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Coverage and Connectivity Probabilities in WSN with Heterogeneous Capability Nodes

G.N. Purohit
Centre for Mathematical Sciences
Banasthali University
Banasthali-304022, INDIA
gn_purohitjaipur @yahoo.co.in

Seema Verma
Department of Electronics, AIM & ACT
Banasthali University
Banasthali-304022, INDIA
seemaverma3@yahoo.com

Megha Sharma*
Department of Computer Science, AIM & ACT
Banasthali University
Banasthali-304022, INDIA
edify44@yahoo.com

Abstract: A sensor network provides flexible communication network, which can be deployed rapidly over wide and/or inaccessible areas. However, the need to gather data from all sensors in the network imposes constraints on the distances between sensors. In a WSN, after collecting information from the environment, sensors need to transmit aggregated data to gateways or information collection nodes. It is important to ensure that every sensor can communicate with the gateways. This leads to the need for sufficient connectivity in addition to optimal coverage. In this paper we consider a heterogeneous sensor network that addresses the problem of coverage and connectivity together, however, in the earlier studies the case of homogeneous sensors was considered. A combination of sensors of higher capabilities (communicating, sensing) and lower capabilities gives better results compared to deployment of only homogeneous sensors. It is ensured that both the coverage and connectivity of the system are maintained. In particular, we are interested in maintaining connected WSNs that effectively cover the Region of Interest (R.O.I). Our sensors can be in alert or sleeping mode. We compute the probabilities for the sensor to stay in the off, sense/receive, and transmit state ensuring coverage and connectivity in the network. We develop the Markov model and its solution for steady state.

Keywords: sensing nodes; heterogeneous; coverage; connectivity; probability

I. INTRODUCTION

Recent advancements in microelectronics, digital signal processing, and low-power RF techniques have enabled the deployment of large wireless sensor networks. Wireless sensor networks can be deployed in areas without infrastructure support, in hostile fields, and under harsh environments. A wireless sensor network consists of many nodes generally communicating through radio waves. The sensors are not integrated into any existing network architecture, so they communicate through a network of ad hoc wireless connections. In the past, sensors were connected by wired lines. Today, this environment is combined with the novel *ad hoc* networking technology to facilitate inter-sensor communication [1, 4]. The flexibility of installing and configuring a sensor network is thus greatly improved.

Coverage and connectivity are the fundamental requirements in wireless sensor networks and can be considered the metrics of interest when targeting quality of service for applications which are considered in many operations of sensor networks, including, clustering, synchronization, query and information discovery, deployment and redeployment. Coverage is the area or the number of targets that can be monitored by a sensor. On the other hand, connectivity ensures that sensor nodes can communicate with each other in order to aggregate data reports to the base stations (sinks). The wireless communication in WSNs can be either ad hoc (multi-hop) or single-hop wireless transmission [3]. We consider a wireless ad hoc network (or sensor network), where each wireless node has a maximum transmission power so that it can send signals to all nodes within its transmission range.

In this paper we focus on the coverage and connectivity issues of heterogeneous two-dimensional networks, where the nodes have different sensing and transmission range. The general probabilistic Markov model is presented, in which each sensor node makes an independent decision regarding which state to be in at a given instance. This model is illustrated using a three state model with a transmit (T), receive/sense (S) and off state (O), and the nodes can be in any of the three states at any given point of time. Node makes transitions between the states which are governed by a set of parameters.

The remaining paper is organized as follows. Section II gives the Markov Model for the state probabilities of a sensor node. In Section III the Sensor Coverage and connectivity are studied. In section IV the Numerical Results are presented graphically. Section V concludes the paper.

II. THE MARKOV MODEL

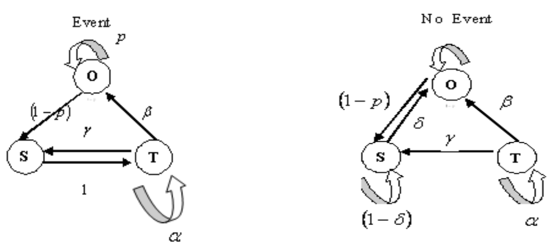
The total number of nodes are represented as $N= n_1+n_2$, where n_1 represents the *Type I nodes*- with stronger sensing capabilities and n_2 represents the *Type II nodes*- with weaker sensing capabilities. Their sensing and transmission ranges are denoted as r_{s1}, r_{s2} and r_{T1} and r_{T2} respectively.

