DOI: http://dx.doi.org/10.26483/ijarcs.v8i9.4930

ISSN No. 0976-5697

Volume 8, No. 9, November-December 2017

International Journal of Advanced Research in Computer Science

RESEARCH PAPER

Available Online at www.ijarcs.info

INVERSE SIGNED DOMINATING FUNCTIONS OF CORONA AND ROOTED PRODUCT GRAPHS

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Abstract: Graph theory is an interesting subject in mathematics. Applications in many fields like Linguistics, Engineering communications, Physical Sciences, Coding theory, Computer networking and Logical Algebra. The theory of domination in graphs has a wide range of applications. Among these applications, the most often discussed is a coding theory and communication networks. Inverse domination theory of graphs which are the important branches of graph theory. In this paper, we study the maximal inverse signed dominating functions of corona product graph of a path with a complete graph and rooted product graph of a path with a cycle.

Keywords: inverse signed dominating functions, inverse signed domination number, corona product graph, rooted product graph.

1. INTRODUCTION

Mostly Product of graphs occurs in discrete mathematics. In 1970, Frucht & Harary [6] introduced a new product on two graphs G_1 and G_2, called corona product denoted by $G_1 \odot G_2$. The corona product of a path P_n with a complete graph K_m is a graph obtained by taking one copy of n-vertex path P_n and n copies of K_m and then joining the i^{th} vertex of P_n to every vertex of i^{th} copy of K_m and it is denoted by $P_n \odot K_m$, where $n>0$ and $m>0$. In 1978, Godsil and McKay [1] introduced a new product on two graphs G_1 and G_2, called rooted product denoted by $G_1 \circ G_2$. In this paper we consider the rooted product graph like, here P_n be a Path graph with n vertices and $C_m (m \geq 3)$ be a cycle with a sequence of n rooted graphs $C_{m1}, C_{m2}, C_{m3}, ---, C_{mn}$. Then by $P_n(C_m)$ we denote the graph obtained by identifying the root of C_{mi} with the i^{th} vertex of P_n. We call $P_n(C_m)$ the rooted product of P_n by C_m and it is denoted by $P_n \circ C_m$. Every i^{th} vertex of P_n is merging with any one vertex in every i^{th} copy of C_m. So in $G = P_n \circ C_m$, P_n contains n vertices and C_m contains (m-1) vertices in each copy of C_m.

In 1995, Dunbar, Hedetniemi, Henning and Slater [4] have studied about "Signed Domination in Graphs". Further we studied about signed domination in [2, 7]. In 1996, Favaron [5] have studied about "Signed domination in regular graphs". In 2010, Zhong-sheng [3] have studied about "On Inverse Signed Total Domination in Graphs". By using signed domination related parameters we can find out inverse signed domination parameters on product graphs.

2. RESULTS ON ROOTED PRODUCT GRAPH

Theorem 2.1: If m is divisible by 3 then the function $f: V \to \{-1, +1\}$ is defined by

$$f(v) = \begin{cases} +1, & \text{if } m \equiv 1 (\text{mod } 3) \text{ vertices in each copy of } C_m \text{ in } G, \\ -1, & \text{otherwise.} \end{cases}$$

is a maximal inverse signed dominating function of a graph $G = P_n \circ C_m$ and inverse signed domination number of G is $\gamma_s^0(G) = \left(\frac{-mn}{3}\right)$.

Proof: Consider the graph $G = P_n \circ C_m$ with $|V|$ number of vertices and $|E|$ number of edges.

Let f be a function defined in the hypothesis. Suppose m is divisible by 3.

Here +1 is assigned to $\left(\frac{m}{3}\right)$ vertices in each copy of C_m in G, -1 is assigned to all other vertices in G.

Case 1: Suppose $v \in P_n$ be such that

(i) As $d(v) = 4$ in G then

$$\sum_{u \in N[v]} f(u) = [(+1)+(-1)] + [(-1)+(-1)+(-1)] = -3.$$

(ii) As $d(v) = 3$ in G then

$$\sum_{u \in N[v]} f(u) = [(+1)+(-1)] + [(-1)+(-1)] = -2.$$

Case 2: Suppose $v \in C_m$ be such that $d(v) = 2$ in G then

$f(v) = -1$, $f(v) = +1$.

