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RESEARCH PAPER

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A NEW HIGHER ORDER SECOND DERIVATIVE BLENDED BLOCK LINEAR MULTISTEP METHODS FOR THE SOLUTIONS STIFF INITIAL VALUE PROBLEMS

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Abstract: This paper is concerned with the accuracy and efficiency of a higher order second derivative blended block linear multistep method for the approximate solution of stiff initial value problems. The main methods were derived by blending of two linear multistep methods using continuous collocation approach. These methods are of uniform order ten. The stability analysis of the block methods indicates that the methods are A–stable, consistent and zero stable hence convergent. Numerical results obtained using the proposed new block methods were compared with those obtained by the well known ODE solver ODE 15s to illustrate the accuracy and effectiveness. The proposed block method is found to be efficient and accurate hence recommended for the solution of stiff initial value problems.

Keywords: Blended Block, Linear multistep methods, Stiff ODEs, continuous collocation.

INTRODUCTION

Mathematical modeling of many problems in real life, Science, Medicine, Engineering and the like gave rise to systems of linear and non linear Differential Equations. In some cases, the differential equations could be solved analytically while in other case like the Holling Tanner equations and the Van Der Pol equations they are too complicated to be solved by analytical methods. Thus solving such problems becomes an uphill task hence the application of numerical methods for approximate solutions to these differential equations.

In this paper, the application of the nine step order ten blended block linear multistep method for the numerical solutions of stiff initial value problems (1) was considered. A potentially good numerical method for the solution of stiff system of ordinary differential equations (ODEs) must have good accuracy and some wide region of absolute stability. One of the first and most important stability requirements for linear multistep methods is A-stability as proposed by Enright (1974). The nine step blended block linear multistep methods is of a high order and A stable hence the application of the method here which makes it suitable for the solution of non linear ODEs.

The solution of stiff system of ODEs has been considered by Chollom et al (2011) where a block hybrid Adams Moulton Method was used and Kumleng et al (2013) where ten step block generalized Adams method was used. Many has discussed the solution of linear and non linear ODEs from different basis functions, among them are Onumanyi et al(1994), Sirisena et al (2004), Kumleng (2012) and so on.

THE NINE STEP BLENDED LINEAR MULTISTEP METHOD

The nine step blended linear multistep method is constructed based on the continuous finite difference approximation approach using the interpolation and collocation criteria described by Lie and Norsett (1981) called multistep collocation (MC) and block multistep methods by Onumanyi et al. (1994,1999). We define based on the interpolation and collocation methods the continuous form of the k- step 2nd derivative new method as

$$y(x) = \sum_{j=1}^{1} a_j(x) y_{n+j} + h \sum_{j=0}^{m-1} b_j(x) f_{n+j} + h^2 l_k(x) y''_{n+k}$$

$$a_{k-1}(x) = \sum_{i=0}^{t+m-1} a_{j,i+1} x^i \qquad j = 0,1,\ldots,t-1$$

$$b_j(x) = \sum_{i=0}^{t+m-1} b_{j,i+1} x^i, \; j = 0,1,2,\ldots,m-1$$

and

$$l_k(x) = \sum_{i=0}^{t+m-1} l_{k,i+1} x^i, \; j = 0,1,2,\ldots,m-1$$

are the continuous coefficients of the method, m is the number of distinct collocation points, h is the step size and from Onumanyi et- al (1994), we obtain our matrices D and C = D-1 by the imposed conditions expressed as DC=I, where

$$D=\begin{bmatrix} 1 & x_n & x_n^2 & \cdots & x_n^{t+m-1} \\ 1 & x_{n+1} & x_{n+1}^2 & \cdots & x_{n+1}^{t+m-1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{n+k-1} & x_{n+k-1}^2 & \cdots & x_{n+k-1}^{t+m-1} \\ 0 & 1 & 2\bar{x}_0 & \cdots & (t+m-1)\bar{x}_0^{\,t+m-2} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 1 & 2\bar{x}_{m-1} & \cdots & (t+m-1)\bar{x}_{m-1}^{\,t+m-2} \\ 0 & 0 & 2 & \cdots & (t+m-2)(t+m-1)\bar{x}_0^{\,t+m-3} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 2 & \cdots & (t+m-2)(t+m-1)\bar{x}_{m-1}^{\,t+m-3} \end{bmatrix} \qquad (7)$$