If r_{S_1} and r_{S_2} are the sensing radii of type I and type II nodes respectively and r_{T_1} and r_{T_2} are the transmission radii of type I and type II nodes respectively, then the combined sensing and transmission radii for both the type I and type II nodes are computed as weighted average of r_{S_1}, r_{S_2} and r_{T_1}, r_{T_2} respectively

$$r_s = \frac{r_{S_1} \times n_1 + r_{S_2} \times n_2}{n_1 + n_2}, \text{ and}$$

$$r_T = \frac{r_{T_1} \times n_1 + r_{T_2} \times n_2}{n_1 + n_2}$$

The three states that a node can remain in are the off (O), the sense/receive (S), and the transmit (T) states. The transition of a node from one state to another depends on its environment, which can be in either of the two states: (i) a sense/ receive event is occurring or (ii) no such event is occurring. The Markov state diagram and the transition probability matrices, E when there is an event and $\overline{\mathrm{E}}$ when there is no event are given below.



$$\begin{array}{c|c} \mathrm{E:} & \begin{array}{ccc} \mathrm{O} & \mathrm{S} & \mathrm{T} \end{array} \\ \begin{array}{c} \mathrm{O} \\ \mathrm{S} \\ \mathrm{T} \end{array} & \begin{bmatrix} p & (1-p) & 0 \\ 0 & 0 & 1 \\ \beta & \gamma & \alpha \end{bmatrix} \end{array} \qquad \begin{array}{c|c} \overline{\mathrm{E}}: & \begin{array}{ccc} \mathrm{O} & \mathrm{S} & \mathrm{T} \end{array} \\ \begin{array}{c} \mathrm{O} \\ \mathrm{S} \\ \mathrm{T} \end{array} & \begin{bmatrix} p & (1-p) & 0 \\ \delta & (1-\delta) & 0 \\ \beta & \gamma & \alpha \end{bmatrix} \end{array}$$

$$\alpha + \beta + \gamma = 1$$

$$0 \le \alpha, \beta, \gamma, \delta, p \le 1$$

Figure 1. Markov state diagram and transition probability matrices

In case of a sensing event, the nodes makes a transition to the transmit state. In case of both sensing and receiving events node always attempts to transmit the sensed event rather than the received event. We denote pr_O, pr_S pr_T as the respective probabilities of the nodes in off, sense/receiving and transmit states. These three probabilities can be collectively denoted as a vector $\mathrm{pr}\,(t) = [pr_O\,,\; pr_S\,,\; pr_T\,]$. Let us assume P_{EV} the probability that there is an event. Then the state probabilities for the node at time $t+1$ are given by

$$\mathrm{pr}(t+1) = \mathrm{pr}(t)\left[P_{EV}\,\mathrm{E} + (1 - P_{EV})\overline{\mathrm{E}}\right] \qquad (1)$$

At any point of time an event can be either sensing or receiving. The probabilities of an event will therefore depend on the probability that a single neighbor is transmitting. If we suppose now, that the, system has equilibrated to steady state, in which

$$\mathrm{pr}(t+1) = \mathrm{pr}(t) = \mathrm{pr}^{\mathrm{S}},$$

where p^{S} the probability in the steady state

We also make the mean field approximation that all the neighbors of the node are in the same steady state and can be treated as independent in which case we can compute P_{EV} as follows.

Let P_{SEV} be the probability of a sensing event and let $P_{R\,EV}$ be the probability of a receiving event. $P_{R\,EV}$ is the probability that exactly one of the node's neighbors is transmitting. We assume that the state probabilities for the neighbors are independent. In this case, if there are **M** neighbors, then, $P_{R\,EV} = M\; pr_T\,(1 - pr_T\,)^{M-1}$. If the transmission radius is $P_{R\,EV}$, then assuming that the sensing disks are in unit torus, the probability that a node is within transmission range of our node is $\pi\, r_T^{\;2}$, and M *h*as a Binomial distribution $\mathrm{PR}[M] = B\left(M\,; N-1, \pi r_T^{\;2}\right)$,

where $B(M\,; N, pr) = \binom{N}{M} pr^{M}\,(1 - pr)^{N-M}$.

Multiplying $P_{R\,EV}$ by $\mathrm{PR}[M]$ and summing over *M*, we finally arrive at the following equation for $P_{R\,EV}$:

$$P_{REV} = (\mathrm{N}-1)\pi\, r_T^{\;2}\, pr_T\, \left(1 - \pi\, r_T^{\;2}\, pr_T\right)^{\mathrm{N}-2} \qquad (2)$$

[

Since, the sensing and receiving events are independent

$$P_{EV} = \Pr[\mathit{Sense\ or\ receive}]$$

$$P_{EV} = P_{SEV} + P_{REV} - P_{SEV}P_{REV}$$, We make use of this expression to solve equation (1)

