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LMS-RLS JOINT ADAPTIVE EQUALIZATION IN WIRELESS COMMUNICATION

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Abstract: Equalization is a most widely used optimization scheme to reduce inter-symbol interference (ISI).Intersymbol interfering environment is a case of misrepresentation of the sent signal at the receiver. Differentsymbols can hinder with one another which results innoise and less relevant, reliable signal. Multipath propagation means a wireless signal from a spreader reaches the receiver through numerous paths. If inter symbol interfering occurs within an arrangement, it must be reduced to the most minimal quantity conceivable. The inter symbol interference is reduced through the use of adaptive algorithms; LMS (Least Mean Square Error) and RLS (Recursive Least Square) equalization procedures being the most prominent ones. In this work, a joint adaptive algorithm is proposed and simulated. A novel method involving both LMS and RLS is used for suppression of Inter symbol interference. The LMS has the benefit of a fast convergence rate but has a significant value of mean square error (MSE).However, the RLS complements it with low value of MSE and a slow convergence rate . Thus, a joint RLS and LMS is proposed The proposed method results in improved BER performance, lesser mean square error and a faster convergence**.**

Keywords: LMS; RLS; MSE;BER.

I. INTRODUCTION

Due to sparseness of frequency spectrum, results in filtration of the transmitted signal to limit its bandwidth with the objective of efficient sharing of frequency resource. Not only this, there exist many channels which are bandpass practically and are dispersive, so that efficient sharing of the frequency resource can be achieved. Moreover, many practical channels are bandpass and, in fact, they often respond differently to inputs with different frequency components, i.e., they are dispersive. To adjust to the time varying properties of a communication channel, we require an adaptive equalizer. For adaptive equalization we apply a same input to an unknown system and to the adaptive equalizer. On comparing the outputs an error signal is generated which is used to manipulate the filter coefficients of an adaptive system. After a considerable number of iterations, the transfer function of an adaptive filter gets converged to unknown systems transfer function. Therefore it can be said that an adaptive equalizer tracks the unknown properties and time varying characteristics of a channel. Therefore,it is capable of removing the noise satisfactorily [1].

II. AN ADAPTIVE FIR FILTER

An adaptive FIR transversal filter which is discrete in time domain and has a filter length N is shown in Figure 1.

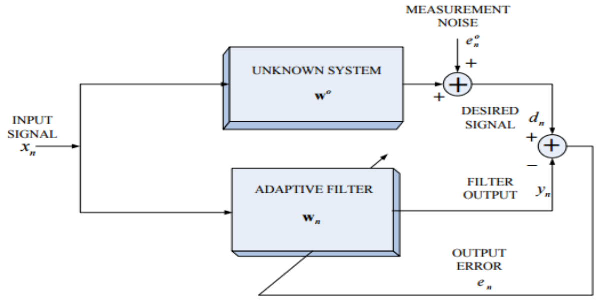

Figure 1.An adaptive filter.

N signal samples $x_n, x_{n-1}, \dots, x_{n-N+1}$ are applied as input to the filter and they are taken from a tap-delay line. The filter coefficients are $w_n(0), w_n(1) \dots w_n(N-1)$, are referred to as the filter weights. A desired response d_n is applied to the filter that helps in the adjustment of the filter weights in such a way so as to make the filter output y_n resemble d_n. e_n is the estimation error that measures the difference between the filter output y_n and the desired response d_n. Therefore we have,

$$e_n = d_n - y_n \quad (1)$$

$$= d_n - w_n^T x_n \quad (2)$$

Where $w_n = [w_n(0), w_n(1) \dots w_n(N-1)]^T$ represents the weight vector.And $x_n = [x_n, x_{n-1} \dots x_{n-N+1})]^T$ represents the input vector.y_n is the product of x_n and w_n. If the desired response y_n and y_n are assumed to be jointly stationary then MSE (mean square error) can be expressed as

$$\varepsilon(w) = E[e_n^2] (3)$$

$$E[d_n^2] = 2w^T P + w^T R w \quad (4)$$

$$R \triangleq E[x_n x_n^T] p \triangleq E[d_n x_n]$$

For determining a point where MSE is minimized, we take derivative of w.r.t w and set it to 0.The result obtained is the Wiener Hopf equation.