$$C=\begin{pmatrix} \alpha_{01} & \alpha_{11} & \cdots & \alpha_{t-1,1} & h\beta_{01} & h_{01} & \cdots & h_{m-1,1} \\ \alpha_{02} & \alpha_{12} & \cdots & \alpha_{t-1,2} & h\beta_{02} & h_{02} & \cdots & h_{m-1,2} \\ . & . & \cdots & . & . & . & . & . \\ . & . & \cdots & . & . & . & . & . \\ . & . & \cdots & . & . & . & . & . \\ \alpha_{0,t+m} & \alpha_{1,t+m} & \cdots & \alpha_{t-1,t+m} & h\beta_{0,t+m} & h_{0,t+m} & \cdots & h_{m-1,t+m} \end{pmatrix}=D^{-1} \qquad (8)$$

respectively. In this case, k=9, t=1 and m=11 and it continuous form expressed in the form of (6) is

$$y(x)=\alpha_8(x)y_{n+8}+h\sum_{j=0}^{m-1}\beta_j(x)f_{n+j}+h^2\lambda_9(x)y''_{n+9} \qquad (9)$$

$$\bar{y}(x_{n+9})=\alpha_8(x)y_{n+8}+h\begin{bmatrix}\beta_0(x)f_n+\beta_1(x)f_{n+1}+\beta_2(x)f_{n+2}+\beta_3(x)f_{n+3}+\beta_4(x)f_{n+4} \\ +\beta_5(x)f_{n+5}+\beta_6(x)f_{n+6}+\beta_7(x)f_{n+7}+\beta_8(x)f_{n+8}+\beta_9(x)f_{n+9}\end{bmatrix}+h^2\lambda_9(x)y''_{n+9} \qquad (3.78)$$

using the approach of Onumanyi et al (1999). The matrix form of

$$D=\begin{bmatrix} 1 & (x_n+8h) & (x_n+8h)^2 & \cdots & (x_n+8h)^{11} \\ 0 & 1 & 2x_n & \cdots & 11x_n^{10} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & 2(x_n+9h) & \cdots & 11(x_n+9h)^{10} \\ 0 & 0 & 2 & \cdots & 110(x_n+9h)^9 \end{bmatrix}$$

Using the Maple software, the inverse of the matrix in (10) is obtained and its elements are used in obtaining the continuous coefficients and substituting these continuous coefficients into (9) yields the continuous form of our new method. The continuous form as:

