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A visit on Maximum Covering and Annulus Problem

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Abstract: Maximum Covering Location Problem sometimes allocate fixed number of facilities to meet maximum demand of customers. We first review the existing works on Maximum Covering Location Problems in facility location. Then we discuss about various annuls problems. In each such problem, a future direction of research is indicated.

Keywords: Computational geometry, Minimum enclosing square, Sweep line algorithm.

I. INTRODUCTION

In Covering Location Problem, objective is to install minimize number of facilities to meet the demand of each customer. This Covering Location Problem is some times referred as Locational Set Covering Problem (LSCP) [19, 41]. In contrast, in Maximum Covering Location Problem (MCLP) [27, 19], the restriction of giving services to all customers is waived due to lack of resources. In realistic point of view, MCLP admits that resources may be insufficient to address the total demand. Here the objective is to install fixed (constant) number of facilities to serve maximum number of customers. The initial research in MCLP starts around early seventy's using non deterministic algorithmic techniques. Later in early eighty's deterministic algorithms are used to deal these problems. Our discussion starts with the important results of MCLP along with some open problem in this field. Then the existing works in various Annulus Problem are discussed and directions for further research are given.

II. MAXIMUM COVERING LOCATION PROBLEM

Maximum Covering Location Problem admits that resources may be insufficient to address the total demand. MCLP was originally stated and solved by Chruch and ReVelle, [36], who offered three initial approaches to the problem. The first approach, dubbed greedy adding (GA), began with the single facility that provided the maximal possible demand coverage and successively added those facilities, one at a time, which incrementally increased the coverage values the most. The second approach, called greedy adding with substitution (GAS), utilized the GA algorithm as the backbone. Besides these heuristics approaches, the author showed that linear programming was effective in producing 0,1 solution for problems upto 55 nodes in about 80 percent of cases. When linear programming produced fractional solutions, it was only necessary to supplement it by modest amounts of branch and bound. Hillsmon [21] as well as Church and ReVell [37] noted the equivalence of a data-modified p-median and MCLP and pointed out that any heuristic (such as vertex substitution) useful for the p-median could also utilized for MCLP. Mauricio G.C. Resende [38] also study MCLP and present a greedy randomized adaptive search procedure (GRASP) that addresses maximum demand, though not necessarily optimum. They also describe a well-known upper bound on the maximum coverage which can be computed by solving a linear program and show that on large instances, the GRASP can produce facility placement

that are nearly optimal. Lorena and Pereira [35] report results obtained with a Lagrangean/surrogate heuristic using a sub-gradient optimization method, as a complement to the dissociated Lagrangean and surrogate heuristic presented in Galvao et al. [7]. Arakaki and Lorena [26] present a constructive genetic algorithm to solve real case instances with up to 500 vertices.

The formulation for planar maximum enclosing problems, where facility can be placed anywhere on the plane, has been also studied by several authors [17, 32, 33, 34]. For the Euclidean distance measure, candidate points would be the points of intersection of circles drawn around the demand points. Similarly, for rectilinear distances, the candidate facility locations would be the points of intersection of diamond shaped boundaries around demand points [17]. In [34], Maherez et al. developed an algorithm for a facility that is *somewhat desirable* and named it ``maximin-minimax'' facility location. Their method involves finding the set of intersection points of any two lines forming the equi-rectilinear distances from the demand points. The techniques used in [17, 32, 33, 34] and in the standard location problem models discussed in books on location theory [23, 24, 29] are based on equidistance shapes. Ventura and Dung [42] studied parts inspection with rectangular and square shapes. Their technique used a Euclidean least-square methods to determine the optimal parameters of the straight lines defining the edged of the part being inspected. Bespamyatnikh and Segal [16] solved covering a set of points by two axis-parallel boxes. No inclination angle or partial covering is proposed in [16].