For the steady state probabilities P^s, we have $\mathrm{pr}(t+1) = \mathrm{pr}(t) = \mathrm{p}^{\mathrm{S}}$. Then equation (1) can be written as:

$$\Pr{}^{\mathrm{S}} = \Pr{}^{\mathrm{S}}\left[P_{EV}\,\mathrm{E} + (1 - P_{EV}\,)\overline{\mathrm{E}}\right] \qquad (3)$$

$$\Pr{}^{\mathrm{S}} = \Pr{}^{\mathrm{S}}\left[\begin{array}{l} \left(\begin{array}{l} P_{SEV} + (\mathrm{N}-1)\mathrm{m}\,(1-\mathrm{m}\,)^{\mathrm{N}-2} \\ -P_{SEV}(\mathrm{N}-1)\mathrm{m}\,(1-\mathrm{m}\,)^{\mathrm{N}-2} \end{array}\right)\mathrm{E} \\ +\left(1 - \left(\begin{array}{l} P_{SEV} + (\mathrm{N}-1)\mathrm{m}\,(1-\mathrm{m}\,)^{\mathrm{N}-2} \\ -P_{SEV}(\mathrm{N}-1)\mathrm{m}\,(1-\mathrm{m}\,)^{\mathrm{N}-2} \end{array}\right)\right)\overline{\mathrm{E}} \end{array}\right]$$

where, $\mathrm{m} = \pi\, r_T^{\;2}\, pr_T$

Since, the sum of all probabilities is always unity, we have

$$1 = \mathrm{pr}.1 = pr_O + pr_S + pr_T \qquad (4)$$

Theorem 1. *The set of non- linear steady state equations for* Pr^S given in (4) has at least one solution.

Proof: Let $\mathrm{TM}(pr)$ be the transition matrix. $\mathrm{TM}(pr) = \left[P_{EV}\mathrm{E} + (1 - P_{EV})\overline{\mathrm{E}}\right]$ as defined in (3). Then, each element of $\mathrm{TM}(pr)$ is greater than or equal to zero, and sum of all elements in a row is 1.i.e., $TM_{ij} \geq 0$ and $\sum_j TMi_j = 1$ for all i. Let Y be the k-dimensional probability simplex,

$$Y = \left\{ y : y_i \geq 0, \sum_i x_i = 1 \right\}.$$

Y is compact, and $f(pr) = [TM(pr)]^T pr$ maps Y onto itself. $P_{EV}(pr)$ is a polynomial in pr, and hence is continuous. Thus, $f(pr)$ is a continuous mapping. Thus the conditions to apply the Brower fixed point theorem are satisfied for $f(pr)$ [2], and so $f(pr)$ has a fixed point.

III. COVERAGE AND CONNECTIVITY

We assume that $\mathrm{N} = \mathrm{n}_1 + \mathrm{n}_2$ are uniformly distributed in the unit torus, U = [0,1] * [0,1]. Let r_{s_1} and r_{s_2} be the sensing radius of type I and type II nodes respectively, and r_{T_1} and r_{T_2} be the transmission radius of type I and type II nodes respectively.

A. *Coverage*

We assume the system to be in steady state, and also that every node can be treated independent, with state probabilities given by P^S. A point $y \in R$ in the R.O.I is said to be covered if there is a node in the sensing state within the sensing radius of *y*. Thus, the probability that a given node is sensing and within the sensing radius of y is $\pi r_{S_1}{}^2 pr_S$ in case of nodes of type I and $\pi r_{S_2}{}^2 p_S$ in case of nodes of type II.

Under the independent assumption that no node can sense an even at *y* is then, given by

$$\left(1 - \pi r_{S_1}{}^2 pr_S\right)^{\mathrm{n}_1} \times \left(1 - \pi r_{S_2}{}^2 pr_S\right)^{\mathrm{n}_2}$$

Which is the probability that y is not covered either by type I nodes or by type II nodes.
We define the coverage function by,

$$f(y) = \begin{cases} 1 & y \text{ is not covered} \\ 0 & y \text{ is covered} \end{cases} \qquad (6)$$

Then, $P[f(y) = 1] = \left(1 - \pi r_{S_1}{}^2 pr_S\right)^{\mathrm{n}_1} \times \left(1 - \pi r_{S_2}{}^2 pr_S\right)^{\mathrm{n}_2}$.
Let A be the area that is not covered, then

$$\mathrm{A} = \int f(y)dy \text{ , so,}$$

$$E[\mathrm{A}] = \int \mathrm{P}[f(y = 1)]dy$$

$$= \left(1 - \pi r_{S_1}{}^2 p_S\right)^{\mathrm{n}_1} \times \left(1 - \pi r_{S_2}{}^2 p_S\right)^{\mathrm{n}_2}$$

Therefore, the expected area covered is

$$1 - E[A] = 1 - \left(\left(1 - \pi r s_1 p_S\right)^{n_1} \times \left(1 - \pi r s_1 p_S\right)^{n_2}\right)$$

Which, after using the fact that log (1-y) ≤-*y*, for *y*<1, leads to the following proposition.

Proposition 2 Let $\pi r_{S_1}{}^2 p_S = \mu(\mathrm{n}_1)/\mathrm{n}_1$

[For type I nodes]

$$\pi r_{S_1}{}^2 p_S = \mu(\mathrm{n}_2)/\mathrm{n}_2$$

[For type II nodes]

Then the expected coverage is given by

$$1 - \left(\left(1 - \mu(\mathrm{n}_1)/\mathrm{n}_1\right)^{\mathrm{n}_1} \times \left(1 - \mu(\mathrm{n}_2)/\mathrm{n}_2\right)^{\mathrm{n}_2}\right) \geq 1 - e^{-\mu(\mathrm{N})}$$

(N=n_1+n_2)

where $\mu(\mathrm{N})$ in general can be interpreted as the expected power used by the sensing nodes.

