DOI: http://dx.doi.org/10.26483/ijarcs.v9i3.6125

ISSN No. 0976-5697

Volume 9, No. 3, May-June 2018

International Journal of Advanced Research in Computer Science

RESEARCH PAPER

Available Online at www.ijarcs.info

SOME FIXED POINT THEOREMS IN FUZZY METRIC SPACE FOR EXPANSION MAPPINGS

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Abstract: In this paper some fixed point theorems for expansion mappings in fuzzy metric spaces are proved under different conditions.

AMS Subject Classification: 47 H10, 54H25

Keywords: Fixed point, Common fixed point, Expansive mapping , Complete Fuzzy metric space.

1. INTRODUCTION

After Zadeh's Pioneering paper [7], where the theory of Fuzzy Sets was introduced, hundreds of examples have been supplied where the nature of uncertainty in the behavior of a given system possesses fuzzy rather than stochastic nature non-stationary fuzzy system described by fuzzy possesses look as their natural extension into the time domina. Since then to use concept of fuzzy in topology and analysis, many authors have expansively developed the theory of fuzzy sets and its applications. Notable are Wang, Gao, Isekey [6], Popa [5], Jain and Jain [4], worked on expansion mappings in metric space. Recently, Agrawal and Chouhan [1] & [2], Bhardwaj, Rajput and Yadava [3] did lot of worke for common fixed point for expansion mapping.

Our object in this paper is, to obtain some result on common fixed point theorems of expansion type's maps on fuzzy metric space, which generalized the result of Wang, Gao, Isekey [6] for metric space.

2. PRILIMINARIES

Definition 2.1: A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if $([0,1],*)$ is an abelian Topological monodies with unit 1 such that

$a * b \geq c * d$ whenever $a \geq c$ and $b \geq d$ for all $a, b, c, d, \in [0,1]$

Example of t-norm are $a * b = a\, b$ and $a * b = min\,\{a, b\}$

Definition 2.2: The 3-tuple $(X, M,*)$ is called a fuzzy metric space if X is an arbitrary set,$*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0,\infty)$ satisfying the following conditions for all $x, y, z \in X$ and $s, t > 0$,

$(FM-1)$: $M(x, y, 0) = 0$

$(FM-2)$: $M(x, y, t) = 1, \forall\, t \succ 0, \Leftrightarrow x = y$

$(FM-3)$: $M(x, y, t) = M(y, x, t)$

$(FM-4)$: $M(x, z, t+s) \geq M(x, y. t) * M(z, y, s)$

$(FM-5)$: $M(x, y, a)$: $[0,1]$ is left continuous

$(FM-6)$: $\lim_{t\to\infty} M(x, y, t) = 1$

In what follows $(X, M,*)$ will denote a fuzzy metric space.

Note that $M(x, y, t)$ can bet thought of as the degree of nearness between x and y with respect to t. We identify $x = y$ with $M\,(x, y, t) = 1$ for all $t > 0$ and $M\,(x, y, t) = 0$ with ∞.

Example: Let (X, d) be a metric space.

Define $a * b = a\, b, or\; a * b = min\,\{a, b\}$ and for all $x, y, \in X$ and $t > 0$,

$$M(x, y, t) = \frac{t}{t + d(x, y)}$$

Then $(X, M,*)$ is a fuzzy metric space. We call this fuzzy metric M induced by the metric d the standard fuzzy metric.

Definition 2.3: Let $(X, M,*)$ is a fuzzy metric space.

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$,
$\lim_{n\to\infty} M(x_n, x, t) = 1$

(ii) A sequence $\{x_n\}$ in X is called a Cauchy sequence if
$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1\ \forall\, t \succ 0$ and $p \succ 0$

(iii) A fuzzy metric space in which every Cauchy sequence is convergent is said to be Complete.

Definition 2.4: A function M is continuous in fuzzy metric space iff whenever

$$x_n \to x, y_n \to y \Longrightarrow \lim_{n\to\infty} M(x_{n+p}, x_n, t) \to M(x, y, t)$$

3. MAIN RESULS

THEOREM (3.1): Let $(X, M,*)$ is a complete fuzzy metric space and F be a self-map of X. The mapping F satisfying the condition;

$$M(Fx, Fy, t) \geq min\left[\begin{matrix}\frac{M(x,F(x),t)M(y,F(y),t)}{M(x,y,t)}, \frac{M(x,F(y),t)M(y,F(x),t)}{M(x,y,t)} \\ M(x, F(x), t)M(y, F(y), t)M(x, y, t)\end{matrix}\right]$$

for, all $x, y \in X$ with $x \neq y$ and F is onto. Then F has a fixed point in X.