$Rw^0=p$ (5)
And Wiener Hopf solution can be written as

$w^0 = R^{-1}p$(6)
The adaptive filter shows an optimal response to such a solution.
For real time applications, we don't use the inverse auto-correlation matrix method to obtain the Wiener solution, instead a recursive adaptive algorithm method is used to get the Wiener solution by making use of a number of iterations. Therefore, an adaptive filter involves an adjustment of the weight vector in accordance with the estimation error. For implementing the adaptation process, an adaptive algorithm updates the weight vector by performing a number of iterations. After sufficient iterations the transfer function of adaptive filter converges to that of the channel.There are many real world applications in which adaptive algorithms have been studied such as speech processing, communications, radar, sonar, or biomedicine, require that the optimal filter or system coefficients need to be adjusted over time depending on the input signal[2].

III. ADAPTIVE ALGORITHMS

A. *LMS Algorithm*

It was invented by Widrow and Ho and is an extremely simple and robust algorithm The LMS algorithm is an approximation of the steepest-descend algorithm which uses aninstantaneous estimate of the gradient vector of the performance function [3].It is regarded as an important member of the family of stochastic gradient algorithms. Before discussing the LMS algorithm, the steepest descent optimization method is to be studied first. We assume the MSE function ε(w) to be continuously differentiable. The gradient vector Δε(w) of ε(w) points in the inverse direction minimum value of MSE. If we increase the weights in the direction opposite to that of the gradient vector by a small amount, the weights will be pushed very near to the minimum point in the weight space. Mathematically, it is represented as

$$w_{n+1} = w_n - \mu \frac{\partial \varepsilon(w)}{\partial w_n}(7)$$

where n represents the iteration number, and μ is the step-size parameter. Substituting w_n into the MSE function , and calculating its derivative yields
$\frac{\partial \varepsilon(w)}{\partial w_n} = 2Rw_n - 2p$Substituting in(7) we get,

$$w_{n+1} = w_n - 2\mu(Rw_n - p)$$

This represents the steepest descent algorithm.
To apply the steepest descent algorithm an exact measurement of the gradient vector is required which may not be feasible practically. Estimation of the gradient vector has to be based on the data available .A way forward is to use the instantaneous gradient vector which is given by:

$$\frac{\partial \hat{\varepsilon}(w_n)}{\partial w_n} = \frac{\partial e_n^2}{\partial w_n}$$

$$=\frac{\partial (d_n - w_n^T x_n)^2}{\partial w_n}$$

$$=-2e_n x_n(8)$$

Making use of stochastic gradient vector ,we get

$$w_{n+1} = w_n + 2\mu e_n x_n \quad (9)$$

This is the update for LMS algorithm.Theconvergence of the LMS algorithm requires convergence of the mean of w_n toward w_0 and also convergence of the variance of elements of w_n to some limited values[3].

When huge number of weights which tend towards infinity are used in an LMS algorithm, Butterweck[4], in his studies derived a limit on the step size parameter,μ.
Thus, $0 < \mu < \frac{1}{NS_{max}}$ is the condition which provides for stability of LMS algorithm.Until this day, a final and exhaustive list of conditions for LMS algorithm convergence is still subject to a lot of debate[5].
A plethora of schemes which deal with utilization of have been proposed[6,7,8,9,10] to meet the desirable features like faster speed of convergence and low steady state error. In recent times it was proved in [11] that LMS algorithm with variable step size which was developed by Kwong and Johnsten in[12] is by far the best of all low complexity variable step-size LMS algorithm available in literature barring a drawback related to noise power measurement.