$\mu(\mathrm{n}_1)/\mathrm{n}_1 \leq 1$ and $\mu(\mathrm{n}_2)/\mathrm{n}_2 \leq 1$

Thus, as long as $\mu(\mathrm{n}_1) \rightarrow \infty$ and $\mu(\mathrm{n}_2) \rightarrow \infty$, the expected coverage approaches 1. In order to achieve a concentration result on coverage, we use a second moment method and compute var (A), to which end we would need E[A^2] . We use the mean filed approximation that our nodes are acting independently in the mean field environment of the neighbors. Then, using a second moment method we have that

Theorem 3. We know N=n_1 +n_2

Let $r_S \leq \dfrac{1}{2\sqrt{2}}$

Then, for any $\in > 0$.

$$\Pr\left[\mathrm{A} \geq 2e^{-\frac{(1-\in)}{10\pi}\mu(\mathrm{N})}\right] < \frac{2\pi \exp\left(\dfrac{\in \mu(\mathrm{N})}{5\pi} + \Theta\left(\dfrac{1}{\mu(\mathrm{N})}\right)\right)}{\mu(\mathrm{N})\left(1 + \Theta\left(\dfrac{1}{\mu(\mathrm{N})}\right)\right)}$$

where $\mu(\mathrm{N}) = \mathrm{N}\pi\, r_S{}^2\, pr_S$,

Proof: We give proof in the appendix.

B. ***Connectivity***

If a sensing event occurs at some position $y \in R$, and we wish to transmit the occurrence of this event to $z \in R$, then we would like to successfully transmit the occurrence of this event for any y, z. i.e., we would want the probability for successful transmission to be high.

A path exists from *y* to *z* if there is a sequence of nodes in the receiving state at locations $l_0, l_1, \ldots, l_M$ such that

T1: $\|y - l_0\| \le r_s$ y can be sensed

T2: $\| l_i - l_{i-1} \| \le r_T$
For i=1,...,M

Therefore the event can be transmitted from l_{i-1} to l_i , and it will be received since l_i is in the receiving state.

T3: $\| l_M - z \| \le r_T$ (l_M can transmit to z)

When, l_0 transmits to l_1 , it is necessary that l_1 is in the sensing state and no their node that is within transmission range of l_1 is also attempting to transmit, and similarly for every path/link l_{i-1} , l_i in the path. If there exists such a contention free path for any y, z then we can say that the sensor network is transmission connected. Connectivity here implies coverage as well i.e., the network covers the area as well. For y to be covered, the same nodes need to cover the area with respect to r_s . But to guarantee that *y* is reachable or can be transmitted to it is necessary that the sensing nodes cover the area with respect to r_T as well. Thus, we apply the results of the previous section on coverage with r_S , r_T replaced by $r = \min\{r_S, r_T\}$. This leads to the following result.

Proposition 4. Let A' be the area that cannot be transmitted to and let A be the area not covered. Then, for any $\in > 0$.

$$\Pr\left[\mathrm{A} \cup \mathrm{A}' \le 2e^{-\frac{(1-\in)}{10\pi}\mu'(\mathrm{N})} \right] \ge 1 - O\left(\frac{e^{-\frac{\in}{5\pi}\mu'(\mathrm{N})}}{\mu'(\mathrm{N})} \right),$$

where $\mu'(\mathrm{N}) = \mathrm{N}\,\pi\, r^{2}\, pr_S$,

Proof: The claim follows from Theorem 3 and the observation that if $r_s \le r_T$ or , then $\mathrm{A} \cup \mathrm{A}' \subseteq \mathrm{A}$, otherwise $\mathrm{A} \cup \mathrm{A}' \subseteq \mathrm{A}'$.

Therefore the coverage results should imply conditions T1 and T3 of path connectivity. Now, we consider requirement T2. For this requirement, it is sufficient that the sensing disk graph obtained by taking disks with radii r_T centered at the sensing nodes be connected. Such results were developed in [5] for the case where N nodes are uniformly scattered on an area *D,* each having radius $r(\mathrm{N})$. A small complication here is that while N nodes are scattered, only about $\mathrm{N}pr_S$ of them are sensing. In [13] the following result is proved.

Theorem 5.([5]) The probability that the random sensing disk graph is connected asymptotically approaches 1 if and only if $\pi r^2(\mathrm{N}) = (\log \mathrm{N} + c(\mathrm{N}))/\mathrm{N}$
where $c(\mathrm{N}) \to \infty$

It is also known that in grid-disk graphs, with unreliable nodes, the results are very similar to the random node placement [9], and in this case it is known that the number of hops required is of order $\sqrt{\mathrm{N}/\log \mathrm{N}}$ .We expect that such results should hold in our case as well.

