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Limiting Behavior of The Queuing System M/G/1

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***Abstract*—** In a bulk service queuing system sometimes customers and servers wait for each other. Such a system is generally called a double-ended queue. We have discussed a problem of bulk queuing system. In this paper, supplementary variable technique has been used in solving M/G/1 and G1/M/1 queuing system. The probability of n unit in the limiting behavior of the queuing system M/G/1.

Keywords- Supplementary variable technique , queuing system, limiting behavior

I. INTRODUCTION

Supplementary variable technique was first used by Kosten (1973), Cox (1955) used a supplementary variable technique to study the queuing system M/G/1. Kendall (1953) considered this technique but preferred to imbedded Markov chain as leading to simpler calculation. In spite of this, supplementary variables have been used by many authors to solve a good number of queuing problems. Jaiswal (1968) makes heavy use of supplementary variables on priority queues, while Kendall's imbedded Markov chain technique is very powerful and elegant, it gives only approximate solutions to queuing problems considered in continuous time. However, in the steady-state case, many problems can be solved by the supplementary variable technique than by the imbedded Markov chain technique.

II. SUPPLEMENTARY VARIABLE TECHNIQUE

In the supplementary variable technique, a non Markovian process in continuous time is made Markovian by the inclusion of one or more supplementary variables. We first consider the queuing system [1]-[5] M/G/1. In M/G/1, the service-time distribution is general [6]-[7]. Let the state of system be defined by a pair of variables, the number N(t) in the system [or $N_{q(t)}$ in the queue] at epoch t, and the elapsed service-time $x(t)$ of the customer who is undergoing service. Then we may study the bivariate Markovian process $\{N(t), x(t)\}$ in order to obtain results for the non-Markovian process $\{N(t)\}$. Thus Cox defines P_n (v, t) to be the joint probability and probability density function of n, the number of customers in the system, including the one being served, and v the elapsed service time of the customer in service. The inclusion of single supplementary variable makes the process Markovian in continuous time. The equation of the process can be written by considering in the usual way, the transitions occurring in the time Δt. In this case $\eta(v)dv$ is taken as the probability that the service is completed in the interval $[v, v + dv]$, conditional upon its remaining in complete upto time v.

III. LIMITING BEHAVIOR OF THE QUEUING SYSTEM M/G/1

In M/G/1 queuing system [8], customers arrive following a Poisson process with mean rate λ and are served one at a time on a First-come-First-served basis. Service times are independently and identically distributed random variables whose probability density function is

$$b(v) = \eta(v) exp\left[-\int_o^v \eta(x)dx\right]$$

With mean

$\mu, 0 < 1/\mu = \int_0^\infty vb(v)dv < \infty$

and the corresponding density function is given by

$$exp\left[\int_0^v \eta(x)dx\right] = 1 - B(v) \qquad \dots (1)$$

We define,

$P_n(x, t)dt + 0(dx), n > 0$

as the probability that at time t there are n customers in the system with the elapsed service time of the customers undergoing service lying between x and x+dx. P0(t) as the probability that the system is empty as time t. Since {N (t), x (t)} is Markovian in continuous time. The equations of the process in the usual Erlangian procedure can be written by considering the transitions occurring in dt.

We see that

$P_0(x, t) \equiv 0$

or in the equilibrium Case

$P_0(x) \equiv 0$

Then considering the empty state,

$$P_0(t+\Delta t) = p_0(t)(1-\lambda\Delta t) + (1-\lambda\Delta t)\int_0^\infty P_1(x,t)\eta(x)dx\Delta t + 0(\Delta t),$$

So that,

$$\frac{dP_0(t)}{2a} = -\lambda P_1(t) + \int_0^\infty P_1(x,t)\eta(x)dx$$

Similarly, we get

$$P_n(x+\Delta t, t+\Delta t) = (1-\lambda\Delta t)(x,t)[1-\eta(x)\Delta t] + \lambda\Delta t P_{n-1}(x,t)[1-\eta(x)\Delta t] + 0(\Delta t)$$

or

$$\frac{P_n(x+\Delta t, t+\Delta t) - P_n(x+\Delta t, t)}{\Delta t} + \frac{P_n(x+\Delta t, t) - P_n(x,t)}{\Delta x} = -[\lambda+\eta(x)]P_n(x,t) + \lambda P_{n-1}(x,t) + \frac{0(\Delta t)}{\Delta t}$$

Taking limit when

$\Delta t \longrightarrow 0$

we get

$$\frac{\partial P_n(x,t)}{\partial t} + \frac{\partial P_n(x,t)}{\partial x} = -[\lambda+\eta(x)]P_n(x,t) + \lambda P_{n-1}(x,t)$$

An equilibrium solution exists when

$\rho = \frac{\lambda}{\pi} < 1$

In this case we can put the limiting derivations with respect to t equal to zero, and by letting