(i) Then $N[v]$ contains 2 vertices of C_m and one vertex of P_n in G.

If $f(v)=-1$ then $\sum_{u\in N[v]} f(u)=(-1)+[(-1)+(+1)]=-1$.

If $f(v)=+1$ then $\sum_{u\in N[v]} f(u)=(+1)+[(-1)+(-1)]=-1$.

(ii) Then $N[v]$ contains 3 vertices of C_m and zero vertex of P_n in G.

If $f(v)=-1$ then $\sum_{u\in N[v]} f(u)=(-1)+[(-1)+(+1)]=-1$.

If $f(v)=+1$ then $\sum_{u\in N[v]} f(u)=(+1)+[(-1)+(-1)]=-1$.

From the above cases the function f is an inverse signed dominating function, because $\sum_{u\in N[v]} f(u)<0, \forall v\in V$.

Now the maximality check for f, define $g: V\to\{-1,+1\}$ by

$$g(v)=\begin{cases}+1, \text{ if any one vertex } v=u_i\in P_n \text{ in } G,\\ +1, \text{ if } \left(\frac{m}{3}\right) \text{ vertices in each copy of } C_m \text{ in } G,\\ -1, \quad \text{otherwise.}\end{cases}$$

Here two cases are followed.

Case 3: Suppose $v\in P_n$ be such that

(i)As $d(v)=4$ in G, then $N[v]$ contains 2 vertices of C_m and three vertices of P_n in G.

Sub case 1: Let $u_i\in P_n$ in i^{th} copy of G then

$\sum_{u\in N[v]} g(u)=[(+1)+(-1)]+[(-1)+(-1)+(+1)]=-1$.

Sub case 2: Let $u_i\notin P_n$ in i^{th} copy of G then

$\sum_{u\in N[v]} g(u)=[(+1)+(-1)]+[(-1)+(-1)+(-1)]=-3$.

(ii)As $d(v)=3$ in G, then $N[v]$ contains 2 vertices of C_m and two vertices of P_n in G.

Sub case 1: Let $u_i\in P_n$ in i^{th} copy of G then

$\sum_{u\in N[v]} g(u)=[(+1)+(-1)]+[(-1)+(+1)]=0$.

Sub case 2: Let $u_i\notin P_n$ in i^{th} copy of G then

$\sum_{u\in N[v]} g(u)=[(+1)+(-1)]+[(-1)+(-1)]=-2$.

Case 4: Suppose $v\in C_m$ be such that $d(v)=2$ in G,

(i)Here $N[v]$ contains 2 vertices of C_m and one vertex of P_n in G then $g(v)=-1$ or $g(v)=+1$.

Sub case 1: Let $u_i\in P_n$ in i^{th} copy of G.

If

$g(v)=-1$ then $\sum_{u\in N[v]} g(u)=(-1)+[(+1)+(+1)]=+1(>0)$.

If

$g(v)=+1$ then $\sum_{u\in N[v]} g(u)=(+1)+[(-1)+(+1)]=+1(>0)$.

Sub case 2: Let $u_i\notin P_n$ in i^{th} copy of G.

If $g(v)=-1$ then $\sum_{u\in N[v]} g(u)=(-1)+[(+1)+(-1)]=-1$.

If $g(v)=+1$ then $\sum_{u\in N[v]} g(u)=(+1)+[(-1)+(-1)]=-1$.

(ii)Here $N[v]$ contains 3 vertices of C_m and zero vertex of P_n in G.

If $g(v)=-1$ then $\sum_{u\in N[v]} g(u)=(-1)+[(+1)+(-1)]=-1$.

If $g(v)=+1$ then $\sum_{u\in N[v]} g(u)=(+1)+[(-1)+(-1)]=-1$.

From the above cases, we get $\sum_{u\in N[v]} g(u)>0$, for some $v\in V$.

This implies that the function g is not an inverse signed dominating function. Hence f is a maximal inverse signed dominating function on G. Now inverse signed total domination number is the sum of the function value of all vertices in G, that is

$$\sum_{u\in V(G)} f(u)=\left(\underbrace{\frac{m}{3}(+1)}_{n-times}\right)+\underbrace{\left((m)-\frac{m}{3}\right)(-1)}_{n-times}=-mn+\frac{2mn}{3}=\frac{-mn}{3}..$$

Therefore $\gamma_s^0(G)=\left(\frac{-mn}{3}\right)$.