Theorem 6. Let $r(\mathrm{N}) = \min\{r_s(\mathrm{N}), r_T(\mathrm{N})\}$
And for any $0 < \in < 1$, let $\mathrm{N}(\in) = (1-\in)\mathrm{N}p_S$,. Let P be the area that is path connected. If
(i) $\pi r^2(\mathrm{N})\mathrm{N}pr_S \to \infty$, and
(ii) $\pi r^2(\mathrm{N})\,\mathrm{N}(\in) = \log(\mathrm{N}(\in) + c(\mathrm{N}(\in)))$,

$$\lim_{m \to \infty} \mathrm{pr}(m) = \infty,$$

Then, for any $\eta > 0, \ \lim\limits_{\mathrm{N} \to \infty} \Pr[|\mathrm{C}| \ge 1 - \eta] = 1.$,

Proof. The proof is provided as an appendix of this paper.

In order to address the contention problem we present a heuristic which we refer to as $\rho - flooding$. We require that in the event that a node needs to transmit the message, the expected number of recipients will be given by $\rho > 1$. In such a case we can see that the particular message will rapidly flood through the network and we can expect the message to spread exponentially fast. Since there are N_S nodes, we can expect that in order of $\log \mathrm{N}_S / \log \rho$ time steps, every member in the network will have received the message. If we simply use $\rho - flooding$, the contention in the network will become uncontrollable. To remove this problem, we would also need to implement a safety mechanism to prevent such over flooding-one approach might be to bound the maximum number of hops a packet is allowed to make. This can be implemented in practice by adding to each packet a hop counter, and setting its maximum allowed value appropriately. Two possibilities are $E[\mathrm{N}]/\log \rho$, the time we expect it takes to flood the whole network, or

$\sqrt{N/\log N_S}$. The requirement of $\rho - flooding$ sets constraints on the allowable parameters in the Markov model, which is what we derive here.

Let's consider the situation when a node is in the transmission state, and let ℓ be any one of the other N-1 nodes. Let Q be the probability to successfully transmit the packet to ℓ, then $\Pr_{SUCCESS} = \pi r_T^2 Q$. To achieve successful transmission given that ℓ is within transmission range, either the first trial was successful, or the first trial was not successful, and some trial after the first trial was successful. Since the process is Markov and since the nodes are independent, the probability that some trial after the first one is successful is also *Q*. Let Q_1 be the probability that we are successful on the first trial given that ℓ is within transmission range. Since the probability to remain transmitting is α, we have $Q = Q_1 + (1 - Q_1)\alpha Q$

or
$$Q = \frac{Q_1}{1 - \alpha + \alpha Q_1} \tag{7}$$

Suppose that ℓ has M neighbors. then we are successful on the first trial if ℓ is in the sensing state and no other neighbor of ℓ is transmitting, which occurs with probability $pr_S(1 - pr_T)^M$. Multiplying $pr_S(1 - pr_T)^M$ by *Pr(M),* summing over M using the fact that M has Binomial distribution $BD(M; N-2, \pi r_T^2)$, we arrive at $Q_1 = pr_S(1 - \pi r_T^2 pr_T)^{N-2}$. Since, there are N-1 nodes to which we could transmit, the expected number of successful transmissions is given by (N-1) $pr_{SUCCESS}$. Requiring that the expected number of successful transmissions is ℓ then leads to the following constraint.

Proposition 7. In order to achieve $\rho - flooding$, the following condition must be satisfied,

$$\rho = \frac{(N-1)\pi r_T^2 \; p r_S \left(1 - \pi r_{T_1}^2 \; pr_T\right)^{(N-2)}}{1 - \alpha + \alpha \; pr_S \left(1 - \pi r_{T_1}^2 \; pr_T\right)^{(N-2)}}$$

IV. NUMERICAL RESULTS

Figure 2 compares the steady state probabilities calculated for different network sizes comprising of different ratios of type I and type II nodes. For the ratio 1:4 of type I and type II nodes the transmission/sensing radius are kept constant at 0.05 and 0.35 for type I and type II nodes respectively, and the sensing event probabilities P_{SEV} are kept constant at 0.4 and 0.65 for type I and type II nodes respectively.
For the ratio 1:3 of type I and type II nodes the transmission/sensing radius are kept constant at 0.1 and 0.3 for type I and type II nodes respectively, and the sensing event probabilities P_{SEV} are kept constant at 0.4 and 0.6 for type I and type II nodes respectively.
For the ratio 1:2 of type I and type II nodes the transmission/sensing radius are kept constant at 0.16 and 0.25 for type I and type II nodes respectively, and the sensing event probabilities P_{SEV} are kept constant at 0.4 and 0.55 for type I and type II nodes respectively.
We can observe that the number of nodes in the off state increases as the network increases and the number of nodes in the sense/receive state decreases with an increase in the network size.