$\lim_{t\to 0} P_n(x,t) = P_n(x)$

and

$\lim_{t\to\infty} P_0(t) \equiv P_0$

we get

$$0 = -\lambda P_0 + \int_0^\infty P_1(x)\eta(x)dx \qquad ...(2)$$

$$\frac{\partial P_n(x)}{\partial x} = -[\lambda+\eta(x)]P_n(x) + \lambda P_{n-1}(x), n \geq 1 \qquad ...(3)$$

These equations are to be solved under the boundary conditions

$$P_n(0) = -\int_0^\infty P_{n+1}(x)\eta(x)dx, n > 1 \qquad ...(4)$$

$$P_1(0) = -\int_0^\infty P_2(x)\eta(x)dx, n + \lambda P_0 \qquad ...(5)$$

and the normalizing condition

$$P_0 + \sum_{n=1}^\infty \int_0^\infty P_n(x)dx = 1 \qquad ...(6)$$

Now the solution can be obtained in closed form with the help of a probability generating function,

$P_0(z;x)$,

Which we define as

$$P_0(z;x) = \sum_{n=1}^\infty P_n(x)z^n \qquad ...(7)$$

Then

$$\frac{\partial P_0(z;x)}{\partial x} = \sum_{n=1}^\infty \frac{\partial P_n(x)}{\partial x} z^n \qquad ...(8)$$

and

$$P_0(z;0) = \sum_{n=1}^\infty P_0(0)z^n \qquad ...(9)$$

We multiply equation (3) by

z^n

and add from n=1 to ∞, then we get

$$\frac{\partial}{\partial x}\sum_{n=1}^\infty P_n(x)z^n = -[\lambda+\eta(x)]\sum_{n=1}^\infty P_n(x)z^n + \lambda\sum_{n=1}^\infty P_{n-1}z^n$$

Using equation (7) and (8), we get

$$\frac{\partial}{\partial x}P_0(z;x) = [\lambda z - \lambda - \eta(x)]P_0(z;x) \qquad ...(10)$$

Similarly from equation (2), (4), (5) and (9), we get

$$zP_0(z;0) = \int_0^\infty P_0(z;x)\eta(x)dx + \lambda z(z-1)P_0 \qquad ...(11)$$

The differential equation (10) on solution we get

$$P_0(z;x) = A\exp\left[\int_0^x [\lambda(z-1) - \eta(y)]dy\right]$$

$$= A\exp[\lambda(z-1)x]\exp\left[-\int_0^x \eta(y)dy\right]$$

$$= A\exp[\lambda(z-1)x][1-B(x)]$$

Where A is constant to be determined from the boundary conditions, the boundary condition, when x=0, we get

$A = P_0(z;0)$

Consequently,

$$P_0(z,x) = P_0(z,0)[1-B(x)]exp[\lambda(z-1)x] \qquad ...(12)$$

Substituting this in equation (11) and simplifying, we get

$$P_0(z,0) = \frac{\lambda z(z-1)P_0}{z - b(\lambda - \lambda z)}$$

Where b(α) is the Laplace Transform of b(v), that is,

$$\mathfrak{b}(\alpha) = \int_0^\infty e^{-av}\, b(v)dv, \quad Re\,\infty > 0.$$

Thus,

$$P_0(z) = \int_0^\infty P_0\,(z,x)dx$$

$$= p_o(z,0)\int_0^\infty e^{-y(1-z)x}[1-B(x)]dx$$

$$= zP_0\frac{\mathfrak{b}(\lambda-\lambda z)-1}{z-\mathfrak{b}(\lambda-\lambda z)}$$

where

$$\int_0^\infty e^{-av}\,[1-B(v)]dv = \frac{1-\mathfrak{b}(\alpha)}{\alpha}$$

So the complete probability generating function of $P_0, n = 0,1,2,\ldots\ldots..$ is $P(z)$, given by

$$P(z) \equiv P_0(z) + P_0$$

$$= \frac{(z-1)\mathfrak{b}(\lambda-\lambda z)P_0}{z-\mathfrak{b}(\lambda-\lambda z)} \qquad \ldots(13)$$

The value of P0 which may be determined by considering the idle time of the server given by

$$P_0 = 1-\rho$$

This completely determines the probability generating function P(Z).

$$\mathfrak{b}(\lambda-\lambda z)$$

The value can be interpreted as the probability generating function of the number of arrivals a service time V of a customer. Assume

$$K_0 = P\,(n\ arriaval\ during\ V)$$

$$= \int_0^\infty P\left(n\ arrival\ during\frac{v}{v}=x\right)b(x)dx$$

IV. CONCLUSION

The theory of Queuing is sufficiently developed so that any of these complications could be included in the model, but the resulting formulae would be complicated and would depend on additional parameters that would be hard to evaluate. In such situations, it is advised to start off with simple models and introduce complications one by one, until sufficient accuracy is obtained.

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