PROOF: Let $x_0 \in X$ since F is onto there is an element x_1 satisfying $x_1 \in F^{-1}(x_0)$. By the same way we can choose, $x_n \in F^{-1}x_{n-1}$, where $(n = 2, 3, 4 - - - - - - - - -)$.

If $x_{m-1} = x_m$ for some m, then x_m is a fixed point of F. Without loss of generality we can suppose $x_{n-1} \neq x_n$ for every n. So,

$$M(x_{n-1}, x_n, t) = M(F(x_n), F(x_{n+1}), t)$$

$$M(F(x_n), F(x_{n+1}), t) \geq min\left[\begin{matrix}\frac{M(x_n,F(x_n),t)M(x_{n+1},F(x_{n+1}),t)}{M(x_n,x_{n+1},t)}, \frac{M(x_n,F(x_{n+1}),t)M(y,F(x_n),t)}{M(x_n,x_{n+1},t)}, \\ M(x_n, F(x_n), t), M(x_{n+1}, F(x_{n+1}), t), M(x_n, x_{n+1}, t)\end{matrix}\right]$$

$M(x_{n-1},x_n,t) = M(x_n,x_{n+1,}t)$

Therefore by well known way $\{x_n\}$ is a Cauchy square in X. Since X is complete $\{x_n\}$, converges to x, for some $x \in X$. since F is onto there exists $y \in X$ such that $y \in F^{-1}(x)$ and for infinitely many $n, x_n \neq x$, for such n

$M(x_n,x,t) = M(F(x_{n-1}),F(y),t)$

$\geq$

$$min\begin{bmatrix}\frac{M(x_{n+1},F(x_{n+1}),t)M(y,F(y),t)}{M(x_{n+1},y,t)},\frac{M(x_{n+1},F(y),t)M(y,F(x_{n+1}),t)}{M(x_{n+1},y,t)},\\ M(x_{n+1},F(x_{n+1}),t),M(y,F(y),t),M(x_{n+1},F(x_{n+1}),t),M(x_{n+1},y,t)\end{bmatrix}$$

$$= min\begin{bmatrix}\frac{M(x_{n+1},x_n,t)M(y,Fy,t)}{M(x_{n+1},y,t)},\frac{M(x_{n+1},x,t)M(y,x_n,t)}{M(x_{n+1},y,t)},\\ M(x_{n+1},x_n,t),M(y,x,t),M(x_{n+1},y,t)\end{bmatrix}$$

On taking limit as, $n \to \infty$

$0 \geq M(y,x,t)$ or $0 \geq \lim_{n\to\infty} M(x_{n+1},y,t)$

So $M(y,x,t) = 0$. And $\lim_{n\to\infty} M(x_{n+1},y,t) = 0$

So, in both cases we get $x = y$. Thus F has a fixed point in X.

This completes the proof.

THEOREM (3.2): Let $(X,M,*)$ is a complete fuzzy metric space and F be a self-map of X. The mapping F satisfying the condition;

$M(Fx,Fy,t) \geq$

$$\begin{bmatrix}\frac{M(x,F(x),t)M(y,F(y),t)+M(x,F(y),t)M(y,F(x),t)}{M(x,y,t)}\\ \frac{[M(y,F(y),t)]^2+M(x,F(y),t)M(y,F(x),t)}{M(x,y,t)},M(x,F(x),t)\\ M(y,F(y),t)\end{bmatrix}$$

for, all $x,y \in X$ with $x \neq y$ and F is onto. Then F has a fixed point in X.

PROOF: Let $x_0 \in X$ since F is onto there is an element x_1 satisfying $x_1 \in F^{-1}(x_0)$.. By the same way we can choose, $x_n \in F^{-1}(x_{n-1})$, where $(n = 2,3,4 --------)$.

