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SOME PROPERTIES ABOUT SMOOTHING, ROUGHEN THE VALUES OF THE INDEX ATTRIBUTE ON THE DECISION BLOCK

Trinh Dinh Thang
Institute of Information Technology,
Hanoi Pedagogical University 2, Viet Nam

Do ThiLanAnh
Institute of Information Technology,
Hanoi Pedagogical University 2, Viet Nam

Abstract: The report proposed and demonstrated some properties about smoothing, roughen the values of the condition index attribute or decision index attributeon the decision block and on the slice of the decisionblock. Every time the condition equivalence class or decision equivalence classon the decision block have been smoothed or roughened then they will partial pullulate or pullulate smoothing, roughing the corresponding classon the slice. From the results foundof the smoothing, roughening the condition equivalence class or decision equivalence classpartial pullulate or pullulate on the slice then the incremental calculation of the support matrices on the slice will be simplerand therefore faster than recalculating these matrices when smoothing, roughing the values of the condition index attribute or decision index attribute.

Keywords: Decision block, smoothing, roughen, index attribute.

I. INTRODUCTION

The study to search for decision laws on the decision table by assessing the measures of decision laws as well as incremental approaches, determining decision laws … has been studied by many groups of authors, such as in [7], [8], … On the other hand, when the decision table is expanded into a decision block, then the study, proposing a model and algorithm to detect decision laws on the decision block has been studied by the authors as in [4], [5], [6]. However, the proposed models and algorithms when smoothing and roughen the values of index attributes on the decision block have not been studied until now.The purpose of this paper is to study the some properties about smoothing, roughen the values of the condition index attribute or decision index attribute on the decision block and on the slice of the decision block.From the results found of the smoothing, roughening the condition equivalence class or decision equivalence class partial pullulate or pullulate on the slice then the incremental calculation of the support matrices on the slice will be simpler and therefore faster than recalculating these matrices when smoothing, roughing the values of the condition index attribute or decision index attribute.

II - THE BASIC CONCEPT

II.1 The block, slice of the block

Definition II.1 [1]

Let $R = (id; A_1, A_2,..., A_n)$ is a finite set of elements, where id is non-empty finite index set, A_i (i=1.. n) is the attribute. Each attribute A_i (i=1.. n) there is a corresponding value domain $dom(A_i)$. A block r on R, denoted r(R) consists of a finite number of elements that each element is a family of mappings from the index set id to the value domain of the attributes A_i (i = 1.. n).

$$t \in r(R) \Leftrightarrow t = \{ t^i : id \rightarrow dom(A_i)\}_{i=1..n}.$$

The block is denoted by r(R) or $r(id; A_1, A_2,..., A_n)$, sometime without fear of confusion we simply denoted r.

Definition II.2 [2],[3]

Let $R = (id; A_1, A_2,..., A_n)$, r(R) is a block over R. For each $x \in id$ we denoted $r(R_x)$ is a block with $R_x = (\{x\}; A_1, A_2,..., A_n)$ such that:

$$t_x \in r(R_x) \Leftrightarrow t_x = \{t^i_x = t^i\big|_x\}_{i=1..n},\ t \in r(R),\ t = \{t^i : id \rightarrow dom(A_i)\}_{i=1..n}$$

where $t^i_x(x) = t^i(x)$, i =1..n.

Then $r(R_x)$ is called a slice of the block r(R) at point x, sometimes we denoted r_x.

Here, for simplicity we use symbols:

$x^{(i)} = (x; A_i)$; $id^{(i)} = \{x^{(i)} \mid x \in id\}$.

We call $x^{(i)}$ ($x \in id$, i = 1..n) are the index attributes of the block scheme $R = (id; A_1,A_2,...,A_n)$.

II.2Information block

DefinitionII.3[4]:*Let block schemeR = (id; $A_1, A_2, \dots, A_n$), ris a block over R. Then, the information block is a tuples of fourelements IB = (U, A, V, f) with Uis a set of objects of r called space objects, $A = \bigcup_{i=1}^{n} id^{(i)}$ is the set of index attributes of the object , $V = \bigcup_{x^{(i)} \in A} V_{x^{(i)}}$, $V_{x^{(i)}}$ is the set of values of the objects corresponding to the index attribute $x^{(i)}$, fis an information function $U \times A \rightarrow V$ satisfy: $\forall u \in U$, $\forall x^{(i)} \in A$ we have $f(u, x^{(i)}) \in V_{x^{(i)}}$.*

We call $f(u, x^{(i)})$ is the value of the object u at the index attribute $x^{(i)}$.

If V contains missing values in at least one index attribute $x^{(i)} \in A$ then we call IB is inadequate information block, In contrast IB is a complete information block, or simply IB is an information block.

DefinitionII.4[4]:*Let block schemeR = (id; $A_1, A_2, \dots, A_n$), r is a block overR, $r_x$is the slice of the block r at the point $x \in id$. Then the slice of the information block at x is a tuples of four elements$IB_x = (U, A_x, V_x, f_x)$ with U is a set of objects of r called space objects, $A_x = \bigcup_{i=1}^{n} x^{(i)}$ is the set of the index* *attributes of the object on the slice at x,* $V_x = \bigcup_{x^{(i)}\in A_x} V_{x^{(i)}}$, $V_{x^{(i)}}$ *is the set of values of the objects corresponding to the index attribute* $x^{(i)}$*,* f_x*is an information function* $U\times A_x \rightarrow V_x$*satisfy:* $\forall u\in U$, $\forall x^{(i)}\in A_x$*we have* $f(u, x^{(i)})\in V_{x^{(i)}}$.

II.3 Relationships are indistinguishable

DefinitionII.5[5]

Letinformation block IB = (U, A, V, f). Then for each index attribute set P⊆ A we define an equivalence relation, signIND(P) defined as follows:

$$IND(P) = \{(u,v)\in U\times U \mid \forall x^{(i)}\in P: f(u,x^{(i)})=f(v,x^{(i)})\},$$

and called non-discriminatory relations:

From the definition we have:

$$\text{IND(P)} = \bigcap_{x^{(i)}\in P} IND(x^{(i)}) .$$

RelationIND(P) divide U into equivalence classes,, constitutes a subdivision of U, sign U/IND(P) or simply U/P.

