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SOLUTION FOR (1+N) DIMENSIONAL NONLINEAR BURGERS' INITIAL VALUE PROBLEM USING ADOMIAN DECOMPOSITION METHOD

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Abstract: In this manuscript, we have represented (1+n) dimensional nonlinear Burgers' initial value problem. Adomian decomposition method (ADM) has been applied to find the solution of this problem. Some examples have also been given to claim the complete convergence of the method for exact solution.

Keywords: (1+n) dimensional Burgers' equation; Adomian decomposition method; Nonlinear partial differential equation, Lagrange multiplier.

1. INTRODUCTION

The researchers are showing their interest in new methods to solve linear – nonlinear, ordinary - partial differential equations with initial and boundary value problems [1-3]. Nonlinear Burgers' equation was firstly reported by J. M. Burgers for a fluid motion model. It is widely used in wave theory, gas and plasma dynamics [4-6]. Adomian decomposition method (ADM) has been introduced by George Adomian in 1980's. It provides an approximation solution with less number of iterations [7].

2. (N+1) DIMENSIONAL NONLINEAR BURGER INITIAL VALUE PROBLEMS

2.1 Problem I

$$w_t = w_{x_1x_1} + w_{x_2x_2} + w_{x_3x_3} + - - - - - + w_{x_nx_n} + ww_{x_1} \quad (01)$$

With initial condition

$$w(x_1, x_2, x_3, - - - - - - -, x_n, 0) = x_1 + 2x_2 + 3x_3 + - - - - - - + nx_n \quad (02)$$

Having exact solution

$$w(x_1, x_2, x_3, - - - - - - -, x_n, t) = \frac{x_1 + 2x_2 + 3x_3 + - - - - - - - + nx_n}{1 - t}, |t| \leq 1 \quad (03)$$

Where $w_{x_kx_k} = \frac{\partial^2 w}{\partial {x_k}^2}$ (04)

Equation (01) can be represents as

$$L_t(w) = f(x_1, x_2, x_3, - - - - - - -, x_n, t) + N(w) \quad (05)$$

Where $L_t = \frac{\partial}{\partial t}$ and $N(w) = ww_{x_1}$ represent linear operator and nonlinear term respectively.

Applying $L_t^{-1}(.) = \int_0^t (.)dt$ in equation (01) and considering initial condition, we get

$$w(x_1, x_2, x_3, - - - - - - -, x_n, t) = (x_1 + 2x_2 + 3x_3 + - - - - - - - + nx_n) + L_t^{-1}(w_{x_1x_1} + w_{x_2x_2} + w_{x_3x_3} + - - - - - + w_{x_nx_n} + ww_{x_1}) \quad (06)$$

Now, in view of Adomian decomposition method, after decomposing the solution w and nonlinear term ww_{x_1} into the series form, equation (06) gives

$$\sum_{m=0}^{\infty} w_m(x_1, x_2, x_3, - - - - - - -, x_n, t) = (x_1 + 2x_2 + 3x_3 + - - - - - - - + nx_n) + L_t^{-1}(w_{x_1x_1} +$$
wx2x2+wx3x3+−−−−+wxnxn+m=0∞Am (07)

Where Adomian polynomials A_m depending upon $w_0, w_1, - - - - - -, w_m$ can be calculated by following

$$A_m = \frac{1}{m!} \frac{\partial^m}{\partial \lambda^m} \left[N\left(\sum_{p=0}^{\infty} w_p \lambda^p\right)\right]_{\lambda=0} \quad m = 0, 1, 2.... \quad (08)$$

Some Adomian polynomials are

$$A_0 = w_0(w_0)_{x_1} \quad (09)$$
$$A_1 = w_0(w_1)_{x_1} + w_1(w_0)_{x_1} \quad (10)$$
$$A_2 = w_0(w_2)_{x_1} + w_1(w_1)_{x_1} + w_2(w_0)_{x_1} \quad (11)$$

And so on

In view of equation no (07), Adomian recursion formula can be obtained as

$$w_0 = (x_1 + 2x_2 + 3x_3 + - - - - - - - + nx_n) \quad (12)$$

$$w_{m+1} = L_t^{-1}(w_{x_1x_1} + w_{x_2x_2} + w_{x_3x_3} + - - - - - + w_{x_nx_n} + Am, m\geq 0) \quad (13)$$

Using equation (13), the solution components can be determined as

$$w_1 = (x_1 + 2x_2 + 3x_3 + - - - - - - - + nx_n)t \quad (14)$$
$$w_2 = (x_1 + 2x_2 + 3x_3 + - - - - - - - + nx_n)t^2 \quad (15)$$
$$w_3 = (x_1 + 2x_2 + 3x_3 + - - - - - - - + nx_n)t^3$$

(16)

And so on....