If $x_{m-1} = x_m$ for some m, then x_m is a fixed point of F. Without loss of generality we can suppose $x_{n-1} \neq x_n$ for every n. So,

$M(x_{n-1},x_n,t) = M(F(x_n),F(x_{n+1}),t)$

$M(F(x_n),F(x_{n+1}),t)$

$\geq$

$$\emptyset\begin{bmatrix}\frac{M(x_n,F(x_n),t)M(x_{n+1},F(x_{n+1}),t)+M(x_n,F(x_{n+1}),t)M(x_{n+1},F(x_n),t)}{M(x_n,x_{n+1,}t)},\\ \frac{M(x_n,F(x_{n+1}),t)M(F(x_n),x_{n+1},t)+[M(x_{n+1},F(x_n),t)]^2}{M(x_n,x_{n+1,}t)}\\ M(x_n,F(x_{n+1}),t),M(x_{n-1},F(x_{n+1}),t),M(x_n,x_{n+1,}t)\end{bmatrix}$$

$M(x_n,x_{n-1,}t) \geq \emptyset\{M(x_n,x_{n+1,}t),M(x_n,x_{n+1,}t)\}$

$M(x_n,x_{n-1,}t) \geq \{M(x_n,x_{n+1,}t)\}$

$M(x_n,x_{n+1,}t) \leq \{M(x_n,x_{n-1,}t)\}$

Therefore $\{x_n\}$ is a Cauchy square in X and X is complete therefore $\{x_n\}$ Converge to x for some x in X. So by continuity of F, we can write

$F(x_n) = x_{n-1} \to F(x)$, as $n \to \infty$

Hence $F(x) = x$

This completes the proof.

THEOREM (3.3): Let $(X,M,*)$ is a complete fuzzy metric space and F be a self-map of X. The mapping F satisfying the condition;

$M(Fx,Fy,t) \geq$

$$min\begin{bmatrix}\frac{M(x,F(x),t)M(y,F(y),t)}{M(x,y,t)},\frac{M(x,F(y),t)M(y,F(x),t)}{M(x,y,t)}\\ M(x,F(x),t),.M(y,F(y),t),M(x,y,t)\end{bmatrix}$$

for, all $x,y \in X$ with $x \neq y$ and F is onto, there exists a point w in X such that

$T(w) = Sup\{T(x): T(x) = M(x,F(x),t), x \in X\}$

Then F has a fixed point in X.

PROOF: Let $w \neq F(w)$, otherwise w is a fixed point of F. Put $x = w$ and $y = F(w)$

$M(F(w),F^2(w),t) \geq$

$$min\begin{bmatrix}\frac{M(w,F(w),t)M(F(w),F(F(w)),t)}{M(w,F(w),t)},\frac{M(w,F(F(w)),t)M(F(w),F(w),t)}{M(w,F(w),t)}\\ M(w,F(w),t),M(F(w),F(F(w)),t),M(w,F(w),t)\end{bmatrix}$$

$= [(F(w),F(F(w)),t)]$ or $M[w,F(w),t]$

$M(F(w),F^2(w),t) \geq (F(w),F(F(w)),t)$ which is not possible

$So\ M(F(w),F^2(w),t) \geq M(w,F(w),t) -----$ (3.3.1)

Similarly on putting $x = F(w)$ and $y = w$, we get

$M(F^2(w),F(w),t)$

$\geq$

$$min\begin{bmatrix}\frac{M(w,F(w),t)M(F(w),F(F(w)),t)}{M(w,F(w),t)},\frac{M(w,F(F(w)),t)M(F(w),F(w),t)}{M(w,F(w),t)}\\ M(F(w),F(F(w)),t),M(w,F(w),t),M(w,F(w),t)\end{bmatrix}$$

$= [(F(w),F(F(w)),t)]$ or $M[w,F(w),t]$

$M(F^2(w),F(w),t) \geq M(w,F(w),t) -----$ (3.3.2)

By (3.3.1) and (3.3.2)

$M(F(w),F^2(w),t) \geq d(w,F(w),t)$

$M(F(w),F^2(w),t) \geq M(w,F(w),t)$

This implies that

$T(F(w)) > T(w)$, giving a contraction.

Hence we must have $F(w) = w$, that is w is a fixed point of F in X.

THEOREM (3.4): Let $(X,M,*)$ is a fuzzy metric space and F be a self-map of X.The mapping F satisfying the condition;

$$M(Fx,Fy,t) \geq \emptyset\begin{Bmatrix}\frac{M(x,F(x),t)M(y,F(y),t)+M(x,F(y),t)M(y,F(x),t)}{M(x,y,t)}\\ M(x,F(x),t),M(y,F(y),t),M(x,y,t)\end{Bmatrix}$$

For, all $x,y \in X$ with $x \neq y$ and F is onto, there exists a point w in X such that

$T(w) = Sup\{T(x): T(x) = d(x,F(x)), x \in X\}$

Then F has a unique fixed point in X.