A closely related problem is to find a placement of one or more geometric objects of same type so that objects cover maximum number of points from a planar set P of n points. These so called problems of maximal covering by convex objects has also received attention of many researchers. Barequet et al. [14] find a translation of a given convex polygon Q to enclose maximum number of points from a planar point set P in $O(nk\log(mk)+m)$ time using $O(m+n)$ space; m is the number of vertices of the convex polygon Q. Barequet et al. [15] define δ -annulus of a convex polygon Q as the closed region defined by all planar points set P at distance at most δ from the boundary of polygon Q. Given convex polygon Q with m vertices and a distance δ, they propose an $O(n^3\log(mn)+m)$ time and $O(m+n)$ space algorithm to compute an arbitrary oriented δ -annulus region that contains maximum number of points from the point set P. Katz and et al. [28] also studied the minsum coverage problem to place undesirable facility within an axis-parallel rectangle of fixed size. They proposed an algorithm that runs in $O(n\log n)$ time and $O(n)$ space. Younies et al. [43] introduce a zero-one mixed integer formulation for maximum covering problem where points are covered by inclined parallelograms in a plane. Sitting directional antennas is one of the applications where parallelogram shapes would be useful. Mahapatra et al. [30] first proposed an optimal $O(n\log n)$ time and $O(n)$ space algorithm for maximal covering by two disjoint axis-parallel unit squares. They also present an $O(n^2)$ time and space algorithm to compute a pair of disjoint or overlapping squares that contains maximum number of points from P. In case of overlapping, the overlapping region does not contains any point from P. Díaz-Báñez et al. [20] proposed algorithms for maximal covering by two disjoint axis-parallel unit squares and circles in $O(n^2)$ and $O(n^3\log n)$ time respectively. Later, the time complexities were improved to $O(n\log n)$ and $O(n^{8/3}\log^2 n)$ respectively [18]. They [18] also find two parallel disjoint squares of arbitrary orientation so that these two squares contain maximum number of points from P. Their algorithm runs in $O(n^3)$ time and $O(n)$ space. However, the problem of computing two arbitrary oriented squares that contain maximum number of points from P is still open. Recently, Mahapatra et al. [31] proposed an $O(n\log^2 n)$ time and $O(n)$ space algorithm that computes a pair of axis-parallel squares containing maximum number of points from P. In this case, a pair of square may be disjoint or overlapping. Some open problems in this context are the query versions of the above problem.

Problem 1 *Given a point set P, an axis parallel square S and a point $p \in P$, locate the square S so that it contains maximum number of points from P and S contains the point p.*

The immediate generalization of this problem is that the square S is arbitrary oriented.

III. ANNULUS PROBLEM

In computational metrology [10], an important task is to find a geometric object that fits nicely to a set of planar points. This immediately leads to the *minimum width annulus* problem. Minimum width annulus problem computes two concentric circles such that all planar points of $P=\{p_1,p_2,\ldots,p_n\}$ are contained by the annulus thus formed and the difference of the two radii is minimum. Wainstein [44] and Roy et al. [39] propose different $O(n^2)$ time algorithm to solve minimum width annulus problem. Ebara et al. [22] prove that the center of the optimal annulus is either a vertex of the closest-point Voronoi diagram of P, or a vertex of the farthest-point Voronoi diagram, or an intersection point of a pair of edges of the two diagrams. Based on this observation, they propose an $O(n^2)$ time algorithm. All such intersections are computed by Guibas and Seidel [25]. One can develop simple $O(n\log n+k)$ time algorithm for computing minimum width annulus; k is the number of these intersections. Parametric search is used by Agarwal et al. [4] to compute a minimum width annulus in $O(n^{\frac{8}{5}+\varepsilon})$ time. Later, Agarwal and Sharir [2] improve the expected running time to $O(n^{\frac{3}{2}+\varepsilon})$. Other variations of minimum width annulus problem depends on the *shape* of the geometric object as well as the *distance metric* used to compute the width of an annulus.

Duncan et al. [8] and Bose et al. [5] consider another variation of minimum width annulus problem. Given the radius r of the *outer* (or inner, or median) circle enclosing the point set, both of these works independently compute the minimum width circular annulus in $O(n\log n)$ time. For a set P of n points in the Euclidean plane, Díaz-Báñez et al. [9] consider the problem of computing an empty annulus A of largest width. For this the points set P is partitioned so that no point $p(\in P)$ lies in the interior of A and width of the annulus is minimum. They propose an $O(n^3\log n)$ time and $O(n)$ space algorithm for this problem.

Barequet et al. [15] first use non-convex object for minimum width annulus problem. They define δ-annulus of a convex polygon Q as the closed region defined by all planar points set P at distance at most δ from the boundary of polygon Q. Given a convex polygon Q with m vertices and a distance δ, they propose an $O(n^3\log(mn)+m)$ time and $O(m+n)$ space algorithm to compute a δ-annulus (of arbitrary orientation) region that contains maximum number of points from P. Gluchshenko et al. [11] present an *optimal* $O(n\log n)$ time algorithm for computing a *rectilinear annulus* of minimum width which to enclose a point set P. See the papers [1, 3, 6] for further study, motivation and applications.

The following variation of annulus problem is still open. One can find a minimum *width rectangular annulus* that encloses the points of P. Rectangular annulus is a pair of rectangles, the inner rectangle fully contained within the outer rectangle and the corresponding sides of the inner and outer rectangles are mutually parallel. Here we need to define *annular distance* as the distance between the corresponding sides of the inner and outer rectangle of a rectangular annulus and width of a rectangular annulus is the maximum distance among the four possible annular distances. This minimum width rectangular annulus can be computed for fixed as well as arbitrary orientation.

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