With each u ∈ U, the equivalence class contains u in relation IND(P), sign $[u]_P$is defined as follows:

$$[u]_P = \{v \in U \mid (u,v) \in IND(P)\}.$$

By this definition we see: two elements u,v∈ U belonging to the same equivalence class if and only ifthey have the same value on every index attribute in P.

DefinitionII.6[5]

Letinformation block IB = (U, A, V, f), P, Q⊆ A is the set of index attributes, $U/P = \{P_1, P_2, \dots, P_m\}$, $U/Q = \{Q_1, Q_2, \dots, Q_n\}$ *is the partition generated by P, Q respectively.Then we say partition by Q is more coarse than partition by P, or partitionby P is smoother than partition by Qif and only if:*

$\forall P_i\in U/P, \exists Q_j\in U/Q: P_i\subseteq Q_j,\ i = 1..m, j = 1..n.$

II.4 Decision block

DefinitionII.7[5]

Letinformation block IB = (U,A,V,f) with U is the space of objects, A = $\bigcup_{i=1}^{n}$ *id. Suppose A is divided into two sets C and D such that:* $C=\bigcup_{i=1,x\in id}^{k} x^{(i)}$, $D=\bigcup_{i=k+1,x\in id}^{n} x^{(i)}$,

then information block IB is called the decision block and denoted by DB=(U,C∪D,V,f), with C is the set of conditional index attributes and D is the set of decision index attributes.

From the definition of the decision block, we see: *C∪D=A, C∩D=∅,*

We can denote the decision block simply by: DB=(U, C∪D).

DefinitionII.8[5]:*Let decision blockDB=(U,C∪D,V,f), with C is the set of conditional index attributes and D is the set of decision index attributes. Then the slice of the block decides at x (x∈id) is a tuples of four elements* $DB_x = (U, C^x\cup D^x, V_x, f_x)$ *with U is the set of objects of r,rcalled the space of objects*

$C^x = \bigcup_{i=1}^{k} x^{(i)}$, $D^x = \bigcup_{i=k+1}^{n} x^{(i)}$, $A_x = C^x\cup D^x$,

$V_x = \bigcup_{x^{(i)}\in A_x} V_{x^{(i)}}$,*is the set of values of the objects corresponding to the index attribute* $x^{(i)}$, f_x*is an information function* $U\times A_x \rightarrow V_x$*satisfy:* $\forall u\in U$, $\forall x^{(i)}\in A_x$*we have:* $f(u, x^{(i)})\in V_{x^{(i)}}$.

Comment:

Let decision blockDB=(U,C∪D,V,f). Then, if id = {x}, the decision block DB degenerate into the decision table as known.

When studying the decision block,people want to find the decisive laws from there. In these decision laws, the conditional part corresponds to the conditional indexattribute, the conclusions will correspond to the decision index attributes.

The decision laws found in the decision block are divided into two categories:

i) The lawsare correct on the block.

ii) The laws are correct on each particular slice of the block.

II.5 The decision laws

DefinitionII.9[5]

*Let decision block*DB=(U, C∪D),*with U is the space of objects:*

$C=\bigcup_{i=1,x\in id}^{k} x^{(i)}$, $D=\bigcup_{i=k+1,x\in id}^{n} x^{(i)}$, *and* $C^x=\bigcup_{i=1}^{k} x^{(i)}$, $D^x=\bigcup_{i=k+1}^{n} x^{(i)}$, $x\in id$.

Then:

$U/C=\{C_1,C_2,\dots,C_m\}$, $U/C^x = \{C_{x1},C_{x2},\dots,C_{xt_x}\}$,

$U/D=\{D_1,D_2,\dots,D_k\}$, $U/D^x = \{D_{x1},D_{x2},\dots,D_{xh_x}\}$,

correspondingly, the partitions are generated by C, C^x*, D,* D^x*. A decision law on a block is denoted by:*

$C_i\rightarrow D_j$, $i=1..m$, $j=1..k$,

and on the slice at point x is denoted by:

$C_{xi}\rightarrow D_{xj}$, $i=1..t_x$, $j=1..h_x$.

PropositionII.1 [5]

*Let decision block*DB=(U, C∪D),*with U is the space of objects:*

$C=\bigcup_{i=1,x\in id}^{k} x^{(i)}$, $D=\bigcup_{i=k+1,x\in id}^{n} x^{(i)}$, *and* $C^x=\bigcup_{i=1}^{k} x^{(i)}$, $D^x=\bigcup_{i=k+1}^{n} x^{(i)}$, $x\in id$.

$U/C=\{C_1,C_2,\dots,C_m\}$, $U/C^x = \{C_{x1},C_{x2},\dots,C_{xt_x}\}$,

$U/D=\{D_1,D_2,\dots,D_k\}$, $U/D^x = \{D_{x1},D_{x2},\dots,D_{xh_x}\}$,

Then: $\forall C_i\in U/C$, $\forall D_j\in U/D$ *we have:*

$C_i = \bigcap_{x\in id} C_{xp_x}$, $D_j = \bigcap_{x\in id} D_{xq_x}$ *with* $p_x\in\{1,2,\dots,t_x\}$, $q_x\in\{1,2,\dots,h_x\}$.

DefinitionII.10[5]

*Let decision block*DB=(U,C∪D), C_i∈U/C, D_j∈U/D, C_{xp} ∈U/C^x, D_{xq} ∈U/D^x,$i=1..m$, $j=1..k$, $p\in\{1,2,\dots,t_x\}$, $q\in\{1,2,\dots,h_x\}$, $x\in id$.*Then, support, accuracy and coverage of decision law*$C_i\rightarrow D_j$*on the block are:*

- Support: $Sup(C_i,D_j) = |C_i\cap D_j|$,

- Accuracy: $Acc(C_i,D_j) = \frac{|C_i \cap D_j|}{|C_i|}$,
- Coverage $Cov(C_i,D_j) = \frac{|C_i \cap D_j|}{|D_j|}$,

and for decision law $C_{xp} \to D_{xq}$ on the slice of the block at point x is:

- Support: $Sup(C_{xp},D_{xq}) = |C_{xp} \cap D_{xq}|$,
- Accuracy: $Acc(C_{xp},D_{xq}) = \frac{|C_{xp} \cap D_{xq}|}{|C_{xp}|}$,
- Coverage: $Cov(C_{xp},D_{xq}) = \frac{|C_{xp} \cap D_{xq}|}{|D_{xq}|}$.