$$
\begin{aligned}
y(t+x_n) = y_{n+8} &+ \left(\tau - \frac{1861\tau^2}{1260h} + \frac{2677\tau^3}{2240h^2} - \frac{348107\tau^4}{580608h^3} + \frac{71957\tau^5}{362880h^4} - \frac{36929\tau^6}{829440h^5} + \frac{6613\tau^7}{967680h^6} - \frac{547\tau^8}{774144h^7} - \frac{41\tau^9}{870912h^8} + \frac{53\tau^{10}}{29030400h^9} - \frac{53\tau^{11}}{29030400h^{10}} - \frac{127588h}{467775} \right) f_n \\
&+ \left(\frac{36\tau^2}{7h} - \frac{4924\tau^3}{735h^2} - \frac{32887\tau^4}{7840h^3} - \frac{556603\tau^5}{352800h^4} + \frac{23281\tau^6}{60480h^5} - \frac{17663\tau^7}{282240h^6} + \frac{17\tau^8}{2520h^7} - \frac{589\tau^9}{270080h^8} + \frac{13\tau^{10}}{705600h^9} - \frac{\tau^{11}}{3104640h^{10}} - \frac{1884544h}{1091475} \right) f_{n+1} \\
&+ \left(\frac{12\tau^2}{h} + \frac{6184\tau^3}{315h^2} - \frac{47543\tau^4}{3360h^3} + \frac{884761\tau^5}{151200h^4} - \frac{26399\tau^6}{17280h^5} + \frac{117\tau^7}{448h^6} - \frac{337\tau^8}{11520h^7} + \frac{47\tau^9}{22680h^8} - \frac{17\tau^{10}}{201600h^9} + \frac{\tau^{11}}{665280h^{10}} + \frac{88192h}{155925} \right) f_{n+2} \\
&+ \left(\frac{112\tau^2}{5h} - \frac{26416\tau^3}{675h^2} + \frac{32801\tau^4}{1080h^3} - \frac{72301\tau^5}{5400h^4} + \frac{7957\tau^6}{2160h^5} - \frac{33121\tau^7}{50400h^6} + \frac{439\tau^8}{5760h^7} - \frac{\tau^9}{180h^8} + \frac{\tau^{10}}{4320h^9} + \frac{\tau^{11}}{237600h^{10}} - \frac{22016h}{5775} \right) f_{n+3} \\
&+ \left(\frac{63t^2}{2h} + \frac{3407t^3}{60h^2} - \frac{58651t^4}{1280h^3} + \frac{303169t^5}{14400h^4} - \frac{416401t^6}{69120h^5} + \frac{89777t^7}{80640h^6} - \frac{6137t^8}{46080h^7} + \frac{517t^9}{51840h^8} - \frac{49t^{10}}{115200h^9} + \frac{t^{11}}{126720h^{10}} + \frac{17936h}{6237} \right) f_{n+4} \\
&+ \left(\frac{168t^2}{5h} - \frac{2776t^3}{45h^2} + \frac{2447t^4}{48h^3} - \frac{52003t^5}{2160h^4} + \frac{30637t^6}{4320h^5} - \frac{27191t^7}{20160h^6} + \frac{191t^8}{1152h^7} - \frac{11t^9}{864h^8} + \frac{t^{10}}{1800h^9} - \frac{t^{11}}{95040h^{10}} - \frac{264448h}{51975} \right) f_{n+5} \\
&+ \left(\frac{28t^2}{h} + \frac{7024t^3}{135h^2} - \frac{188741t^4}{4320h^3} + \frac{454351t^5}{21600h^4} - \frac{109421t^6}{17280h^5} + \frac{24827t^7}{20160h^6} - \frac{1787t^8}{11520h^7} + \frac{79t^9}{6480h^8} - \frac{47t^{10}}{86400h^9} + \frac{t^{11}}{95040h^{10}} + \frac{372352h}{155925} \right) f_{n+6} \\
&+ \left(\frac{144t^2}{7h} - \frac{4048t^3}{105h^2} + \frac{9169t^4}{280h^3} - \frac{201493t^5}{12600h^4} + \frac{10573t^6}{2160h^5} - \frac{1957t^7}{2016h^6} + \frac{5027t^8}{40320h^7} - \frac{227t^9}{22680h^8} + \frac{23t^{10}}{50400h^9} - \frac{t^{11}}{110880h^{10}} - \frac{543232h}{155925} \right) f_{n+7} \\
&+ \left(\frac{1161t^2}{140h} + \frac{1222513t^3}{78400h^2} - \frac{3337891t^4}{250880h^3} + \frac{3073541t^5}{470400h^4} - \frac{3896567t^6}{1935360h^5} + \frac{4542079t^7}{11289600h^6} - \frac{67231t^8}{1290240h^7} + \frac{42883t^9}{10160640h^8} - \frac{877t^{10}}{4515840h^9} - \frac{481t^{11}}{124185600h^{10}} + \frac{549644h}{1091475} \right) f_{n+8} \\
&+ \left(\frac{4t^2}{9h} + \frac{796t^3}{945h^2} - \frac{65911t^4}{90720h^3} + \frac{164099t^5}{453600h^4} - \frac{2939t^6}{25920h^5} + \frac{2797t^7}{120960h^6} - \frac{53t^8}{17280h^7} + \frac{139t^9}{544320h^8} - \frac{11t^{10}}{907200h^9} - \frac{t^{11}}{3991680h^{10}} + \frac{4736h}{93555} \right) f_{n+9} \\
&+ \left(\frac{9}{2}t^2 - \frac{7129t^3}{840h} - \frac{6515t^4}{896h^2} - \frac{4523t^5}{1260h^3} + \frac{285t^6}{256h^4} - \frac{3013t^7}{13440h^5} + \frac{15t^8}{512h^7} - \frac{29t^9}{12096h^8} + \frac{t^{10}}{8960h^8} - \frac{t^{11}}{443520h^{10}} - \frac{4736h^2}{10395} \right) g_{n+9}
\end{aligned}
\qquad (11)
$$