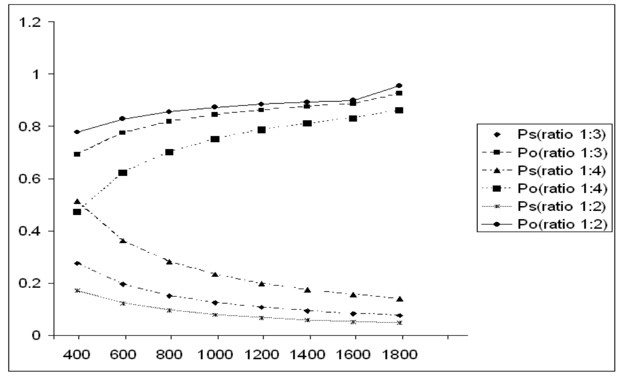


Fig.ure 2. The steady state probabilities for different network sizes and different ratios type I and type II sensor nodes. The probabilities for off state and sense/receive state are shown, whereas the probability for the transmission state is $pr_T = 1 - pr_O - pr_S$

Figure 3 presents the coverage and connectivity results. We can observe that the overall coverage and connectivity is well maintained with the increasing number of nodes. Although the number of nodes in the sense/receive state decreases with the increase in network size, as can be seen in Fig. 2 but the number of sensing nodes if high enough to maintain the coverage and connectivity with a high probability, i.e., the number of sensing nodes is optimal as required.

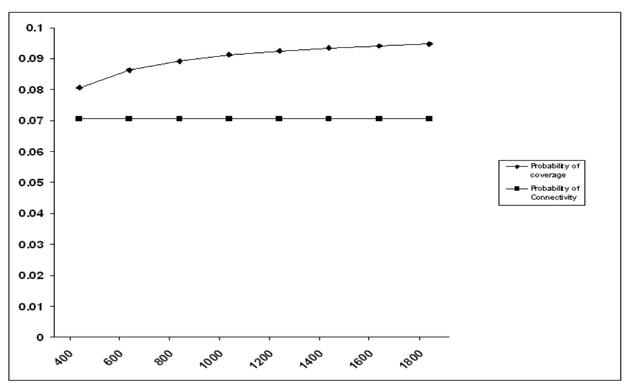


Figure 3. The coverage and connectivity graphs for different network sizes for the ration 1:3 of type I and type II nodes.

Figure 4 compares the steady state probabilities by varying the sensing event probabilities, and keeping the network size constant for a fixed ratio of type I and II nodes.

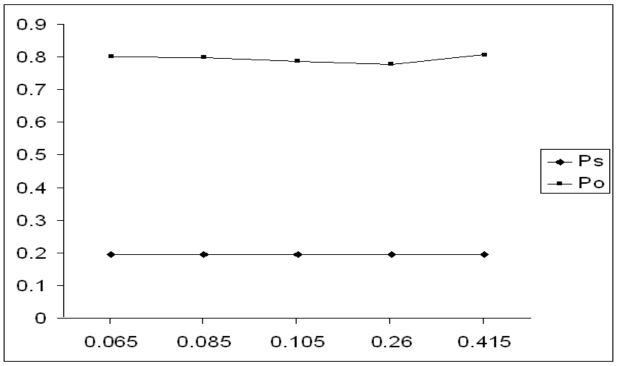


Figure 4. The Steady state probabilities for different values of sensing event probability, for a network of size 600, for the ratio 1:3 of type I and type II nodes

V. CONCLUSION

The probabilities for staying in the off, sense/receive, transmit state for a heterogeneous (the nodes have different sensing range and transmission range) two dimensional network are calculated and represented graphically which ensure both coverage and connectivity. A general probabilistic Markov model is proposed in which each sensor node makes an independent decision regarding which state to be in at a given time and is illustrated using a three state model with a transmit, receive/sense and off state, and the nodes can be in either of the three states at any given point of time viz. transmit, receive/sense and off state.

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VII. APPENDIX

Proof of Theorem 3. We can inscribe a square of side $\Delta = r_S\sqrt{2}$ in a circle of radius r_S . The coverage by disks will then be no less then the coverage by the squares. Let A' be the area not covered by the squares, then G≤U. Defining the coverage function $f_{\mathrm{A}}(y)$ for the squares analogously to (4), we find that $E[\mathrm{A}] = (1-\Delta^2 pr_S)\mathrm{N}$