PROOF: Let $w \neq F(w)$, otherwise w is a fixed point of F. Put $x = w$ and $y = F(w)$

$M(F(w),F^2(w),t) \geq$

$$\emptyset\begin{bmatrix}\frac{M(w,F(w),t)M(F(w),F(F(w)),t)+M(w,F(F(w)),t)M(F(w),F(w),t)}{M(w,F(w),t)}\\ M(w,F(w),t),M(F(w),F(F(w)),t),M(w,F(w),t)\end{bmatrix}$$

$M(F(w),F^2(w),t) \geq$

$\emptyset\{M(F(w),F^2(w),t),M(w,F(w),t)\}$

$M(F(w),F^2(w),t) \geq \{M(w,F(w),t)\} -----$

$-$(3.4.1)

Similarly on putting $x = F(w)$ and $y = w$, we get

$M(F(w),F^2(w),t) \geq$

$$\emptyset\begin{bmatrix}\frac{M(w,F(w),t)M(F(w),F(F(w)),t)+M(w,F(F(w)),t)M(F(w),F(w),t)}{M(w,F(w),t)}\\ M(F((w))F(F(w)),t),M(w,F(w),t),M(w,F(w),t)\end{bmatrix}$$

$M(F(w),F^2(w),t) \geq$

$\emptyset\{M(F(w),F^2(w),t),M(w,F(w),t)\}$

$M(F(w),F^2(w),t) \geq \{M(w,F(w),t)\} -----$

$-$(3.4.2)

It is clear by (3.4 .1) and (3.4.2) that

$T(F(w)) > T(w)$, giving a contraction

Hence we must have $F(w) = w$, that is w is a fixed point of F in X.

THEOREM (3.5): Let $(X, M,*)$ is a complete fuzzy metric space and F be a self-map of X. The mapping F satisfying the condition;

$$M(F(x), F(y), t) \geq min\begin{bmatrix} M(x, F(x), t)M(y, F(y), t), M(x, F(y), t), M(y, F(x), t), \\ M(x, F(x), t)M(x, y, t), M(x, F(y), t)M(x, y, t) \end{bmatrix}^{\frac{1}{2}}$$

For each $x \neq y$ and $x, y \in X$ and F is onto and then F has a fixed point.

PROOF: Let $x_0 \in X$ since F is onto there is an element x_1 satisfying $x_1 \in F^{-1}(x_0)$. By the same way we can choose, $x_n \in F^{-1}x_{n-1}$, where $(n = 2, 3, 4 ---------)$.

If $x_{m-1} = x_m$ for some m, the x_m is a fixed point of F. Without loss of generality we can suppose $x_{n-1} \neq x_n$ for every n. So,

$$M(x_{n-1}, x_n, t) = M(F(x_n), F(x_{n+1}), t)$$

$$M(x_{n-1}, x_n, t) \geq min\begin{bmatrix} M(x_n, F(x_n), t)M(x_{n+1}, F(x_{n+1}), t), M(x_n, F(x_{n+1}), t)M(x_{n+1}, F(x_n), t) \\ M(x_n, F(x_n), t)M(x_n, x_{n+1}, t), M(x_n, F(x_{n+1}), t)M(x_n, x_{n+1}, t) \end{bmatrix}^{\frac{1}{2}}$$

$$M(x_{n-1}, x_n, t) \geq [M(x_n, x_{n-1}, t)M(x_{n+1}, x_n, t)]^{\frac{1}{2}}$$

$$M(x_{n-1}, x_n, t) \geq [M(x_{n+1}, x_n, t)]$$

$$[M(x_{n+1}, x_n, t)] \leq M(x_{n-1}, x_n, t)$$

Therefore by well known way $\{x_n\}$ is a Cauchy square in X. Since X is complete $\{x_n\}$ converges to x, for some $x \in X$. since F is onto there exists $y \in X$ such that $y \in F^{-1}(x)$ and for infinitely many n, $x_n \neq x$, for such n