Evaluating the continues scheme (11) at $t = 0, h, 2h, 3h, 4h, 5h, 6h, 7h, 9h$

gives the nine discrete methods which constitute the nine step blended block linear multistep method.

$$
\begin{aligned}
y_n - y_{n+8} &- \frac{127588}{467775} hf_n - \frac{1884544}{1091475} hf_{n+1} + \frac{88192}{155925} hf_{n+2} - \frac{22016}{5775} hf_{n+3} + \frac{17936}{6237} hf_{n+4} - \frac{264448}{51975} hf_{n+5} \\
&+ \frac{28184}{155925} hf_{n+6} - \frac{543232}{155925} hf_{n+7} + \frac{549644}{1091475} hf_{n+8} + \frac{4736}{93555} hf_{n+9} - \frac{4736}{10395} h^2 y''_{n+9}
\end{aligned}
$$

$$
\begin{aligned}
y_{n+1} = y_{n+8} &+ \frac{1570597}{273715200} hf_n - \frac{60109}{178200} hf_{n+1} - \frac{1590491}{1140480} hf_{n+2} - \frac{88837}{190080} hf_{n+3} - \frac{37482011}{22809600} hf_{n+4} + \frac{357749}{950400} hf_{n+5} \\
&- \frac{644317}{5702400} hf_{n+6} - \frac{76409}{114048} hf_{n+7} - \frac{11422159}{18247680} hf_{n+8} - \frac{79919}{8553600} hf_{n+9} + \frac{117943}{760320} h^2 y''_{n+9}
\end{aligned}
$$

$$
\begin{aligned}
y_{n+2} = y_{n+8} &- \frac{19}{246400}hf_n + \frac{387}{26950}hf_{n+1} \\
&- \frac{734}{1925}hf_{n+2} - \frac{2424}{1925}hf_{n+3} \\
&- \frac{23013}{30800}hf_{n+4} - \frac{95}{77}hf_{n+5} \\
&- \frac{1563}{1925}hf_{n+6} - \frac{2196}{1925}hf_{n+7} \\
&- \frac{379163}{862400}hf_{n+8} - \frac{1}{3850}hf_{n+9} \\
&+ \frac{171}{3080}h^2 y''_{n+9}
\end{aligned}
$$

$$
\begin{aligned}
y_{n+3} = y_{n+8} &+ \frac{18875}{76640256}hf_n - \frac{625}{17436}hf_{n+1} \\
&- \frac{625}{228096}hf_{n+2} - \frac{12655}{29568}hf_{n+3} \\
&- \frac{7397125}{6386688}hf_{n+4} - \frac{240875}{266112}hf_{n+5} \\
&- \frac{1671875}{1596672}hf_{n+6} - \frac{785375}{798336}hf_{n+7} \\
&- \frac{89540755}{1788264}hf_{n+8} - \frac{6625}{2395008}hf_{n+9} \\
&- \frac{93325}{1064448}h^2 y''_{n+9}
\end{aligned}
$$