Theorem 2.2: If m=3k+1 or 3k+2 is not divisible by 3 then inverse signed domination number of G is

$$\gamma_s^0(G)=\begin{cases}n\left[2\left\lfloor\frac{m}{3}\right\rfloor-m\right], & \text{if } m=3k+1.\\ n\left[2\left\lceil\frac{m}{3}\right\rceil-m\right], & \text{if } m=3k+2.\end{cases}$$

Proof: Consider the graph $G=P_n\circ C_m$ with $|V|$ number of vertices and $|E|$ number of edges.

Case I: Suppose m=3k+1

Let $f:V\to\{-1,+1\}$ be a function defined by

$$f(v)=\begin{cases}+1, \text{ if } m\equiv 1(\text{mod } 3) \text{ vertices in each copy of } C_m \text{ in } G,\\ -1, \quad \text{otherwise.}\end{cases}$$

Here +1 is assigned to $\left\lfloor\frac{m}{3}\right\rfloor$ vertices in each copy of C_m in G, -1 is assigned to all other vertices in G.

Case 1: Suppose $v\in P_n$ be such that

(i) As $d(v)=4$ in G then

$\sum_{u\in N[v]} f(u)=[(+1)+(-1)]+[(-1)+(-1)+(-1)]=-3$.

(ii) As $d(v)=3$ in G then

$\sum_{u\in N[v]} f(u)=[(+1)+(-1)]+[(-1)+(-1)]=-2$.

Case 2: Suppose $v \in C_m$ be such that $d(v)=2$ in G then $f(v)=-1$, $f(v)=+1$.

(i) Here $N[v]$ contains 2 vertices of C_m and one vertex of P_n in G.

If $f(v)=-1$ then $\sum_{u\in N[v]} f(u)=(-1)+[(-1)+(-1)]=-3$.

If $f(v)=+1$ then $\sum_{u\in N[v]} f(u)=(+1)+[(-1)+(-1)]=-1$.

(ii) Here $N[v]$ contains 3 vertices of C_m and zero vertex of P_n in G.

If $f(v)=\pm 1$ then $\sum_{u\in N[v]} f(u)=(-1)+[(-1)+(+1)]=-1$.

From the above cases the function f is an inverse signed dominating function, because $\sum_{u\in N[v]} f(u)<0, \forall v\in V$.

Now the maximality check for f, define $g: V \to \{-1,+1\}$ by

$$g(v)=\begin{cases} +1, \text{ if any one vertex } v=u_i\in P_n \text{ in } G, \\ +1, \text{ if } \left\lfloor \frac{m}{3} \right\rfloor \text{ vertices in each copy of } C_m \text{ in } G, \\ -1, \quad \text{otherwise.} \end{cases}$$

Here two cases are followed.

Case 3: Suppose $v \in P_n$ be such that

(i)As $d(v)=4$ in G, then $N[v]$ contains 2 vertices of C_m and three vertices of P_n in G.

Sub case 1: Let $u_i \in P_n$ in i^{th} copy of G then

$$\sum_{u\in N[v]} g(u)=[(+1)+(-1)]+[(+1)+(-1)+(-1)]=-1.$$

Sub case 2: Let $u_i \notin P_n$ in i^{th} copy of G then

$$\sum_{u\in N[v]} g(u)=[(+1)+(-1)]+[(-1)+(-1)+(-1)]=-3.$$

(ii)As $d(v)=3$ in G, then $N[v]$ contains 2 vertices of C_m and two vertices of P_n in G.

Sub case 1: Let $u_i \in P_n$ in i^{th} copy of G then

$$\sum_{u\in N[v]} g(u)=[(+1)+(-1)]+[(-1)+(+1)]=0.$$

Sub case 2: Let $u_i \notin P_n$ in i^{th} copy of G then

$$\sum_{u\in N[v]} g(u)=[(+1)+(-1)]+[(-1)+(-1)]=-2.$$

Case 4: Suppose $v \in C_m$ be such that $d(v)=2$ in G,

(i)Here $N[v]$ contains 2 vertices of C_m and one vertex of P_n in G then $g(v)=-1 \text{ or } +1$.