From this definition, we have:

$$0 \le Acc(C_i,D_j) \le 1,\ 0 \le Acc(C_{xp},D_{xq}) \le 1,$$

$$\sum_{j=1}^{n} Acc(C_i,D_j)=1, \quad \sum_{q=1}^{h_x} Acc(C_{xp},D_{xq})=1,$$

$$0 \le Cov(C_i,D_j) \le 1,\ 0 \le Cov(C_{xp},D_{xq}) \le 1,$$

$$\sum_{i=1}^{m} Cov(C_i,D_j)=1, \quad \sum_{p=1}^{t_x} Cov(C_{xp},D_{xq})=1.$$

We can represent the measure of the decision laws on the block in the form of the following measurement matrices:

- Matrix of support:

$Sup(C, D) = Sup(C_i,D_j)_{mxk}=$

$$= \begin{pmatrix} Sup(C_1,D_1) & ... & Sup(C_1,D_k) \\ & ... & \\ Sup(C_m,D_1) & ... & Sup(C_m,D_k) \end{pmatrix}$$

- Matrix of Accuracy:

$AccC, D) = Acc(C_i,D_j)_{mxk}=$

$$= \begin{pmatrix} Acc(C_1,D_1) & ... & Acc(C_1,D_k) \\ & ... & \\ Acc(C_m,D_1) & ... & Acc(C_m,D_k) \end{pmatrix}$$

- Matrix of coverage:

$CovC, D) = Cov(C_i,D_j)_{mxk}=$

$$= \begin{pmatrix} Cov(C_1,D_1) & ... & Cov(C_1,D_k) \\ & ... & \\ Cov(C_m,D_1) & ... & Cov(C_m,D_k) \end{pmatrix}$$

With the decision laws on the slices of the blocks, we also have the same support, accuracy, and coverage matrix.

DefinitionII.11[5]

Let decision block $DB=(U,C\cup D)$, $C_i \in U/C$, $D_j \in U/D$ is the conditional equivalence class and decision equivalence class generated by C, Dcorresponding, $C_i \to D_j$ is the decision lawon the block DB, i =1..m, j=1..k.

- *If $Acc(C_i \to D_j) = 1$ then $C_i \to D_j$ is called certain decision law.*
- *If $0 < Acc(C_i \to D_j) < 1$ then $C_i \to D_j$ is called uncertain decision law.*

PropositionII.2 [5]

*Let decision block*DB=(U, C∪D),*with U is the space of objects:*

$$C= \bigcup_{i=1, x\in id}^{k} x^{(i)},\ D= \bigcup_{i=k+1, x\in id}^{n} x^{(i)}.$$

Then $\forall C_i \in U/C$, $\forall D_j \in U/D$, (i =1..m, j=1..n) we have:

i) $Acc(C_i \to D_j) = \frac{Sup(C_i,D_j)}{\sum_{q=1}^{n} Sup(C_i,D_q)}$,

ii) $Cov(C_i \to D_j) = \frac{Sup(C_i,D_j)}{\sum_{p=1}^{m} Sup(C_p,D_j)}$.

DefinitionII.12[5]

Let decision block $DB=(U,C\cup D)$, $C_i \in U/C$, $D_j \in U/D$, i =1..m, j=1..kis the conditional equivalence class and decision equivalence class generated by C,Dcorresponding; α, $\beta$are two given thresholds (α, $\beta \in (0,1)$). If $Acc(C_i,D_j) \ge \alpha$ and $Cov(C_i,D_j) \ge \beta$ then we call $C_i \to D_j$ is the decision lawmeaning.

DefinitionII.13[5]

*Let decision block*DB=(U,C∪D,V,f),*with U is the space of objects, $a \in C\cup D$, V_a is the set of existing values of the index attribute a. Suppose $Z=\{x_s \in U|\ f(x_s,a) = z\}$ is the set of objects whose z value is on the index attribute a. If Z is partitioned into two sets W and Y such that: $Z=W\cup Y$, $W\cap Y=\varnothing$ with $W=\{x_p \in U|\ f(x_p,a) = w,\ w \notin V_a\}$, $Y==\{x_q \in U|\ f(x_q,a) = y,\ y \notin V_a\}$, thenwe say the z value of the index attribute a is smoothed to two new values w and y.*

DefinitionII.14[5]

*Let decision block*DB=(U,C∪D,V,f),*with U is the space of objects, $a \in C\cup D$, V_a is the set of existing values of the index attribute a. Suppose $f(x_p,a)=w$, $f(x_q,a)=y$ are respectively the values of x_p, x_q on the index attribute a ($p\neq q$). If at any one time we have: $f(x_p,a)=f(x_q,a)=z$, ($z \notin V_a$) thenwe say the two values w, y of a are roughened to the new value z.*

Theorem II.1[6]

*Let decision block*DB= (U, C∪D, V, f),*with U is the space of objects, $a \in C\cup D$, V_a is the set of existing values of the index attribute a.Then, two equivalent classes E_p, E_q (E_p, $E_q \in U/E$, $E \in \{C,D\}$) is made rough into new equivalent class E_s if and only if $\forall a_j \neq a$: $f(E_p,a_j) = f(E_q,a_j)$.*

TheoremII.2[6]

*Let decision block*DB= (U, C∪D, V, f),*with U is the space of objects, $a \in C\cup D$, V_a is the set of existing values of the index attribute a. Then, equivalent class E_s ($E_s \in U/E$, $E \in \{C,D\}$) smoothed into two new equivalents classes E_p, E_q if and only if we can put: $f(E_p,a)=w$, $f(E_q,a)=y$ và $E_p \cup E_q= E_s$, $w, y \notin V_a$, $w\neq y$.*

Theorem II.3 [6]

*Let decision block*DB=(U, C∪D),*with U is the space of objects:*

$$C= \bigcup_{i=1, x\in id}^{k} x^{(i)},\ D= \bigcup_{i=k+1, x\in id}^{n} x^{(i)},\ \text{and}\ C^x=\bigcup_{i=1}^{k} x^{(i)},\ D^x= \bigcup_{i=k+1}^{n} x^{(i)},\ x \in id.$$

$U/C=\{C_1,C_2,...,C_m\}$, $U/C^x= \{C_{x1},C_{x2},...,C_{xt_x}\}$,

$U/D=\{D_1,D_2,...,D_k\}$, $U/D^x= \{D_{x1},D_{x2},...,D_{xh_x}\}$,

α, $\beta$are two given thresholds (α, $\beta \in (0,1)$).