$$
\begin{aligned}
y_{n+4} = y_{n+8} &- \frac{17}{267300}hf_n + \frac{40}{43659}hf_{n+1} \\
&- \frac{1048}{155925}hf_{n+2} + \frac{1952}{51975}hf_{n+3} \\
&- \frac{68939}{155925}hf_{n+4} - \frac{59536}{51975}hf_{n+5} \\
&- \frac{28184}{31185}hf_{n+6} - \frac{167072}{155925}hf_{n+7} \\
&+ \frac{2041553}{4365900}hf_{n+8} - \frac{568}{467775}hf_{n+9} \\
&+ \frac{731}{10395}h^2 y''_{n+9}
\end{aligned}
$$

$$
\begin{aligned}
y_{n+5} = y_{n+8} &- \frac{689}{7884800}hf_n - \frac{117}{107800}hf_{n+1} \\
&+ \frac{3167}{492800}hf_{n+2} - \frac{6141}{246400}hf_{n+3} \\
&+ \frac{6219}{7884800}hf_{n+4} - \frac{125897}{246400}hf_{n+5} \\
&- \frac{520923}{492800}hf_{n+6} - \frac{244791}{246400}hf_{n+7} \\
&+ \frac{27347371}{55193600}hf_{n+8} - \frac{17}{7040}hf_{n+9} \\
&+ \frac{16659}{197120}h^2 y''_{n+9}
\end{aligned}
$$

$$
\begin{aligned}
y_{n+6} = y_{n+8} &- \frac{589}{29937600}hf_n + \frac{571}{2182950}hf_{n+1} \\
&- \frac{52}{31185}hf_{n+2} + \frac{8}{1155}hf_{n+3} \\
&- \frac{55093}{2494800}hf_{n+4} + \frac{3419}{51975}hf_{n+5} \\
&- \frac{74849}{155925}hf_{n+6} - \frac{988}{891}hf_{n+7} \\
&- \frac{6115919}{1390880}hf_{n+8} - \frac{997}{935550}hf_{n+9} \\
&+ \frac{5609}{83160}h^2 y''_{n+9}
\end{aligned}
$$

$$
\begin{aligned}
y_{n+7} = y_{n+8} &- \frac{30887}{38201280}hf_n - \frac{4181}{4365900}hf_{n+1} \\
&+ \frac{211151}{39916800}hf_{n+2} - \frac{120719}{6652800}hf_{n+3} \\
&+ \frac{7012483}{159667200}hf_{n+4} - \frac{21995}{266112}hf_{n+5} \\
&- \frac{5632757}{39916800}hf_{n+6} \\
&- \frac{10946503}{19958400}hf_{n+7} \\
&- \frac{2400779867}{4470681600}hf_{n+8} \\
&- \frac{190073}{59875200}hf_{n+9} \\
&+ \frac{530113}{5322240}h^2 y''_{n+9}
\end{aligned}
$$

$$
\begin{aligned}
y_{n+9} = y_{n+8} &- \frac{1129981}{191600640}hf_n - \frac{2395}{349272}hf_{n+1} \\
&- \frac{1468913}{39916800}hf_{n+2} + \frac{89609}{739200}hf_{n+3} \\
&- \frac{44073661}{159667200}hf_{n+4} \\
&+ \frac{3117517}{6652800}hf_{n+5} - \frac{5150479}{7983360}hf_{n+6} \\
&+ \frac{17760409}{19958400}hf_{n+7} \\
&+ \frac{1130650853}{4470681600}hf_{n+8} \\
&+ \frac{13119671}{59875200}hf_{n+9} \\
&- \frac{3250433}{5322240}h^2 y''_{n+9}
\end{aligned}
$$

STABILITY ANALYSIS OF THE NEW METHODS

In this section, we consider the analysis of the newly constructed methods. Their convergence is determined and their regions of absolute stability plotted.