$$M(x_n, x, t) = M(F(x_{n+1}), F(y), t)$$

$$\geq \begin{bmatrix} M(x_{n+1}, F(x_{n+1}), t)M(y, F(y), t), M(x_{n+1}, F(y), t)M(y, F(x_{n+1}), t) \\ M(x_{n+1}, F(x_{n+1}), t)M(x_{n+1}, y, t), M(x_{n+1}, F(y), t)M(x_n, y, t) \end{bmatrix}^{\frac{1}{2}}$$

$$M(x_n, x, t) \geq min\begin{bmatrix} M(x_{n+1}, x_n, t)M(y, x, t), M(x_{n+1}, x, t)M(y, x_n, t), \\ M(x_{n+1}, x_n, t)M(x_{n+1}, y, t), M(x_n, x, t)M(x_{n+1}, y, t) \end{bmatrix}^{\frac{1}{2}}$$

On taking limit as, $n \to \infty$, $M(x_n, x, t) \to 0$.

So we have $M(x, y, t) = 0$, which implies that $x = y$.

Hence F has a fixed point.

THEOREM (3.6): Let $(X, M,*)$ is a complete fuzzy metric space, $f: X \to X$, satisfy the conditions:,

$$M(f(x), f)(y), t) \geq 2\,min\begin{bmatrix} \frac{M(x,f(x),t)M(x,f(y),t)}{M(x,f(x),t)+M(y,f(y),t)+M(x,f(y),t)M(y,f(x),t)} \\ + \frac{M(x,f(y),t)M(y,f(x),t)}{M(x,f(x),t)+M(y,f(y),t)+M(x,f(y),t)M(y,f(x),t)} \end{bmatrix}$$

for all $x, y \in X$, with $x \neq y$, Then F has a fixed point.

PROOF: Let $x_0 \in X$ since F is onto there is an element x_1 satisfying $x_1 \in F^{-1}(x_0)$. By the same way we can choose, $x_n \in F^{-1}x_{n-1}$, where $(n = 2, 3, 4 ---------)$.

If $x_{m-1} = x_m$ for some m, the x_m is a fixed point of F. Without loss of generality we can suppose $x_{n-1} \neq x_n$ for every n.

$$M(x_{n-1}, x_n, t) = M(F(x_n), F(x_{n+1}), t)$$

$$\geq 2\,min\begin{bmatrix} \frac{M(x_{n-1},x_n,t)M(x_n,x_{n+1},t)}{M(x_{n-1},x_n,t)+M(x_n,x_{n+1},t)+M(x_{n-1},fx_{n+1},t)M(x_{n+1},fx_n,t)} + \\ \frac{M(x_{n-1},x_n,t)M(x_n,x_{n+1},t)}{M(x_{n-1},x_n,t)+M(x_n,x_{n+1},t)+M(x_{n-1},fx_{n+1},t)M(x_{n+1},fx_n,t)} \end{bmatrix}$$

$$M(x_{n-1}, x_n, t)M(x_n, x_{n+1}, t) \geq 2M(x_n, x_{n+1}, t)$$

Therefore $\{x_n\}$ converge to some $x \in X$.

Since f is onto there exists $y \in X$ such that $y \in F^{-1}(x)$ and for infinitely many n, $x_n \neq x$, for such

$$M(x_{n+1}, x_n, t) \leq M(x_n, x_{n-1}, t)$$

$$M(x_n, x, t) = m(f(x_{n-1}), f(y), t)$$

$$\geq 2\,min\begin{bmatrix} \frac{M(x,y,t)T(x_n,x_{n+1},t)}{M(x,y,t)+M(x_n,x_{n+1},t)+M(x_{n+1},f(y),t)M(y,fx_{n+1},t)} + \\ \frac{M(x_{n+1},f(y),t)M(y,fx_{n+1},t)}{M(x,y,t)+M(x_n,x_{n+1},t)+M(x_{n+1},f(y),t)M(y,fx_{n+1},t)} \end{bmatrix}$$

And

$$M(x_n, x, t)[M(x, y, t) + M(x_n, x_{n+1}, t) + M(x_{n+1}, f(y), t)M(y, fx_{n+1}, t)]$$

$$\geq 2[M(x, y, t)M(x_n, x_{n+1}, t) + M(x_{n+1}, f(y), t)M(y, fx_{n+1}, t)]$$

Since, $M(x_n, x, t) \to 1$ as $n \to \infty$, we have $M(x, y, t) = 1$ as $x_n \neq x_{n+1}$

Therefore $x = y$, so f has a fixed point.