Suppose that if $C_i \to D_j$ is the decision lawmeaning on the decision block then it is also the decision lawmeaningon any slice of the decision block at $x \in id$.

III. RESEARCH RESULTS

III.1 Smoothing, roughening the conditional equivalenteclases on the decision block and on the slice.

PropositionIII.1

*Let decision block*DB= (U, C∪D, V, f),$a=x^{(i)}\in C$, V_a*is the set of existing values of the conditional index attribute a, The z value of a is smoothed to two new values w and y.*

$C=\bigcup_{i=1,x\in id}^{k} x^{(i)}$, $D=\bigcup_{i=k+1,x\in id}^{n} x^{(i)}$, and

$C^x=\bigcup_{i=1}^{k} x^{(i)}$, $D^x=\bigcup_{i=k+1}^{n} x^{(i)}$, $x\in id$.

$U/C=\{C_1,C_2,\ldots,C_m\}$, $U/C^x=\{C_{x1},C_{x2},\ldots,C_{xt_x}\}$,

$U/D=\{D_1,D_2,\ldots,D_k\}$, $U/D^x=\{D_{x1},D_{x2},\ldots,D_{xh_x}\}$,

Suppose that if the conditional equivalence class $C_s\in U/C$, ($f(C_s,a)=z$) smoothed into two new conditionalequivalents classesC_p,C_q($f(C_p,a)=w$, $f(C_q,a)=y$, with $w,y\notin V_a$) thenon the slicer_x, exists equivalence class C_{xi}satisfy: $C_s\subseteq C_{xi}$, also smoothed into two new conditional equivalents classes $C_{xi'}$ and $C_{xi''}$ satisfy: $C_p\subseteq C_{xi'}$, $C_q\subseteq C_{xi''}$ ($f(C_{xi'},a)=w$, $f(C_{xi''},a)=y$).

We say on the slice r_x thenC_{xi}*is smoothed sympathetic partiallysmoothed into two new conditionalequivalents classes$C_{xi'}$ and $C_{xi''}$* by the smoothing of C_s*into two new conditionalequivalents classesC_p,C_q.*

Prove

Assuming we have: *$C_s\in U/C$, ($f(C_s,a)=z$) smoothed into two new conditionalequivalents classesC_p,C_q ($f(C_p,a)=w$, $f(C_q,a)=y$,with $w,y\notin V_a$).* Because *$C_s\in U/C$,* applying the results of clause I.1 we have: $C_s=\bigcap_{x\in id} C_{xp_x}$ *,thence inferred* $\exists C_{xi}\in U/C^x$*satisfy*: $C_s\subseteq C_{xi}$. On the other hand, by C_s*smoothed into two conditionalequivalents classesC_p and $C_q$so according to theorem I.2 we have:* $C_s = C_p\cup C_q \Rightarrow C_p, C_q\subseteq C_{xi}$*with $f(C_p,a)=w$, $f(C_q,a)=y$.*

Finally, we assign each element $u\in C_{xi}\setminus C_s$at the index attribute a either w or y thenwe have a subdivision of C_{xi}*into two new conditionalequivalents classes$C_{xi'}$ and $C_{xi''}$satisfy: $f(C_{xi'},a)=w$, $f(C_{xi''},a)=y$and*$C_{xi} = C_{xi'}\cup C_{xi''}$.

The result is on the slice r_x then*theconditionalequivalent class C_{xi}satisfy: $C_s\subseteq C_{xi}$, also smoothed into two conditionalequivalents classes$C_{xi'}$ and $C_{xi''}$ satisfy: $C_p\subseteq C_{xi'}$, $C_q\subseteq C_{xi''}$ ($f(C_{xi'},a)=w$, $f(C_{xi''},a)=y$)and*$C_{xi} = C_{xi'}\cup C_{xi''}$.

PropositionIII.2

Let decision blockDB=(U,C∪D), $a=x^{(i)}\in C$, $V_a$is the set of existing values of the conditional index attribute a, The z value of a is smoothed to two new values w and y.

$C=\bigcup_{i=1,x\in id}^{k} x^{(i)}$, $D=\bigcup_{i=k+1,x\in id}^{n} x^{(i)}$, and $C^x=\bigcup_{i=1}^{k} x^{(i)}$

$D^x=\bigcup_{i=k+1}^{n} x^{(i)}$, $x\in id$

$U/C=\{C_1,C_2,\ldots,C_m\}$, $U/C^x=\{C_{x1},C_{x2},\ldots,C_{xt_x}\}$,

$U/D=\{D_1,D_2,\ldots,D_k\}$, $U/D^x=\{D_{x1},D_{x2},\ldots,D_{xh_x}\}$,

$C_s\in U/C$, $C_{xi}\in U/C^x$, $C_s\subseteq C_{xi}$, $D_{xj}\in U/D^x$, s=1..m, i=1..t_x, j=1..h_x. Suppose that if C_s($f(C_s,a)=z$) smoothed into two conditionalequivalents classes $C_p$andC_q($f(C_p,a)=w$, $f(C_q,a)=y$ andon the slicer_x, C_{xi}*is smoothed sympathetic partially into two new conditionalequivalents classes$C_{xi'}$ and$C_{xi''}$ then:*

i) $C_{xi} = C_{xi'}\cup C_{xi''}$,

ii) $\forall D_{xj}\in U/D^x$: $Sup(C_{xi},D_{xj}) = Sup(C_{xi'},D_{xj}) + Sup(C_{xi''},D_{xj})$, with j=1,2,…,$h_x$.

Prove

i) From the smoothing of the conditional equivalence class C_{xi}we have: $C_{xi}= C_{xi'}\cup C_{xi''}$.

ii) Assuming we have: C_{xi}*is smoothed sympathetic partiallyinto two new conditionalequivalents classes$C_{xi'}$ and$C_{xi''}$*

$\Rightarrow C_{xi}= C_{xi'}\cup C_{xi''}$ and$C_{xi'}\cap C_{xi''} = \varnothing$..

Other way: *$\forall D_{xj}\in U/D^x$: $Sup(C_{xi},D_{xj})=|C_{xi}\cap D_{xj}|$* $= |(C_{xi'}\cup C_{xi''})\cap D_{xj}| = |(C_{xi'}\cap D_{xj})\cup(C_{xi''}\cap D_{xj})|$.