Convergence

The convergence of the new block methods is determined using the approach by Fatunla (1991) and Chollom *et.al* (2007) for linear multistep methods, where the block methods are represented in a single block, r point multistep method of the form

$$\rho(r) = \sum_{i=0}^{k} \alpha_j r^j \qquad (4.1)$$

Zero Stability of the BBLMM for k=9

To determine the zero stability of the BBLMM we use the approach of Ehigie (2007) for linear multistep methods where he expressed the methods in the matrix form as shown below.

Following the work of Ehigie and Okunuga (2014), we observed that the seven step block method is zero stable as the roots of the equation $det\left(r\left(A - Cz - D1z^2\right) - B\right) = 0$ are less than or equal to 1. Since the block method is consistent and zero-stable, the method is convergent (Henrici 1962).

These new methods are consistent since their orders are 11, it is also zero-stable, above all, there are A – stable as can be seen in figure 1. The new ten step discrete methods that constitute the block method have the following orders and error constants as shown below.

The nine step blended block multistep methods has uniform order of $(10,10,10,10,10,10,10,10,10)^T$ and error constants of

$$C_{12}=\left(\frac{-855}{100000},\frac{285}{1000000},\frac{-934}{1000000},\frac{-293}{100000},\frac{-514}{1000000},\frac{352}{10000000},\frac{645}{1000000},\frac{788}{1000000},\frac{-801}{1000000}\right)^T$$

REGIONS OF ABSOLUTE STABILITY OF THE METHODS

The absolute stability regions of the newly constructed blended block linear multistep methods (8) and (12) are plotted using Ehigie (2007) by reformulating the methods into a characteristic equation of the form

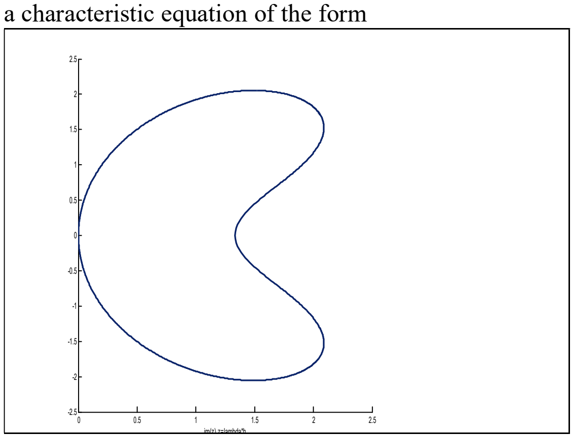

Fig 1: Absolute Stability Region For BBLMM For K=9.

This absolute stability region is A –stable since it consist of the set of points in the complex plane outside the enclosed figure.

NUMERICAL EXAMPLES

We report here a numerical example on stiff problem taken from the literature using the solution curve. In comparison, we also report the performance of the new blended block linear multistep methods and the well-known Matlab stiff ODE solver ODE15S on the same problems and on the same axes.

Problem1 Oregonator (Chemical Reaction) Problem

The oregonator chemical reaction model is a theoretical model of autocatalytic reaction.

It is a chemical dynamics of the oscillatory reaction. It is a reaction between $HBrO_2, Br^-$ and $Ce(IV)$ described by Noyes and Field (1974). The Oregonator model is expressed mathematically by the following