Sub case 1: Let $u_i \in P_n$ in i^{th} copy of G.

If $g(v)=-1 \Rightarrow \sum_{u\in N[v]} g(u)=(-1)+[(-1)+(+1)]=-1$.

If $g(v)=+1 \Rightarrow \sum_{u\in N[v]} g(u)=(+1)+[(+1)+(-1)]=+1(>0)$.

Sub case 2: Let $u_i \notin P_n$ in i^{th} copy of G.

If $g(v)=-1 \Rightarrow \sum_{u\in N[v]} g(u)=(-1)+[(-1)+(-1)]=-3$.

If $g(v)=+1 \Rightarrow \sum_{u\in N[v]} g(u)=(+1)+[(-1)+(-1)]=-1$.

(ii)Here $N[v]$ contains 3 vertices of C_m and zero vertex of P_n in G.

If $g(v)=-1 \Rightarrow \sum_{u\in N[v]} g(u)=(-1)+[(-1)+(+1)]=-1$.

If $g(v)=+1 \Rightarrow \sum_{u\in N[v]} g(u)=(+1)+[(-1)+(-1)]=-1$.

From the above cases, we get $\sum_{u\in N[v]} g(u)>0, \text{for some } v\in V$.

This implies that the function g is not an inverse signed dominating function. Hence f is a maximal inverse signed dominating function on G. Now inverse signed total domination number is the sum of the function value of all vertices in G, that is

$$\sum_{u\in V(G)} f(u)=\left(\underbrace{\left\lfloor \frac{m}{3} \right\rfloor(+1)}_{n-times}\right)+\underbrace{\left((m)-\left\lfloor \frac{m}{3} \right\rfloor\right)(-1)}_{n-times}=-mn+2n\left\lfloor \frac{m}{3} \right\rfloor.$$

Therefore $\gamma_s^0(G)=n\left[2\left\lfloor \frac{m}{3} \right\rfloor-m\right]$.

Case II: Suppose m=3k+2

Let $f: V \to \{-1,+1\}$ be a function defined by

$$f(v)=\begin{cases} +1, \text{ if } m\equiv 1 (\text{mod } 3) \text{ vertices in each copy of } C_m \text{ in } G, \\ -1, \quad \text{otherwise.} \end{cases}$$

Here +1 is assigned to $\left\lceil \frac{m}{3} \right\rceil$ vertices in each copy of C_m in G, -1 is assigned to all other vertices in G.

Case 1: Suppose $v \in P_n$ be such that

(i) As $d(v)=4$ in G then

$$\sum_{u\in N[v]} f(u)=[(+1)+(+1)]+[(-1)+(-1)+(-1)]=-1.$$

(ii) As $d(v)=3$ in G then

$$\sum_{u\in N[v]} f(u)=[(+1)+(+1)]+[(-1)+(-1)]=0.$$

Case 2: Suppose $v \in C_m$ be such that $d(v)=2$ in G.

(i) Here $N[v]$ contains 2 vertices of C_m and one vertex of P_n in G then $f(v)=+1$.

If $f(v)=+1 \Rightarrow \sum_{u\in N[v]} f(u)=(+1)+[(-1)+(-1)]=-1$.

(ii) Here $N[v]$ contains 3 vertices of C_m and zero vertex of P_n in G then $f(v)=-1$ and $f(v)=+1$.

If $f(v)=-1 \Rightarrow \sum_{u\in N[v]} f(u)=(-1)+[(-1)+(+1)]=-1$.

If $f(v)=+1 \Rightarrow \sum_{u\in N[v]} f(u)=(+1)+[(-1)+(-1)]=-1$.

From the above cases the function f is an inverse signed dominating function, because $\sum_{u\in N[v]} f(u)\le 0, \forall v\in V$.

Now maximality check for f, define $g: V \to \{-1,+1\}$ by

$$g(v)=\begin{cases} +1, \text{ if any one vertex } v=u_i\in P_n \text{ in } G, \\ +1, \text{ if } \left\lceil \frac{m}{3} \right\rceil \text{ vertices in each copy of } C_m \text{ in } G, \\ -1, \quad \text{otherwise.} \end{cases}$$

Here two cases are followed.