. $E[\mathrm{A}^2] = \int dy \int dz\, f_{\mathrm{A}}(y) f_{\mathrm{A}}(z)$.The $f_{\mathrm{a}}(y) f_{\mathrm{A}}(z)$ term in the integrand is the probability that both points *y* and *z* are not covered. Let A_W denote the square centered at the point $W \in R$. Then the probability that both points *y* and *z* (in the integrand) are not covered is given by the probability that all the sensing squares are outside $\mathrm{A}_x \cup \mathrm{A}_y$, so $E[\mathrm{A}^2] = \int dy \int dz \left(1 - p_{S_1}\left|\mathrm{A}_x \cup \mathrm{A}_y\right|\right)^{\mathrm{N}}$. In the integral, let $y = (y_1, y_2)$ and $z = (z_1, z_2)$ If $|y_1 - z_1| \ge \Delta$ or $|y_2 - z_2| \ge \Delta$ then $\left|\mathrm{A}_y \cup \mathrm{A}_z\right| = 2\Delta^2$. Otherwise, $\left|\mathrm{A}_y \cup \mathrm{A}_z\right| = 2\Delta^2 . - (\Delta - |y_1 - z_1|)(\Delta - |y_2 - z_2|)$. Fix z in the y integral. The area over which z can range with A_z disjoint from A_y is $1 - 4\Delta^2$. This area thus contributes $(1-4\Delta^2)(1-2\Delta^2)^{\mathrm{N}}$ to the integral. Over the remaining area, changing coordinate in the z integral so that its origin lies at *y,* this contribution to the integral (over the area when two squares overlap) becomes

$$I = 4\int dy \int dz \left(1 - 2pr_S\Delta^2 + pr_S(\Delta - z_1)(\Delta - z_2)\right)^{\mathrm{N}},$$
$$0 \le z_1, z_2 \le \Delta$$

A computation to perform these integrals then leads to the following result, after adding the contribution from the part of the integral over the region where A_y and A_z are disjoint.

$$E[\mathrm{A}^2] = (1 - 2pr_S\Delta^2)^{\mathrm{N}} \left(1 + 4pr_S\Delta^2 \sum_{i=1}^{\mathrm{N}} \binom{\mathrm{N}}{i} \frac{\lambda^{\mathrm{i}}}{(i+1)^2}\right),$$

where $\lambda = pr_S\Delta^2 / (1 - 2pr_S\Delta^2)$. Using the fact that $\mathrm{var}(\mathrm{A}) = E[\mathrm{A}^2] - E[\mathrm{A}]^2$ and $E[\mathrm{A}]^2 = (1 - 2pr_S\Delta^2)^{\mathrm{N}}(1 + pr_S\Delta^2\lambda)^{\mathrm{N}}$, we arrive at

$$\mathrm{var}(\mathrm{A}) = \left(1 - 2pr_S\Delta^2\right)^{\mathrm{N}} \sum_{i=1}^{\mathrm{N}} \binom{\mathrm{N}}{i} \lambda^i$$

$$\left(\frac{4pr_S\Delta^2}{(i+1)^2} - \left(pr_S\Delta^2\right)^i\right),$$

$$\leq 4pr_S\Delta^2\left(1 - 2pr_S\Delta^2\right)\mathrm{N}\sum_{i=1}^{\mathrm{N}} \binom{\mathrm{N}}{i} \frac{\lambda^i}{(i+1)^2},$$

$$\leq 4pr_S\Delta^2 e - 2\mathrm{N}p_S\Delta^2 \sum_{i=1}^{\mathrm{N}} \binom{\mathrm{N}}{i} \frac{\lambda^i}{(i+1)^2}.$$

Let $F(i) = \binom{\mathrm{N}}{i} \frac{\lambda i}{(i+1)^2}$, then we can bound the sum by $\mathrm{n}_1 \max_i F(i)$, so we bound $F(i)$. $F(i)$ is a very sharply peaked function of *i*. Its maximum occurs at i^* for which $F(i^*)/F(i^*-1) \geq 1$ and $F(i^*+1)/F(i^*) < 1$. since this condition can be solved for i^* to give $i^* = \mathrm{N}\lambda/(1+\lambda) + \Theta(1/\mathrm{N}\lambda)$. Using the fact that $\binom{\mathrm{N}}{i^*} \leq \left(en/i^*\right)^{i^*}$, we get the following bound,

$$\frac{4pr_S\Delta^2 \exp\left(-2\mathrm{N}\Delta^2 p_S + \frac{\mathrm{N}\lambda(1+\log(1+\lambda))}{1+\lambda} + \Theta\left(\frac{1}{\mathrm{N}\lambda}\right)\right)}{\frac{\mathrm{N}\lambda}{1+\lambda}\left(1 + \Theta\left(\frac{1}{\mathrm{N}\lambda}\right)\right)}$$

Noting that for $r \leq 1/2\sqrt{2}$, $\Delta \leq \frac{1}{2}$, hence $(1+\log(1+\lambda)) -$, we get that

$$\mathrm{var}(\mathrm{A}) \leq \frac{4\exp\left(-\frac{\mathrm{N}\Delta^2 pr_S}{10} + \Theta\left(\frac{1}{\mathrm{N}\Delta^2 pr_S}\right)\right)}{\mathrm{N}\Delta^2 pr_S\left(1 + \Theta\left(\frac{1}{\mathrm{N}\Delta^2 pr_S}\right)\right)},$$

Since $\mathrm{N}\,\Delta^2 pr_S = \frac{2}{\pi}\mu(\mathrm{N})$,

we have that,

$$\mathrm{var}(\mathrm{A}) \leq \frac{2\pi\exp\left(-\frac{\mu(\mathrm{N})}{5\pi} + \Theta\left(\frac{1}{\mu(\mathrm{N})}\right)\right)}{\mu(\mathrm{N})\left(1 + \Theta\frac{1}{\mu(\mathrm{N})}\right)},$$

Since $E[\mathrm{A}] \leq e - \frac{2}{\pi}\mu(\mathrm{N}) \leq e - \frac{(1-\in)}{10\pi}\mu(\mathrm{N})$.