Now we find a common fixed point theorem for expansion mappings.

THEOREM (3.7): Let $(X, M,*)$ is a complete fuzzy metric space, if the mapping $G, F: X \to X$, satisfy the conditions:

$$M(Gx, Gy, t) \geq min\begin{bmatrix} \frac{M(x,Gx,t)M(y,fy,t)}{M(x,y,t)}, \frac{M(x,fy,t)M(y,Gx,t)}{M(x,y,t)}, \\ M(x, Gx, t), M(y, fy, t), M(x, y, t) \end{bmatrix}$$

for all $x, y \in X$, with $x \neq y$ and $G(X) \subseteq X, T(X) \subseteq X$

Then G and F have a common fixed point.

PROOF: Let x_0 be any point of X, we define a sequence $\{x_n\}$ recurrently as follows:

$x_0 = Gx_1, x_1 = Fx_2, X_{2n} = Gx_{2n+1}, x_{2n+1} = Fx_{2n+1},$

Now for same, $n \geq 0$, if $x_{2n+1} = x_{2n}$

Since, $M(x_{2n}, x_{2n+1}, t) = M(x_{2n+1}, x_{2n+2}, t)$

If $x_{2n+1} \neq x_{2n+2}$, Then we can write by the definition.

$$M(Gx_{2n+1}, Fx_{2n+2}, t) \geq min\begin{bmatrix} \frac{M(x_{2n+1},Gx_{2n+1},t)M(x_{2n+2},Fx_{2n+2},t)}{M(x_{2n+1},x_{2n+2},t)}, \\ \frac{M(x_{2n+1},Fx_{2n+2},t)M(x_{2n+2},Gx_{2n+1},t)}{M(x_{2n+1},x_{2n+2},t)}, \\ M(x_{2n+1}, Gx_{2n+1}, t)M(x_{2n+2}, Fx_{2n+2}, t), M(x_{2n+1}, x_{2n+2}, t) \end{bmatrix}$$

$$= min\begin{bmatrix} \frac{M(x_{2n+1},x_{2n},t)M(x_{2n+2},x_{2n+1},t)}{d(x_{2n+1},x_{2n+2},t)}, M(x_{2n+1}, x_{2n}, t), \\ M(x_{2n+2}, x_{2n+1}, t), M(x_{2n+1}, x_{2n+2}, t) \end{bmatrix}$$

$$M(x_{2n}, x_{2n+1}, t) \geq M(x_{2n+1}, x_{2n+2}, t)$$

Then we must have $M(Fx_{2n+2}, x_{2n+1}, t) = 1$.

This implies, $x_{2n+1} = x_{2n+2}$ a contradiction.

Thus we have $x_{2n} = x_{2n+2}$

Similarly, if $x_{2k+1} = x_{2k+2}$, $(k \geq 0)$.

We get that $x_{2k} = x_{2k+2} = x_{2k+2} = x_{2k+3} = \cdots \dots \dots$

If follow that G and F have common fixed point.

Next suppose $x_{2n+1} \neq x_{2n+2}$ and $x_{2n+2} \neq x_{2n+3}$ for all $n \geq 0$,

So, by the given definition

$$M(x_{2n}, x_{2n+1}, t) \geq M(x_{2n+1}, x_{2n+2}, t) \geq M(x_{2n+1}, x_{2n+3}, t)$$

If follow that $\{x_n\}$ is a Cauchy sequence. By completeness of X there is some point z in X which $\{x_n\}$ converges to z

By the condition there is a point w in X such that $Gw = z$

Since we can suppose $w \neq x_{2n+2}$ for infinitely many n we can write,

$$M(z, x_{2n+1}, t) = M(Gw, \mathrm{F}x_{n+2}, t)$$

$$\geq min\left[\frac{M(w,Gw,t)M(x_{2n+2},\mathrm{F}x_{2n+2},t)}{M(z,x_{2n+2},t)}, M(w, Gw, t)M(x_{2n+2}, \mathrm{F}x_{2n+2}, t), M(w, x_{2n+2}, t)\right]$$

As $n \to \infty$ we obtain

$$0 \geq M(w, z, t)$$

Therefore it implies that $z = w = Gz$ $i.e. Gz = z$. Similarly, $Fz = z$.

This completes the proof.

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