Case 3: Suppose $v\in P_n$ be such that

(i)As $d(v)=4$ in G, then $N[v]$ contains 2 vertices of C_m and three vertices of P_n in G.

Sub case 1: Let $u_i\in P_n$ in i^{th} copy of G then

$$\sum_{u\in N[v]} g(u)=[(+1)+(+1)]+[(-1)+(-1)+(+1)]=+1(>0).$$

Sub case 2: Let $u_i\notin P_n$ in i^{th} copy of G then

$$\sum_{u\in N[v]} g(u)=[(+1)+(+1)]+[(-1)+(-1)+(-1)]=-1.$$

(ii)As $d(v)=3$ in G, then $N[v]$ contains 2 vertices of C_m and two vertices of P_n in G.

Sub case 1: Let $u_i\in P_n$ in i^{th} copy of G then

$$\sum_{u\in N[v]} g(u)=[(+1)+(+1)]+[(-1)+(+1)]=+2(>0).$$

Sub case 2: Let $u_i\notin P_n$ in i^{th} copy of G then

$$\sum_{u\in N[v]} g(u)=[(+1)+(+1)]+[(-1)+(-1)]=0.$$

Case 4: Suppose $v\in C_m$ be such that $d(v)=2$ in G,

(i)Here $N[v]$ contains 2 vertices of C_m and one vertex of P_n in G then $g(v)=+1$.

Sub case (1): Let $u_i\in P_n$ in i^{th} copy of G then

$$\sum_{u\in N[v]} g(u)=(+1)+[(-1)+(-1)]=-1.$$

Sub case (2): Let $u_i\notin P_n$ in i^{th} copy of G then

$$\sum_{u\in N[v]} g(u)=(+1)+[(-1)+(-1)]=-1.$$

(ii)Then $N[v]$ contains 3 vertices of C_m and zero vertex of P_n in G then $g(v)=-1$ and $g(v)=+1$.

If $g(v)=-1 \Rightarrow \sum_{u\in N[v]} g(u)=(-1)+[(-1)+(+1)]=-1$.

If $g(v)=+1 \Rightarrow \sum_{u\in N[v]} g(u)=(+1)+[(-1)+(-1)]=-1$.

From the above cases, we get $\sum_{u\in N[v]} g(u)>0$, for some $v\in V$.

This implies that the function g is not an inverse signed dominating function. Hence f is a maximal inverse signed dominating function on G. Now inverse signed domination number is the sum of the function value of all vertices in G, that is

$$\sum_{u\in V(G)} f(u)=\Bigg(\underbrace{\left\lceil \frac{m}{3}\right\rceil (+1)}_{n-times}\Bigg)+\underbrace{\left((m)-\left\lceil \frac{m}{3}\right\rceil\right)(-1)}_{n-times}=-mn+2n\left\lceil \frac{m}{3}\right\rceil.$$

Therefore $\gamma_s^0(G)=n\left[2\left\lceil \frac{m}{3}\right\rceil-m\right]$.

3. RESULTS ON CORONA PRODUCT GRAPH

Theorem 3.1: A function $f: V\to\{-1,+1\}$ is defined by

$$f(v_i)=\begin{cases} +1, \text{ if } 1\le i\le \left(\frac{m}{2}\right) \text{ of each copy of } K_m \text{ in } G, \\ -1, \quad \text{otherwise.} \end{cases}$$

is a maximal inverse signed dominating function of a graph $G=P_n \odot K_m$ and inverse signed domination number is $\gamma_s^0(G)=-n$, if m is even.

Proof: Consider the graph $G=P_n \odot K_m$ with $|V|$ number of vertices and $|E|$ number of edges.

Let f be a function defined in the hypothesis.

Case 1: Let $v_i\in P_n$ be such that $d(v_i)=m+2$ in G then

$$\sum_{u\in N[v_i]} f(u)=(-1)+(-1)+(-1)+\left[\left(\frac{m}{2}\right)(-1)+\left(\frac{m}{2}\right)(+1)\right]=-3.$$

Case 2: Let $v_i\in P_n$ be such that $d(v_i)=m+1$ in G then

$$\sum_{u\in N[v_i]} f(u)=(-1)+(-1)+\left[\left(\frac{m}{2}\right)(-1)+\left(\frac{m}{2}\right)(+1)\right]=-2.$$

Case 3: Let $v_i\in K_m$ be such that $d(v_i)=m$ in G and $f(v_i)=-1$ or $+1$.