We have: $C_{xi'}\cap C_{xi''}=\varnothing \Rightarrow (C_{xi'}\cap D_{xj})\cap(C_{xi''}\cap D_{xj}) = \varnothing$.

Inferred: *$Sup(C_{xi},D_{xj})$* $=|(C_{xi'}\cap D_{xj})\cup(C_{xi''}\cap D_{xj})| = |(C_{xi'}\cap D_{xj})|+|(C_{xi''}\cap D_{xj})| =$ *$Sup(C_{xi'},D_{xj}) + Sup(C_{xi''},D_{xj})$.*

So we infer: *$\forall D_{xj}\in U/D^x$: $Sup(C_{xi},D_{xj}) = Sup(C_{xi'},D_{xj}) + Sup(C_{xi''},D_{xj})$, with j=1,2,…,$h_x$.*

From this result we see: *row corresponding to theconditionalequivalence classC_{xi}in the support matrix for slice$r_x$will be split into two new lines corresponding to two new conditionalequivalents classes$C_{xi'}$ and$C_{xi''}$.*

Therefore, to calculate the value of the elements of these two new rows in the support matrix with slice $r_x$thenwe first calculate the values $Sup(C_{xi}, D_{xj})$ with j=1,2,…,h_x. From there, we infer the values $Sup(C_{xi''}, D_{xj})$ is the subtraction between $Sup(C_{xi}, D_{xj})$ and $Sup(C_{xi'}, D_{xj})$ with j=1,2,…,h_x.

PropositionIII.3

*Let decision block*DB= (U, C∪D, V, f), *$a=x^{(i)}\in C$, $V_a$is the set of existing values of the conditional index attribute a, the w and y values of a are roughened to the new value z.*

$C=\bigcup_{i=1,x\in id}^{k} x^{(i)}$, $D=\bigcup_{i=k+1,x\in id}^{n} x^{(i)}$, and$C^x=\bigcup_{i=1}^{k} x^{(i)}$, $D^x=\bigcup_{i=k+1}^{n} x^{(i)}$, $x\in id$.

$U/C=\{C_1,C_2,\ldots,C_m\}$, $U/C^x=\{C_{x1},C_{x2},\ldots,C_{xt_x}\}$,

$U/D=\{D_1,D_2,\ldots,D_k\}$, $U/D^x=\{D_{x1},D_{x2},\ldots,D_{xh_x}\}$,

Suppose, if two conditionalequivalents classes$C_p,C_q\in U/C$, ($f(C_p,a)=w$, $f(C_q,a)=y$) is made rough into new conditionalequivalent class $C_s\in U/C$ ($f(C_s,a)=z$) thenon the slice$r_x$exists two conditionalequivalents classesC_{xi}, C_{xj}satisfy: $C_p\subseteq C_{xi}$, $C_q\subseteq C_{xj}$, also is made rough into new conditionalequivalent classC_{xk}satisfy: $C_s\subseteq C_{xk}$.

We say on the slice r_x then*thetwo conditionalequivalents classesC_{xi}, $C_{xj}$is made rough sympathetic into C_{xk}*by the roughening of*two conditionalequivalents classesC_p,C_q to*C_s.

Prove

Assumingwe have: *C_p, $C_q\in U/C$, ($f(C_p,a)=w$, $f(C_q,a)=y$)*, applying the results of proposition I.1 we infer on the slice*$r_x$exists two conditionalequivalents classesC_{xi}, C_{xj}satisfy: $C_p\subseteq C_{xi}$, $C_q\subseteq C_{xj}$*. From there we have: *$f(u,a)=w$ with $u\in C_p\subseteq C_{xi}\Rightarrow f(C_{xi},a)=w$, In the same way we also have: $f(u,a)=y$ with $u\in C_q\subseteq C_{xj}\Rightarrow f(C_{xj},a)=y$.*

On the other hand, assuming we have: *two conditionalequivalents classes$C_p,C_q\in U/C$is made rough into new conditionalequivalent class$C_s\in U/C$,* according to the results of theorem I.1 then we have:

$\forall a_j \neq a,\ a_j \in C$: $f(C_p,a_j) = f(C_q,a_j) \Rightarrow \forall a_j \neq a,\ a_j \in C^x$: $f(C_p,a_j) = f(C_q,a_j)$(1)

In slices $r_x$thenwe have:

$C_p \subseteq C_{xi} \in U/C^x \Rightarrow \forall a_j \neq a,\ a_j \in C^x$: $f(C_p,a_j) = f(C_{xi},a_j)$(2)

Same, we also have:

$C_q \subseteq C_{xj} \in U/C^x \Rightarrow \forall a_j \neq a,\ a_j \in C^x$: $f(C_q,a_j) = f(C_{xj},a_j)$(3)

From (1), (2) and (3) we infer:

$\forall a_j \neq a,\ a_j \in C^x$: $f(C_{xi},a_j) = f(C_{xj},a_j)$.

Therefore, apply the necessary and sufficient conditions in the statement of the theorem I.1, we have*two conditionalequivalents classes C_{xi}, $C_{xj}$is made rough sympathetic into C_{xk}*by the roughening of*two conditionalequivalents classesC_p,C_q to*C_s.

From the nature of the rough work *two conditionalequivalents classesC_{xi}, C_{xj}*toC_{xk} we have:

$$C_{xk} = (C_{xi} \cup C_{xj}) \supseteq (C_p \cup C_q) = C_s.$$

From that: $C_s \subseteq C_{xk}$.

PropositionIII.4

Let decision blockDB=(U,C∪D), $a=x^{(i)} \in C$, $V_a$is the set of existing values of the conditional index attribute a, the w and y values of a are roughened to the new value z

$$C= \bigcup_{i=1,x\in id}^{k} x^{(i)},\ D= \bigcup_{i=k+1,x\in id}^{n} x^{(i)}, \text{ and } C^x=\bigcup_{i=1}^{k} x^{(i)},\ D^x= \bigcup_{i=k+1}^{n} x^{(i)},\ x\in id.$$

$U/C=\{C_1,C_2,...,C_m\}$, $U/C^x= \{C_{x1},C_{x2},...,C_{xt_x}\}$,

$U/D=\{D_1,D_2,...,D_k\}$, $U/D^x= \{D_{x1},D_{x2},...,D_{xh_x}\}$,

C_p, $C_q \in U/C$, ($f(C_p,a)=w$, $f(C_q,a)=y$), $D_{xh} \in U/D^x$, $h=1..h_x$. Suppose, if C_p, C_q is made rough into new conditionalequivalent class C_s,($f(C_s,a)=z$)and on the slice $r_x$two conditionalequivalents classesC_{xi}, C_{xj}($C_p \subseteq C_{xi}$, $C_q \subseteq C_{xj}$)is made rough sympathetic intoC_{xk}then:

i) $C_{xi} \cup C_{xj} = C_{xk}$

ii) $\forall D_{xh} \in U/D^x$: $Sup(C_{xi},D_{xh})+Sup(C_{xj},D_{xh}) = Sup(C_{xk},D_{xh})$, vớih=1,2,...,$h_x$.