$$y_1' = 77.27[y_2 + y_1(1 - 8.375\times 10^{-6}y_1 - y_2)]$$

$$y_2' = \frac{1}{77.27}[y_3 - (1+y_1)y_2]$$

$$y_3' = 0.161(y_1 - y_2)$$

$$y_1(0) = 0, y_2(0) = 1, y_3(0) = 1$$

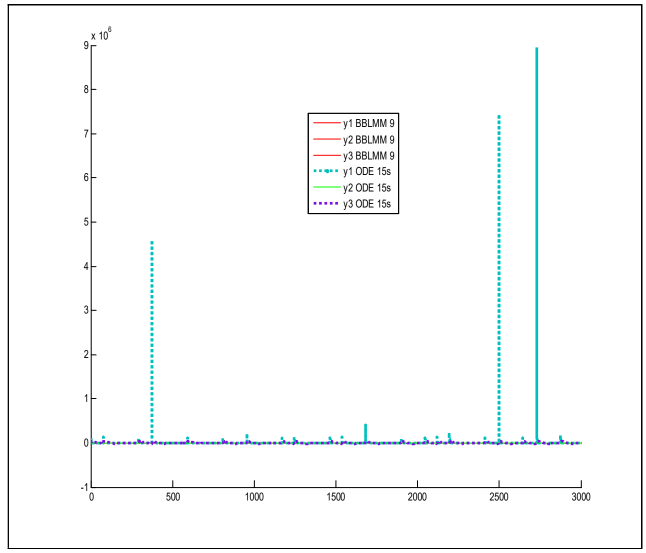

Fig 2 :Solution Curve Of The Problem 1 Computed By Nine Step BBLMM

Problem 2: Irregular Heartbeat and Lidocaine Model

The irregular heartbeat and Lidocaine model is expressed mathematically by the following

$$y_1' = -0.09\,y_1 + 0.038\,y_2$$

$$y_2' = 0.066\,y_1 - 0.038\,y_2$$

$$y_1(0) = y_2(0) = y_0$$

$$y_0 = \text{Maximum Safe Dosage} = 3\text{mw}/$$

$$0 \le x \le 700, \quad h = 0.1$$

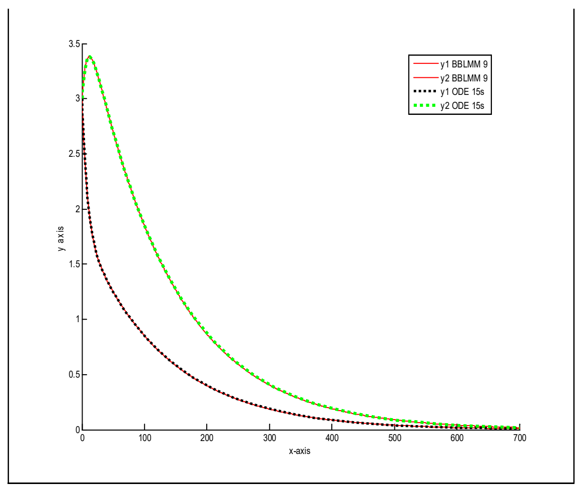

Fig 3 : Solution Curve Of The Problem 2 Computed By Nine Step BBLMM

CONCLUSION

Problem 1 which is a famous chemical reaction with periodic solutions and a highly stiff Ordinary Differential Equation whose solutions change rapidly over many orders of magnitudes. The solution curves in Figures 3 plotted within the range of $0 \le x \le 7000$ with step size of 0.1 shows active oscillatory regime for different values. The solution curves using the BBLMM compare favourably with results obtained using the variable step size code ODE (15s).

Problem 2 is a model of the relationship between Lidocaine and Irregular Heart beat. Lidocaine belong to a group of drugs known as anti-arrhythmic which work by preventing sodium from being pumped out on the cells of the heart to help the heart beat normally. From our solution curves, it was observed that normalcy in the heart beat can be attained with the use of Lidocaine within the correct dosage. Our solution curves coincide with the solutions of ODE 15s.

The numerical results from figures 2 and 3 reveal the accuracy of the newly constructed higher order blended block linear multistep methods (BBLMM) for step numbers 9. It can be seen clearly from the curve that our new methods perform favourably better than the well known ODE15S for the problems solved in problem 1and 2. It was also observed that the new methods have better stability regions than the conventional Adams Moulton method for step number 9.

RECOMMENDATIONS

This method is recommended for the solution of stiff system of ODEs since they are A-stable which implies a wider range of stability for effective performance.