B. *RLS Algorithm*

Despite many advantageous properties of LMS algorithm, the major drawback is its slow convergence which may limit its use in applications where fast convergence is aimed at. In this regard, the exact least squares algorithms will be more useful.
Unlike the LMS algorithm minimizes the error generated per iteration only based on the current values of input data, the RLS algorithm is used to find an exact least squares solution per iteration utilizing all past data[5].In comparison to LMS algorithm,the RLS algorithm is less beneficial when we talk in terms of complexity[13][14].
It's a basic representative of the recursive algorithms class based on the theory of Kalmans filtering [15], time averaging [15] and the method of least squares[15].A detailed description andderivation of the RLS algorithm can be found in [15].
The cost function of the least squares is shown below:

$$J_n = \frac{1}{2}\sum_{k=1}^{n} r^{n-k} e_k^2(10)$$

Where e_k is the error at output and $r \in [0,1]$ is the exponential weighting factor.
To minimize the cost function is same as to find the derivativewith respect to w_n which yields
$\frac{\partial J_n}{\partial w_n} = \sum_{k=1}^{n} r^{n-k} x_k x_k^T w_n - \sum_{k=1}^{n} r^{n-k} x_k d_k(11)$
Estimation of autocorrelation matrix R and cross correlation pleads to estimated values of R_n and p_n

$$R_n = \sum_{k=1}^{n} r^{n-k} x_k x_k^T(12)$$

$p_n = \sum_{k=1}^{n} r^{n-k} x_k d_k(13)$
Putting R_n and p_n in the above equations and by setting the result to 0 we get:

$$R_n w_n = p_n(14)$$

Addition of a regularizing term $\delta r^n \|w_n\|^2$ to the cost function and then solving the normal equation gives least squares solution.

$$w_n = R_n^{-1} p_n \quad (15)$$

Here δ is the positive regularization parameter.
In recursive form R_n and p_n are expressed as:

$$R_n = \gamma R_{n-1} + x_n x_n^T$$
$$p_n = \gamma p_{n-1} + x_n d_n$$

Putting these values in least squares solution

$$w_n = R_n^{-1} p_n$$

results in

$$w_n = (\gamma R_{n-1} + x_n x_n^T)^{-1} (\gamma p_{n-1} + x_n d_n)(16)$$

On application of matrix inversion lemma to the above equation and defining priori as

$$e_n^{'} = d_n - x_n^T w_{n-1}$$

We get

$$w_n = w_{n-1} + G_n x_n e_n^{'}(17)$$

The gain matrix is defined as $G_n = \frac{r^{-1} R_{n-1}^{-1}}{1+r^{-1} x_n^T R_{n-1}^{-1} x_n}$

The above equations are updating forms of RLS algorithm and initially the weight vector is set to 0.

In the RLS Algorithm the estimate of previous samples of output signal, error signal and filter weight is required that leads to higher memory requirements[16].

IV. JOINT ADAPTIVE EQUALIZATION

Two major families of adaptive algorithms, i.e., the LMS algorithms and the RLS algorithms, have been described. Properties of the LMS algorithm show its simplicity, robustness and good tracking capabilities as well as its slow convergence and sensitivity to the step-size parameter. Properties of the RLS algorithm show its fast convergence independent of the input eigenvalue spread, but also show its high computational complexity, numerical instability and poor tracking capabilities. The LMS algorithm and the RLS algorithm are extreme implementations in terms of the simplicity and convergence.The LMS has the advantage of low computation complexity but its convergence is very slow.However, the RLS is opposite with fast convergence and high computation. They are exactly complementary. Thus, a jointLMS and RLS is proposed.In the first stage, the RLS algorithm is initially used toobtain the weight coefficients of relative balance state. Then,the LMS algorithm works to equalize received signals andupdating weights each N-symbols.

The joint RLS and LMSalgorithm repeatedly runs with low complexity and fastconvergence. In addition, the configuration of proposedalgorithm should carefully consider the orders of RLS andLMS. It's important for RLS and LMS algorithms to inheritweights from each other and exert their respective advantagesat the same time.Joint RLS and LMS algorithm is mainly based on Minimum Mean Square Error (MMSE) criterion in stable working mode.The results obtained are compared on the basis of parameters likeBER performance ,Convergence rate ,Mean square error.