Prove

i) Suppose we have: $x \in C_{xi} \cup C_{xj} \Rightarrow x \in C_{xi}$or $x \in C_{xj}$. If $x \in C_{xi}$ thenfrom *thetwo conditionalequivalents classes*C_{xi}, C_{xj}*is made rough into conditionalequivalent class*$C_{xk} \Rightarrow f(x,a) = f(C_{xi},a)=f(C_{xk},a)=z$.

On the other hand, applying the results of theorem 2.1 we have $\forall a_j \neq a$: $f(C_{xi},a_j) = f(C_{xj},a_j)= f(C_{xk},a_j) \Rightarrow f(x,a_j) = f(C_{xi},a_j) = f(C_{xj},a_j)= f(C_{xk},a_j) \Rightarrow x \in C_{xk}$. Totally similar, when $x \in C_{xj}$we also prove that $x \in C_{xk}$.

So inference: $(C_{xi} \cup C_{xj}) \subseteq C_{xk}$. (5)

On the contrary, suppose $x \in C_{xk}$, because $C_{xi}$and C_{xj}*is made rough into* C_{xk}applying the results of theorem 2.1 we have: $\forall a_j \neq a$: $f(C_{xi},a_j)= f(C_{xj},a_j)= f(C_{xk},a_j) \Rightarrow f(x,a_j) = f(C_{xi},a_j) = f(C_{xj},a_j)$. On the other hand, because$x \in C_{xk} \Rightarrow f(x,a)=z$ but z *is made rough from* w and y $\Rightarrow f(x,a)=w$ or $f(x,a)=y$.

- If $f(x,a)=w \Rightarrow f(x,a)=f(C_{xi},a)= w \Rightarrow x \in C_{xi}$.
- If $f(x,a)=y \Rightarrow f(x,a)=f(C_{xj},a)= y \Rightarrow x \in C_{xj}$.

So $x \in C_{xi}$or $x \in C_{xj} \Rightarrow x \in C_{xi} \cup C_{xj}$.

Therefore, from $x \in C_{xk} \Rightarrow x \in C_{xi} \cup C_{xj}$.

So: $C_{xk} \subseteq (C_{xi} \cup C_{xj})$ (6)

Combined (5) and (6) we have: $C_{xi} \cup C_{xj} = C_{xk}$.

ii) BecauseC_{xi}, C_{xj}*arethe conditionalequivalents classes*, so we have:$C_{xi} \cap C_{xj} = \emptyset$.

On the other hand: $\forall D_{xh} \in U/D^x$: $Sup(C_{xk},D_{xh})=|C_{xk} \cap D_{xh}| = |(C_{xi} \cup C_{xj}) \cap D_{xh}| = |(C_{xi} \cap D_{xh}) \cup (C_{xj} \cap D_{xh})|$.

We have: $C_{xi} \cap C_{xj} = \emptyset \Rightarrow (C_{xi} \cap D_{xh}) \cap (C_{xj} \cap D_{xh}) = \emptyset$.

Inferred: $Sup(C_{xk},D_{xh})= |(C_{xi} \cap D_{xh}) \cup (C_{xj} \cap D_{xh})| = |(C_{xi} \cap D_{xh})|+|(C_{xj} \cap D_{xh})| = Sup(C_{xi},D_{xh}) + Sup(C_{xj},D_{xh})$.

So inference: $\forall D_{xh} \in U/D^x$: $Sup(C_{xi},D_{xh})=Sup(C_{xi},D_{xh}) + Sup(C_{xj},D_{xh})$ with $h=1,2,...,h_x$.

Thus, we see two rows of matrix of support on the slice r_x,corresponding to the two conditionalequivalents classesC_{xi}, $C_{xj}$is combined into a new row corresponding to the conditionalequivalent class C_{xk}. The value of each element of the new line corresponds to $C_{xk}$is the total value of two elements of two lines corresponding to C_{xi}andC_{xj}.

III.2 Smoothing, roughening thedecisionequivalenceclassesonthedecision block and ontheslice.

PropositionIII.5

*Let decision block*DB= (U, C∪D, V, f), *$a=x^{(i)} \in D$, $V_a$is the set of existing values of the decision index attribute a, the z value of a is smoothed to two new values w and y.*

$$C= \bigcup_{i=1,x\in id}^{k} x^{(i)},\ D= \bigcup_{i=k+1,x\in id}^{n} x^{(i)}, \text{ and } C^x=\bigcup_{i=1}^{k} x^{(i)},\ D^x= \bigcup_{i=k+1}^{n} x^{(i)},\ x\in id.$$

$U/C=\{C_1,C_2,...,C_m\}$, $U/C^x= \{C_{x1},C_{x2},...,C_{xt_x}\}$,

$U/D=\{D_1,D_2,...,D_k\}$, $U/D^x= \{D_{x1},D_{x2},...,D_{xh_x}\}$,

Suppose that if decision equivalent class$D_s \in U/D$ ($f(D_s,a)=z$) smoothed into two decisionequivalents classesD_p,D_q ($f(D_p,a)=w$, $f(D_q,a)=y$, with$w,y \notin V_a$) thenon the slice r_x, exists decision equivalence classD_{xi}satisfy: $D_s \subseteq D_{xi}$, also smoothed into two new decision equivalents classes$D_{xi'}$ and$D_{xi''}$ satisfy: $D_p \subseteq D_{xi'}$, $D_q \subseteq D_{xi''}$ ($f(D_{xi'},a)=w$, $f(D_{xi''},a)=y$).