REFERENCES

[1]. Butcher, J.C., (1966). On the Convergence of Numerical Solutions to Ordinary Differential Equations. *Math. Comp.* **20**, 1 -10.

[2]. Chollom J.P., (2005): A study of Block Hybrid Methods with link of two-step Runge Kutta Methods for first order Ordinary Differential Equations. PhD Thesis (Unpublished) University of Jos,Jos Nigeria.

[3]. Chollom J.P., Ndam, J.N.and Kumleng G.M., (2007):some properties of block linear multistep methods. Science world journal, 2(3), 11-17.

[4]. Chollom J.P., Olatunbusin I.O. and Omagwu S., (2012): A Class of A-Stable Block Explicit Methods for the Solution of Ordinary Differential Equations. *Research Journal of Mathematics and Statistics*, **4**(2), 52-56.

[5]. Ehigie, J.O., Okunuga, S.A., Sofoluwe, A.B. & Akanbi, M. A. (2010). Generalized 2-step Continuous Linear Multistep Method of Hybrid Type for the Integration of Second Order Ordinary Differential Equations. *Scholars Research Library (Archives of Applied Science Research)*, *2*(6), 362–372.

[6]. Enright, W. H. (1972). Numerical Solution of Stiff Differential Equations (pp. 321–331). *Dept of Computer Science, University of Toronto, Toronto, Canada,.*

[7] Enright, W.H (1974). Second Derivative Multistep Methods for Stiff Ordinary Differential Equations. SIAM Journals on Numerical Analysis, 11 (2), 376-391.

[8] Ezzeddine, A.K. and Hojjati, G., (2012). Third derivative multistep methods for stiff systems,International journal of nonlinear science, 14, 443-450.

[9] Gamal, A.F., Ismail, K. & Iman, H. I. (1999). A New Efficient Second Derivative multistep Method for Stiff System. *Applied Mathematical Modeling*, *23*, 279–288.

[10] Henrici, P. (1962). *Discrete variable methods in Ordinary Differential Equations* (p. 407).John Willey, New York.

[11]. Kumleng, G.M., Sirisena, U.W., & Chollon, J. P. (2012). A Class of A-Stable Order Four and Six linear Multistep Methods for Stiff Initial Value Problems. *Mathematical Theory and Modeling*, *3*(11), 94–102.

[12]. Kumleng, G.M., Sirisena, U.W.W, Dang, B. C. (2013). A Ten Step Block Generalized Adams Method for the Solution of the Holling Tanner and Lorenz Models. *African Journal of Natural Sciences*, *16*, 63–70.

[13]. Lie, I. & Norset, R. (1989). Super convergence for multistep collocation. *Mathematics of Computation*, *52*(185), 65–79.

[14].Lotka a. j. (1925) element of physical biology. Baltimore, Williams and wilkins company

[15]. Mehdizadeh, M., Khalsarai, N., Nasehi, O. & Hojjati, G. (2012). A Class of Second Derivative multistep methods for stiff systems. *Journal of Acta*, *15*(2012), 209–222.

[16]. Onumanyi, P., Awoyemi, D.O., Jator, S.N.& Sirisena, U. W. (1994). New Linear Multistep Methods with continuous coefficients for first order IVPs. *Journal of NMS*, *31*(1), 37–51.

[17]. Onumanyi, P., Sirisena, U.W. & Jator, S. N. (1999). Continuous finite difference approximations for solving differential equations. *International Journal of Computer and Mathematics*, *72*(1), 15–27.

[18]. Rosenzweng M. L. and MacArthur R. H. (1963) Graphical representatation and stability conditions of pradator-prey interactions. A.M. Nat. 97, 209-223

[19]. Sahi, R.K., Jator, S.N. & Khan, N. A. (2012). A Simpson's Type Second Derivative Method for Stiff Systems. *International Journal of Pure and Applied Mathematics*, *81*(4), 619–633.