V. RESULTS AND DISCUSSION

On simulation in MATLAB 2015A by applying different algorithms we get the results discussed in the coming section.

The Performance of the filter can be evaluatedby analyzing the convergence rate, MSE and BER .TheBER can bedefined asthe ratio of the number of bit errors detected in the receiver to the total no. of bits transmitted.The convergence speed is the rate at which the filter gets converged to its resultant state. A faster convergence rate is a desirable feature of an adaptive system.The mean square error (MSE) is a performance metric indicating how well a system can adapt to a given solution. A small minimum MSE is an indication that the adaptive system has accurately modeled, predicted, adapted and/or converged to a solution for the system.

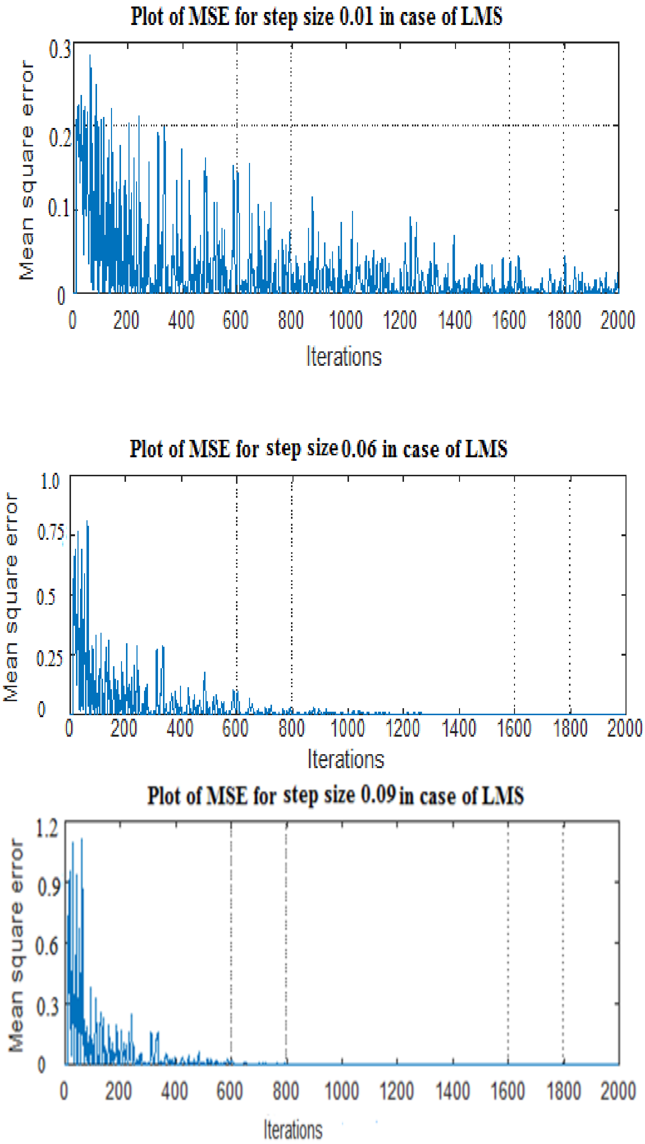

Figure 2. Plot of MSE for different step sizes in case of LMS

Figure 2.shows that in case of LMS ,when step size is less, the mean square error (MSE) is less but the rate of convergence is slow. On increasing the step size we obtain a fast response to changes incurred but at the same time MSE increases considerably. Therefore it is essential to maintain a tradeoff between MSE and convergence rate. For that the value of step size is to be chosen carefully.Convergence rate and MSE are therefore not, however, independent of all of the other performance characteristics. There will be a tradeoff, in other performance criteria, for an improved convergence rate and there will be a decreased convergence performance for an increase in other performance. Same happens in case of MSE.