We say on the slice r_x then*decision equivalent class*D_{xi}*is smoothed sympathetic partially into two new decisionequivalents classes$D_{xi'}$ and $D_{xi''}$* by the smoothing of D_s*into two new decisionequivalents classesD_p,D_q.*

Proving this clause is similar to the proof of the proposition II.1.

PropositionIII.6

Let decision blockDB=(U,C∪D), $a=x^{(i)} \in D$, $V_a$is the set of existing values of the decision index attribute a, the z value of a is smoothed to two new values w and y.

$$C= \bigcup_{i=1,x\in id}^{k} x^{(i)},\ D= \bigcup_{i=k+1,x\in id}^{n} x^{(i)}, \text{ and } C^x=\bigcup_{i=1}^{k} x^{(i)},\ D^x= \bigcup_{i=k+1}^{n} x^{(i)},\ x\in id.$$

$U/C=\{C_1,C_2,...,C_m\}$, $U/C^x= \{C_{x1},C_{x2},...,C_{xt_x}\}$,

$U/D=\{D_1,D_2,...,D_k\}$, $U/D^x= \{D_{x1},D_{x2},...,D_{xh_x}\}$,

$D_s \in U/D$, $D_{xi} \in U/D^x$, $D_s \subseteq D_{xi}$, $C_{xj} \in U/C^x$, $s=1..k$, $i=1..h_x$, $j=1..t_x$. Suppose that if decision equivalent classD_s($f(D_s,a)=z$) smoothed into two decisionequivalents classesD_p, D_q ($f(D_p,a)=w$, $f(D_q,a)=y$ and on the slice r_x, D_{xi}*is smoothed sympathetic partially into two new*

decisionequivalents classes $D_{xi'}$ *and* $D_{xi''}$ *then:*

i) $D_{xi} = D_{xi'} \cup D_{xi''}$,

ii) $\forall C_{xj} \in U/C^x$: $Sup(C_{xj},D_{xi}) = Sup(C_{xj},D_{xi'}) + Sup(C_{xj},D_{xi''})$, with $j=1,2,\ldots,t_x$.

Prove

i) From the smoothing of the decision equivalent class D_{xi} we see that: $D_{xi} = D_{xi'} \cup D_{xi''}$.

ii) Assuming we have: D_{xi} *is smoothed sympathetic partially into two new decisionequivalents classes* $D_{xi'}$ *and* $D_{xi''}$

$\Rightarrow D_{xi} = D_{xi'} \cup D_{xi''}$ and $D_{xi'} \cap D_{xi''} = \varnothing$..

Other way: $\forall C_{xj} \in U/C^x$: $Sup(C_{xj},D_{xi}) = |C_{xj} \cap D_{xi}| = |C_{xj} \cap (D_{xi'} \cup D_{xi''})| = |(C_{xj} \cap D_{xi'}) \cup (C_{xj} \cap D_{xi''})|$.

We have: $D_{xi'} \cap D_{xi''} = \varnothing \Rightarrow (C_{xj} \cap D_{xi}) \cap (C_{xj} \cap D_{xi''}) = \varnothing$.

Come on: $Sup(C_{xj},D_{xi}) = |(C_{xj} \cap D_{xi'}) \cup (C_{xj} \cap D_{xi''})| = |(C_{xj} \cap D_{xi'})| + |(C_{xj} \cap D_{xi''})| = Sup(C_{xj},D_{xi'}) + Sup(C_{xj},D_{xi''})$.

So we infer: $\forall C_{xj} \in U/C^x$: $Sup(C_{xj},D_{xi}) = Sup(C_{xj},D_{xi'}) + Sup(C_{xj},D_{xi''})$, with $j=1,2,\ldots,t_x$.

From this result we see: *columncorresponding to thedecisionequivalence class* D_{xi} *in the support matrix for slice* r_x *will be split into two new columns corresponding to two new decisionequivalents classes* $D_{xi'}$ and $D_{xi''}$.

Therefore, to calculate the value of the elements of these two new *columns* in the support matrix with slice r_x then we first calculate the values $Sup(C_{xj}, D_{xi})$ with $j=1,2,\ldots,t_x$. From there, we infer the values $Sup(C_{xj}, D_{xi''})$ is the subtraction between $Sup(C_{xj}, D_{xi})$ and $Sup(C_{xj}, D_{xi'})$ with $j=1,2,\ldots,t_x$.

PropositionIII.7

Let decision block DB= (U, C∪D, V, f), $a=x^{(i)} \in D$, V_a *is the set of existing values of the decision index attribute a, the w and y values of a are roughened to the new value z.*

$$C=\bigcup_{i=1,x\in id}^{k} x^{(i)},\ D=\bigcup_{i=k+1,x\in id}^{n} x^{(i)},\ \text{and}\ C^x=\bigcup_{i=1}^{k} x^{(i)},$$

$$D^x=\bigcup_{i=k+1}^{n} x^{(i)},\ x\in id.$$

$U/C=\{C_1,C_2,\ldots,C_m\}$, $U/C^x=\{C_{x1},C_{x2},\ldots,C_{xt_x}\}$,

$U/D=\{D_1,D_2,\ldots,D_k\}$, $U/D^x=\{D_{x1},D_{x2},\ldots,D_{xh_x}\}$,

Suppose, if two decisionequivalents classes D_p,D_q, *(*$f(D_p,a)=w$, $f(D_q,a)=y$*) is made rough into new decisionequivalent class* $D_s \in U/D$ *(* $f(D_s,a)=z$ *) then on the slice* r_x *exists two decisionequivalents classes* D_{xi}, D_{xj} *satisfy:* $D_p \subseteq D_{xi}$, $D_q \subseteq D_{xj}$, *also is made rough into new decision equivalent class* D_{xk} *satisfy:* $D_s \subseteq D_{xk}$.

We say on the slice r_x then*two decisionequivalents classes* D_{xi}, D_{xj} *is made rough sympathetic into* D_{xk} by the roughening of thetwo decision equivalents classes D_p,D_q *todecision equivalent class* D_s.

Proving this clause is similar to the proof of the proposition II.3.

