Y' = (1-c) \cdot P_1^t + c \cdot P_2^t$$

In this case, the feasible set is not convex. Thus we must check the feasibility of each child before accepting it. We set the maximum component of each child to be 1. Then we check if the children satisfy the constraints. If both children are feasible, then we replace the parents with them. If not, we keep the feasible child if it exists, and keep the other parent still.

Step 6: Update the chromosomes by mutation operations with a predetermined probability of mutation $P_m$. In a similar manner to the process of selecting parents for crossover operation, we repeat the following steps from $t=1$ to pop-size: generating a random number $r$ from the interval $[0,1]$, the chromosome $U_t$ and $P_t$ is selected as a parent if $r<P_m$. For each selected parent, denoted by $U_t=\{u_1^t,u_2^t,\cdots,u_n^t\}$ and $P_t=\{p_1^t,p_2^t,\cdots,p_n^t\}$, we mutate it in the following way. For each selected parent, we randomly select one $u_i^t$ and $p_i^t$ of this chromosome and regenerate their values. Then set the maximum $u_i^t$ to be 1 and check the feasibility of it.

Step 7: Repeat Step 3 to Step 6 for $N$ times, where $N$ is a sufficiently large integer.

Step 8: Report the best chromosome $U_t$ and $P_t$ as the optimal solution.

## V. NUMERICAL EXAMPLES

In order to illustrate the effectiveness of the proposed genetic algorithm, let us consider Example 1 in Table 1 ($\xi$ is a discrete random fuzzy variable taking values in $\{1,3,4,8,10,11\}$ with $p=\{1,1,1,1,1,1\}$ and expected value $E[\xi]$ satisfying $E[\xi]\le 4$) for the comparison of the algorithm with respect to different parameters (Gao and You [3]). We compare the solutions when different parametric values of $P_c$, $P_m$, $pop-size$ and $N$ are taken in the genetic algorithm. The results are shown in Fig and the errors shown in Table 2 are calculated by the formula: (actual value-optimal value)/optimal value$\times 100\%$.

Table 1

| Example | $\{x_1,x_2,x_3,x_4,x_5,x_6\}$ | $E[\xi]$ | $\mathrm{Cr}\wedge\mathrm{Pr}$ | $H_{\max}$ |
|---|---|---|---|---|
| 1 | $\{1,3,4,8,10,11\}$ | $\le 4$ | $\{0.6462,0.3538,0.3432,0.2810,0.2755,0.2744\}$ | 3.7127 |
| 2 | $\{1,2,3,4,5,6\}$ | $\le 3$ | $\{0.5619,0.4381,0.4224,0.3859,0.3821,0.3713\}$ | 4.0436 |

Table 2

| | pop-size | $P_c$ | $P_m$ | $N$ | $\mathrm{Cr}\wedge\mathrm{Pr}$ | $H_{\max}$ | Error$(\%)$ |
|---|---|---|---|---|---|---|---|
| 1 | 160 | 0.7 | 0.4 | 1000 | $\{0.5179,0.4821,0.4821,0.2239,0.2225,0.2133\}$ | 3.6576 | 1.48 |
| 2 | 190 | 0.7 | 0.3 | 1000 | $\{0.6999,0.3000,0.2997,0.3001,0.3001,0.2998\}$ | 3.6648 | 1.29 |
| 3 | 160 | 0.7 | 0.3 | 1000 | $\{0.5506,0.4494,0.4402,0.2373,0.2373,0.2373\}$ | 3.7058 | 0.19 |
| 4 | 160 | 0.8 | 0.3 | 3000 | $\{0.5963,0.4037,0.4037,0.2833,0.2185,0.2185\}$ | 3.6696 | 1.16 |
| | $u=\{u_1,u_2,u_3,u_4,u_5,u_6\}$ | | | | $p=\{p_1,p_2,p_3,p_4,p_5,p_6\}$ | | |