If

$$f(v_i)=\pm 1 \Rightarrow \sum_{u\in N[v_i]} f(u)=(-1)+\left[\left(\frac{m}{2}\right)(-1)+\left(\frac{m}{2}\right)(+1)\right]=-1.$$

Hence for all the above possibilities, we get $\sum_{u\in N[v_i]} f(u)<0, \forall v_i\in V$. This implies that the function f is an inverse signed dominating function. Now the maximality check for f, define $g: V\to\{-1,+1\}$ by

$$g(v_i)=\begin{cases} +1, \quad \text{if } 1\le i\le \frac{m}{2} \text{ of each copy of } K_m \text{ in } G, \\ +1, \quad \text{if } v_i=v_k\in P_n \text{ in } G, \\ -1, \quad \text{otherwise.} \end{cases}$$

Case 1: Let $v_i\in P_n$ be such that $d(v_i)=m+2$ in G.

(i) If $v_k \in N[v_i] \Rightarrow \sum_{u\in N[v_i]} g(u) = 1+(-1)+(-1)+\left[\left(\frac{m}{2}\right)(+1)+\left(\frac{m}{2}\right)(-1)\right] = -1.$

(ii) If $v_k \notin N[v_i] \Rightarrow \sum_{u\in N[v_i]} g(u) = (-1)+(-1)+(-1)+\left[\left(\frac{m}{2}\right)(+1)+\left(\frac{m}{2}\right)(-1)\right] = -3.$

Case 2: Let $v_i \in P_n$ be such that $d(v_i) = m+1$ in G.

(i) If $v_k \in N[v_i] \Rightarrow \sum_{u\in N[v_i]} g(u) = 1+(-1)+\left[\left(\frac{m}{2}\right)(+1)+\left(\frac{m}{2}\right)(-1)\right] = 0.$

(ii) If $v_k \notin N[v_i] \Rightarrow \sum_{u\in N[v_i]} g(u) = (-1)+(-1)+\left[\left(\frac{m}{2}\right)(+1)+\left(\frac{m}{2}\right)(-1)\right] = -2.$

Case 3: Let $v_i \in K_m$ be such that $d(v_i) = m$ in G and $g(v_i) = \pm 1$.

(i) Let $v_k \in N[v_i] \Rightarrow \sum_{u\in N[v_i]} g(u) = 1+\left[\left(\frac{m}{2}\right)(+1)+\left(\frac{m}{2}\right)(-1)\right] = 1.$

(ii) Let $v_k \notin N[v_i] \Rightarrow \sum_{u\in N[v_i]} g(u) = (-1)+\left[\left(\frac{m}{2}\right)(+1)+\left(\frac{m}{2}\right)(-1)\right] = -1.$

This implies that g is not an inverse signed dominating function because $\sum_{u\in N[v_i]} g(u) > 0$, for some $v_i \in V$.

Hence f is a maximal inverse signed dominating function on G. Now inverse signed domination number is the sum of the function value of all vertices in G, that is

$$\sum_{u\in V(G)} f(u) = \underbrace{(-1)+---+(-1)}_{n-times} + \left[\underbrace{\left(\frac{m}{2}\right)(+1)+\left(\frac{m}{2}\right)(-1)}_{n-times}\right] = -n.$$

Finally $\gamma_s^0(G) = -n$, if m is even.

Theorem 3.2: A function $f: V \to \{-1,+1\}$ is defined by

$$f(v_i) = \begin{cases} +1, & \text{if } 1 \le i \le \left(\frac{m+1}{2}\right) \text{ of each copy of } K_m \text{ in } G, \\ -1, & \text{otherwise.} \end{cases}$$

is a maximal inverse signed dominating function of a graph $G = P_n \square K_m$ and inverse signed domination number is $\gamma_s^0(G) = 0$, if m is odd.

Proof: Let f be a function defined in the hypothesis.