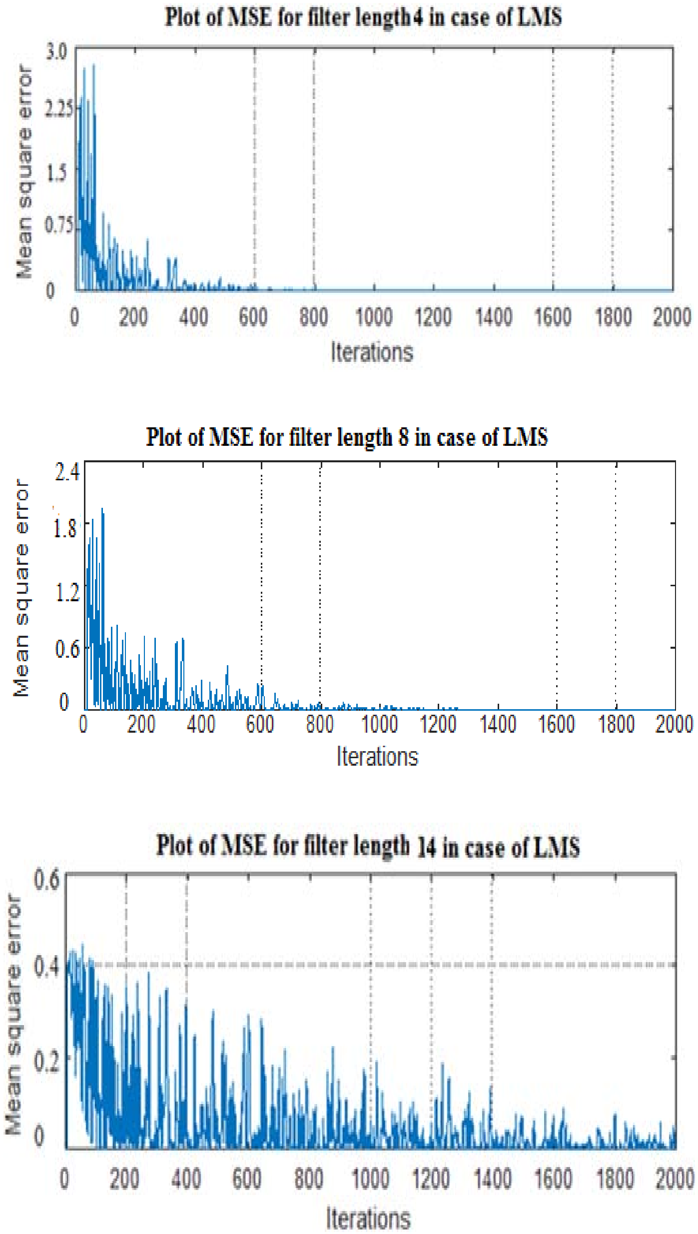

Figure 3. Plot of MSE for different filter lengths in case of LMS

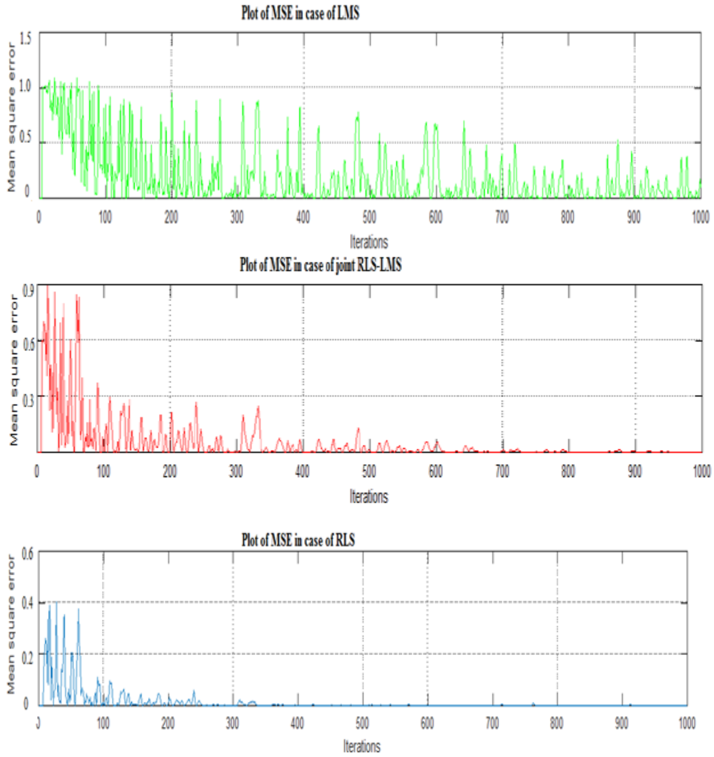

Figure 4.MSE performance of RLS, proposed algorithm and LMS

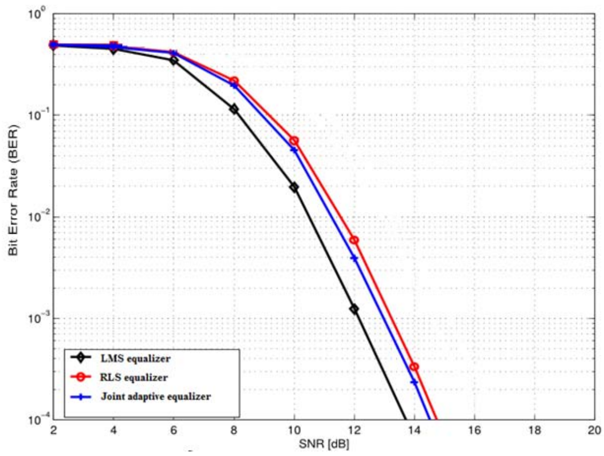

Figure 5. BER performance of channel equalization using RLS,LMS and proposed scheme.

Figure 3.shows that in case of LMS algorithm when filter length is 4 , the value of MSE is less but it takes more time to converge. If we increase the length of the filter the MSE increases but it has a faster response. Like the case of step size there should be a balance between these two parameters.

Figure 4.shows the MSE performance of RLS, proposed algorithm and LMS.As we can clearly see our proposed algorithm offers a middle path between RLS and LMS, thereby balancing MSE and convergence rate.

In figure 5 we can clearly see that the BER performance of the proposed algorithm is much better than the RLS algorithm and slightly lesser than the LMS scheme. Since LMS has better tracking abilities than RLS, it's BER performance is better. Hence the BER performance of Joint RLS-LMS lies in between the two.
The results obtained can be summarized in the table I:

Table I.

Sr. No.	*Algorithm*	*Convergence rate*	*Mean square error*	*BER Performance*
1	RLS	More	Less	Less than LMS
2	Joint LMS-RLS	In between	In between	In between
3	LMS	Less	More	Better

VI. CONCLUSION AND FUTURE SCOPE

Channel equalization is typically developed as an effective way of anti-fading method. Among various adaptive algorithms, LMS and RLS are most widely used for appropriate complexity and good performance the features of LMS and RLS algorithms are compared. The LMS has the advantage of low computation complexity but its convergence is very slow. However, the RLS is opposite with fast convergence and high computation. They are exactly complementary. Thus, a joint RLS and LMS is proposed.

In theProposed algorithm,the RLS algorithm is initially used toobtain the weight coefficients of relative balance state. Then,the LMS algorithm works to equalize received signals andupdating weights each N-symbols. The joint RLS and LMSalgorithm repeatedly runs with low complexity and fast convergence. In addition, the configuration of proposed algorithm should carefully consider the orders of RLS and

LMS. It's important for RLS and LMS algorithms to inherit weights from each other and exert their respective advantages at the same time.Joint RLS and LMS algorithm is mainly based on Minimum Mean Square Error (MMSE) criterion in stable working mode. The average BER is used as the performance measure for the various digital signal processing techniques discussed in this dissertation. The average frame error-rate and, more generally, the statistical distribution of the BER's are also useful performance measures, and should be investigated in the future. In addition to multipath mitigation techniques, error control mechanisms are also necessary for establishing a reliable wireless communication link. Error detecting codes and automatic request for retransmission (ARQ) protocols should be investigated in the future. Other variants of the recursive least-squares (RLS) algorithm should be investigated. The combination of the RLS and least-mean-square (LMS) algorithms should be studied.

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