We can now apply the Markov inequality to A to get

$$\mathrm{PR}\left[\mathrm{A} \geq 2\mathrm{e} - \frac{(1\text{-}\in)}{10\pi}\mu(\mathrm{N})\right] < \frac{2\pi\exp\left(-\frac{\in\mu(\mathrm{N})}{5\pi} + \Theta\frac{1}{\mu(\mathrm{N})}\right)}{\mu(\mathrm{N})\left(1 + \Theta\left(\frac{1}{\mu(\mathrm{N})}\right)\right)}$$

Noting that $\Pr[\mathrm{A} \geq v] \leq \Pr[\mathrm{A} \geq v]$ for any v, we get the required bound.

Proof of Theorem 6. Conditions T1 and T3 of path connectivity for a large enough area (of size $\geq 1-\eta$) are implied by condition *(i)*in the t1heorem and Proposition 4. It remains to show that the disk graph obtained from nodes in the sensing state is connected with probability 1 in the limit. Let n_{S1} and n_{S2} be the number of type I and type II sensing nodes (randomly scattered). Then, on account of the independence assumption, N_S is a binomial random variable, , and so the Chernoff bound, [6], gives $P[\mathrm{N}_S < (1-\in)\mathrm{N}pr_S] < \exp(-\mathrm{N}pr_S \in^2/2)$. Since $\mathrm{N}\,pr_S \to \infty$, we have that $\Pr[\mathrm{N_S} \geq (1-\in)\mu] \to 1$. Let $\Pr[\mathrm{T2}]$ be the probability that condition T2 holds, and let $\mathrm{N}_S(\in) = (1-\in)\mathrm{N}pr_S$

Then,

$$\Pr[\mathrm{T2}] \geq \Pr[\mathrm{T2} \mid \mathrm{N}_S \geq \mathrm{N}_S(\in)]\Pr[\mathrm{N}_S \geq \mathrm{N}_S(\in)]$$

$$c(\mathrm{N}_S) = \pi r^2\mathrm{N}_S - \log\mathrm{N}_S \to \infty,$$

because $\mathrm{N}_S \geq (1-\in)\mathrm{Nr}p_S$, and $\mathrm{N}\,pr_S \to \infty$, and so from theorem 5, have that $\Pr[\mathrm{T2}|\mathrm{N}_S \geq \mathrm{N}_S(\in)] \to 1.$ Since, we also have that $\Pr[\mathrm{N}_S \geq \mathrm{N}_S(\in)] \to 1.$ and $\Pr[n_{S2} \geq n_{S2}(\in)] \to 1.$, we then have that $\Pr[\mathrm{T2}] \to 1.$ So there is sufficiently large area for which we have that $\Pr[\mathrm{T1}] = 1 - e_1(\mathrm{N})$, $\Pr[\mathrm{T2}] = 1 - e_2(\mathrm{N})$, and $\Pr[\mathrm{T3}] = 1 - e_3(\mathrm{N})$, and for that area, where $e_i(\mathrm{N}) \to 0$. By the union bound, $\Pr[\approx T1 \vee \approx \mathrm{T2} \vee \approx \mathrm{T3}] \leq e_1(\mathrm{N}) + e_2(\mathrm{N}) + e_2(\mathrm{N}) \to 0,$ hence we conclude that $\Pr[\approx T1 \vee \mathrm{T2} \vee \mathrm{T3}] \to 1$ for a sufficiently large area, proving that the network is path connected on a sufficiently large area, with probability 1 in the asymptotic limit.

AUTHORS

Prof. G. N. Purohit is a Professor in Department of Mathematics & Statistics at Banasthali University (Rajasthan). Before joining Banasthali University, he was Professor and Head of the Department of Mathematics, University of Rajasthan, Jaipur. He had been Chief-editor of a journal.His present interest is in O.R., Discrete Mathematics and Communication networks. He has published around 40 research papers in various journals.

Mrs. (Dr.) Seema Verma obtained her M.Tech and Ph.D degree from Banasthali University in 2003 and 2006 respectively. She is a associate Professor of Electronics. Her research areas are VLSI Design, communication Networks. She has published around 18 research papers in various Journals.

Megha Sharma received the B.C.A and M.C.A degree from I.G.N.O.U in 2004 and 2008, respectively. She is currently working towards a Ph.D degree in computer Science at the Banasthali University of Rajasthan. Her research interests include wireless sensor networks with a focus on the coverage of wireless sensor networks.