According to ADM, the final solution in series form is

$$w(x_1, x_2, x_3, ------, x_n, t) = (x_1 + 2x_2 + 3x_3 + ------ + nx_n)(1 + t + t^2 + t^3 + ------) \quad (17)$$

Which is same as the exact solution (03).

2.2 Problem II

Reconsider the equation (01)

$$w_t = w_{x_1x_1} + w_{x_2x_2} + w_{x_3x_3} + ---- + w_{x_nx_n} + ww_{x_1} \quad (18)$$

With different initial condition

$$w(x_1, x_2, x_3, ------, x_n, 0) = ex_1 + e^2x_2 + e^3x_3 + ------ + e^nx_n \quad (19)$$

Having exact solution

$$w(x_1, x_2, x_3, ------, x_n, t) = \frac{ex_1 + e^2x_2 + e^3x_3 + ------ + e^nx_n}{1-t}, |t| \leq 1 \quad (20)$$

Where $w_{x_kx_k} = \frac{\partial^2 w}{\partial {x_k}^2}$ (21)

Equation (18) can be represents as

$$L_t(w) = f(x_1, x_2, x_3, ------, x_n, t) + N(w) \quad (22)$$

Where $L_t = \frac{\partial}{\partial t}$ and $N(w) = ww_{x_1}$ represent linear operator and nonlinear term respectively.

Applying $L_t^{-1}(.) = \int_0^t (.)dt$ in equation (18) and considering initial condition, we get

$$w(x_1, x_2, x_3, ------, x_n, t) = (ex_1 + e^2x_2 + e^3x_3 + ------ + e^nx_n) + L_t^{-1}(w_{x_1x_1} + w_{x_2x_2} + w_{x_3x_3} + ---- + w_{x_nx_n} + ww_{x_1}) \quad (23)$$

Now, in view of Adomian decomposition method, after decomposing the solution w and nonlinear term ww_{x_1} into the series form, equation (23) gives

$$\sum_{m=0}^{\infty} w_m(x_1, x_2, x_3, ------, x_n, t) = (ex_1 + e^2x_2 + e^3x_3 + ------ + e^nx_n) + L_t^{-1}\big(w_{x_1x_1} + wx2x2+wx3x3+----+wxnxn+m=0\infty Am \quad (24)$$

Where Adomian polynomials A_m depending upon $w_0, w_1, -----, w_m$ can be calculated by following

$$A_m = \frac{1}{m!} \frac{\partial^m}{\partial \lambda^m} \left[N\left(\sum_{p=0}^{\infty} w_p \lambda^p\right)\right]_{\lambda=0} \quad m = 0, 1, 2.... \quad (25)$$

Some Adomian polynomials are

$$A_0 = w_0(w_0)_{x_1} \quad (26)$$

$$A_1 = w_0(w_1)_{x_1} + w_1(w_0)_{x_1} \quad (27)$$

$$A_2 = w_0(w_2)_{x_1} + w_1(w_1)_{x_1} + w_2(w_0)_{x_1} \quad (28)$$

And so on

In view of equation no (24), Adomian recursion formula can be obtained as

$$w_0 = (ex_1 + e^2x_2 + e^3x_3 + ------ + e^nx_n) \quad (29)$$

$$w_{m+1} = L_t^{-1}\big(w_{x_1x_1} + w_{x_2x_2} + w_{x_3x_3} + ---- + w_{x_nx_n} + Am, m\geq 0 \quad (30)$$

Using equation (30), the solution components can be determined as

$$w_1 = (ex_1 + e^2x_2 + e^3x_3 + ------ + e^nx_n)t \quad (31)$$

$$w_2 = (ex_1 + e^2x_2 + e^3x_3 + ------ + e^nx_n)t^2 \quad (32)$$

$$w_3 = (ex_1 + e^2x_2 + e^3x_3 + ------ + e^nx_n)t^3 \quad (33)$$

And so on....

According to ADM, the final solution in series form is

$$w(x_1, x_2, x_3, ------, x_n, t) = (ex_1 + e^2x_2 + e^3x_3 + ------ + e^nx_n)(1 + t + t^2 + t^3 + ------) \quad (34)$$

Which is same as the exact solution (20).

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