Case 1: Let $v_i \in P_n$ be such that $d(v_i) = m+2$ in G then

$$\sum_{u\in N[v_i]} f(u) = (-1)+(-1)+(-1)+\left[\left(\frac{m+1}{2}\right)(+1)+\left(\frac{m-1}{2}\right)(-1)\right] = -2.$$

Case 2: Let $v_i \in P_n$ be such that $d(v_i) = m+1$ in G then

$$\sum_{u\in N[v_i]} f(u) = (-1)+(-1)+\left[\left(\frac{m+1}{2}\right)(+1)+\left(\frac{m-1}{2}\right)(-1)\right] = -1.$$

Case 3: Let $v_i \in K_m$ be such that $d(v_i) = m$ in G and

$f(v_i) = \pm 1 \Rightarrow \sum_{u\in N[v_i]} f(u) = (-1)+\left[\left(\frac{m+1}{2}\right)(+1)+\left(\frac{m-1}{2}\right)(-1)\right] = 0.$

Hence for all the above possibilities, we get

$$\sum_{u\in N[v_i]} f(u) \le 0, \ \forall v_i \in V.$$

This implies that the function f is an inverse signed dominating function.

Now the maximality check for f, define $g: V \to \{-1,+1\}$ by

$$g(v_i) = \begin{cases} +1, & \text{if } 1 \le i \le \frac{m+1}{2} \text{ of each copy of } K_m \text{ in } G, \\ +1, & \text{if } v_i = v_k \in P_n \text{ in } G, \\ -1, & \text{otherwise.} \end{cases}$$

Case 1: Let $v_i \in P_n$ be such that $d(v_i) = m+2$ in G.

(i) If $v_k \in N[v_i] \Rightarrow \sum_{u\in N[v_i]} g(u) = 1+(-1)+(-1)+\left[\left(\frac{m-1}{2}\right)(-1)+\left(\frac{m+1}{2}\right)(+1)\right] = 0.$

(ii) If $v_k \notin N[v_i] \Rightarrow \sum_{u\in N[v_i]} g(u) = (-1)+(-1)+(-1)+\left[\left(\frac{m-1}{2}\right)(-1)+\left(\frac{m+1}{2}\right)(+1)\right] = -2.$

Case 2: Let $v_i \in P_n$ be such that $d(v_i) = m+1$ in G.

(i) If $v_k \in N[v_i] \Rightarrow \sum_{u\in N[v_i]} g(u) = 1+(-1)+\left[\left(\frac{m-1}{2}\right)(-1)+\left(\frac{m+1}{2}\right)(+1)\right] = 1.$

(ii) If $v_k \notin N[v_i] \Rightarrow \sum_{u\in N[v_i]} g(u) = (-1)+(-1)+\left[\left(\frac{m-1}{2}\right)(+1)+\left(\frac{m+1}{2}\right)(-1)\right] = -3.$

Case (3): Let $v_i \in K_m$ be such that $d(v_i) = m$ in G and $g(v_i) = \pm 1$.

(i) Let $v_k \in N[v_i] \Rightarrow \sum_{u\in N[v_i]} g(u) = 1+\left[\left(\frac{m-1}{2}\right)(-1)+\left(\frac{m+1}{2}\right)(+1)\right] = 2$

.

(ii) Let $v_k \notin N[v_i] \Rightarrow \sum_{u\in N[v_i]} g(u) = (-1)+\left[\left(\frac{m-1}{2}\right)(-1)+\left(\frac{m+1}{2}\right)(+1)\right] = 0.$

This implies that g is not an inverse signed dominating function because $\sum_{u\in N[v_i]} g(u) > 0$, for some $v_i \in V$.

Hence f is a maximal inverse signed dominating function on G. Now inverse signed domination number is the sum of the function value of all vertices in G, that is

$$\sum_{u\in V(G)} f(u) = \underbrace{(-1)+---+(-1)}_{n-times} + \left[\underbrace{\left(\frac{m+1}{2}\right)(+1)+\left(\frac{m-1}{2}\right)(-1)}_{n-times}\right] = 0$$

.

Finally $\gamma_s^0(G) = 0$, if m is odd.

4. CONCLUSION

It is interesting to study the inverse signed dominating functions of corona product graph of complete graph with a path and rooted product graph of a path with cycle. This work gives the scope for an extensive study of various inverse dominating functions of these graphs.

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