PropositionIII.8

Let decision block $DB=(U,C\cup D)$, $a=x^{(i)} \in D$, V_a *is the set of existing values of the decision index attribute a, the w and y values of a are roughened to the new value z.*

$$C=\bigcup_{i=1,x\in id}^{k} x^{(i)},\ D=\bigcup_{i=k+1,x\in id}^{n} x^{(i)},\ \text{and}\ C^x=\bigcup_{i=1}^{k} x^{(i)},$$

$$D^x=\bigcup_{i=k+1}^{n} x^{(i)},\ x\in id.$$

$U/C=\{C_1,C_2,\ldots,C_m\}$, $U/C^x=\{C_{x1},C_{x2},\ldots,C_{xt_x}\}$,

$U/D=\{D_1,D_2,\ldots,D_k\}$, $U/D^x=\{D_{x1},D_{x2},\ldots,D_{xh_x}\}$,

D_p, $D_q \in U/D$, $(f(D_p,a)=w$, $f(D_q,a)=y)$, $C_{xh} \in U/C^x$, $h=1..t_x$. *Suppose, if two decisionequivalents classes* D_p, D_q *is made rough into new decision equivalent class* D_s, *(* $f(D_s,a)=z$ *) and on the slice* r_x *two decisionequivalents classes* D_{xi}, D_{xj} *(*$D_p \subseteq D_{xi}$, $D_q \subseteq D_{xj}$*)is made rough sympathetic into* D_{xk} *then:*

i) $D_{xi} \cup D_{xj} = D_{xk}$

ii) $\forall C_{xh} \in U/C^x$: $Sup(C_{xh},D_{xi})+Sup(C_{xh},D_{xj}) = Sup(C_{xh},D_{xk})$, with $h=1,2,\ldots,t_x$.

Prove

i) Suppose we have: $u \in D_{xi} \cup D_{xj} \Rightarrow u \in D_{xi}$ or $u \in D_{xj}$. If $u \in D_{xi}$ thenby two decision equivalence classes D_{xi}, D_{xj} *is made rough intodecision equivalent class* $D_{xk} \Rightarrow f(u,a) = f(D_{xi},a)=f(D_{xk},a)=z$.

On the other hand, apply the results of the theorem 2.1 we have $\forall a_r \neq a$: $f(D_{xi},a_r) = f(D_{xj},a_r) = f(D_{xk},a_r) \Rightarrow f(u,a_r) = f(D_{xi},a_r) = f(D_{xj},a_r) = f(D_{xk},a_r) \Rightarrow u \in D_{xk}$. Completely similar, if $u \in D_{xj}$ then we also proved $u \in D_{xk}$.

So inference: $(D_{xi} \cup D_{xj}) \subseteq D_{xk}$. (7)

On the contrary, suppose $u \in D_{xk}$, because D_{xi} and D_{xj} *is made rough into* D_{xk} should apply the results of the theorem 2.1 we have: $\forall a_r \neq a$: $f(D_{xi},a_r) = f(D_{xj},a_r) = f(D_{xk},a_r) \Rightarrow f(u,a_r) = f(D_{xi},a_r) = f(D_{xj},a_r)$. On the other hand, by $u \in D_{xk} \Rightarrow f(u,a)=z$ but z made rough from w and y $\Rightarrow f(u,a)=w$ or $f(u,a)=y$.

- If $f(u,a)=w \Rightarrow f(u,a)=f(D_{xi},a)= w \Rightarrow u \in D_{xi}$.
- If $f(u,a)=y \Rightarrow f(u,a)=f(D_{xj},a)= y \Rightarrow u \in D_{xj}$.

So $u \in D_{xi}$ or $u \in D_{xj} \Rightarrow u \in D_{xi} \cup D_{xj}$.

Therefore, from $u \in D_{xk} \Rightarrow u \in D_{xi} \cup D_{xj}$.

So: $D_{xk} \subseteq (D_{xi} \cup D_{xj})$. (8)

Combined (7) and (8) we have: $D_{xi} \cup D_{xj} = D_{xk}$.

ii) Because D_{xi}, D_{xj} aredecision equivalence classes, so we have: $D_{xi} \cap D_{xj} = \varnothing$.

On the other hand: $\forall C_{xh} \in U/C^x$: $Sup(C_{xh},D_{xk}) = |C_{xh} \cap D_{xk}| = |(D_{xi} \cup D_{xj}) \cap C_{xh}| = |(D_{xi} \cap C_{xh}) \cup (D_{xj} \cap C_{xh})|$.

We have: $D_{xi} \cap D_{xj} = \varnothing \Rightarrow (D_{xi} \cap C_{xh}) \cap (D_{xj} \cap C_{xh}) = \varnothing$.

Inferred: $Sup(C_{xh},D_{xk}) = |(C_{xh} \cap D_{xi}) \cup (C_{xh} \cap D_{xj})| = |(C_{xh} \cap D_{xi})| + |(C_{xh} \cap D_{xj})| = Sup(C_{xh},D_{xi}) + Sup(C_{xh},D_{xj})$.

So inference: $\forall C_{xh} \in U/C^x$: $Sup(C_{xh},D_{xi})+Sup(C_{xh},D_{xj}) = Sup(C_{xh},D_{xk})$, with $h=1,2,\ldots,t_x$.

Thus, we see two columns of the support matrix on the slice r_x corresponds to two decision equivalence classes D_{xi}, D_{xj} *is made rough sympathetic into* a new column corresponding to the decision equivalent class D_{xk}. The value of each element of the new column corresponds to D_{xk} is the total value of two elements of two columns corresponding to two decision equivalence classes D_{xi} and D_{xj}.

IV. CONCLUSIONS

From the initial results on the decision block, the paper proposes and demonstrates some of the results of the relationship between roughing, smoothing the values of conditional attributes or decisionsfor conditionalequivalents classesordecision equivalence classeson the decision blocks and on the slices. The smoothing of conditionalequivalents classes or decision equivalence classes on the decision blockshave a sympathetic partially the smoothing of conditionalequivalents classes or decision equivalence classesrespectively on the slice. The roughening of conditionalequivalents classes or decision equivalence classes on the decision blockshave a sympathetic the roughening of conditionalequivalents classes or decision equivalence classes on the slice. From these results, calculation of support matrix on the slicesame is define as the calculation of the support matrix on the block whenthe smoothing, roughening of conditionalequivalents classes or decision equivalence classes.

In special cases, the index set id = {x},the information blocks degenerate into information systemsthenthese results coincide with the results reported by many authors for the information system. Onthebasis of theseresultswe can studythe reverse relationshipbetweenslices of information block withthat block itself, in case theobjects of theinformation block are changed...,someotherresultsmay be considered in individual cases of information blocks...,itaddsthetheoreticalresults of theexploitation of decision rules oninformation blocks.

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