| | | |
|---|---|---|
| 1 | $\{1.0000, 0.9642, 0.9642, 0.4477, 0.4450, 0.4266\}$ | $\{0.9447, 0.6913, 0.6655, 0.9015, 0.9361, 0.6108\}$ |
| 2 | $\{1.0000, 0.6000, 0.9999, 0.6001, 0.6001, 0.5995\}$ | $\{0.8845, 0.9664, 0.2997, 0.3405, 0.8631, 0.4502\}$ |
| 3 | $\{1.0000, 0.8987, 0.8804, 0.5832, 0.4746, 0.4746\}$ | $\{0.9621, 0.9829, 0.8863, 0.2373, 0.7496, 0.9978\}$ |
| 4 | $\{1.0000, 0.8074, 0.8074, 0.5943, 0.4371, 0.4371\}$ | $\{0.6935, 0.6410, 0.5838, 0.2833, 0.3524, 0.3824\}$ |

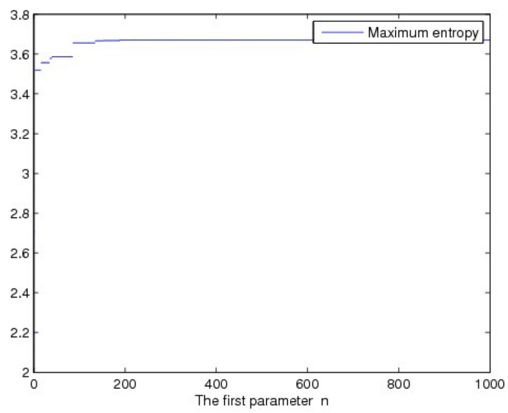


Figure 1

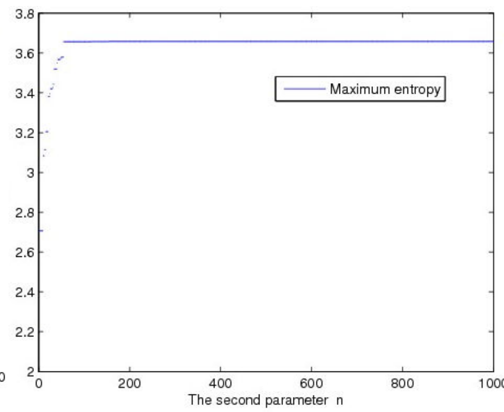


Figure 2

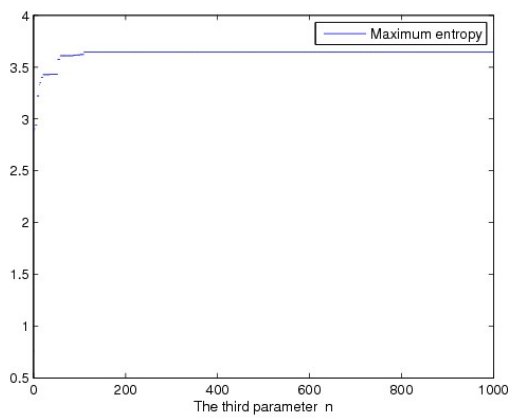


Figure 3

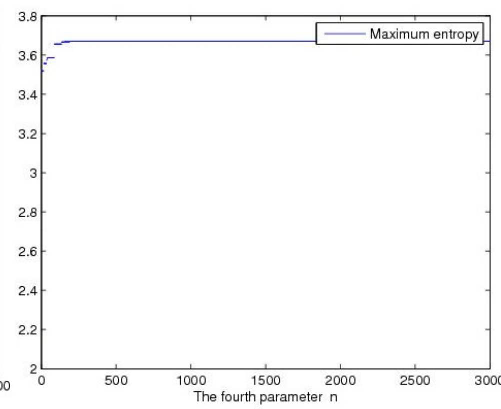


Figure 4

It follows from Table that the error does not exceed $3\%$, which shows that the proposed algorithm is effective to solve the above model.

## VI. CONCLUSIONS

In this paper, we promote the idea of Gao and You [3] to solve this problem for maximum Entropy functions of discrete random fuzzy variables and design a genetic algorithm to solve this problem. Along with the improvement of uncertainty theory, we can use this method to solve many uncertain events such as toward fuzzy. In the future work, we will continue focus on this area.

## VII. ACKNOWLEDGMENT

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