DOI: http://dx.doi.org/10.26483/ijarcs.v8i8.4676

ISSN No. 0976-5697

Volume 8, No. 8, September-October 2017

International Journal of Advanced Research in Computer Science

RESEARCH PAPER

Available Online at www.ijarcs.info

GLC AND GLC** CONTINUOUS FUNCTIONS: A CONCEPTUAL FLAW

P.L. Powar
Department of Mathematics and Computer Science
R.D. University, Jabalpur,India

Pratibha Dubey
Department of Mathematics, St. Aloysius College,
Jabalpur,India

Abstract : The concept of generalized locally closed sets (glc-sets), GLC**-sets followed by the notion of GLC and GLC**-continuous maps was initiated by Balachandran et al. (Generalized locally closed sets and GLC-continuous functions, Indian J. pure appl. Math 27(3): 235-244, 1996). In the present work, it has been established that the collection of glc-sets and the collection of GLC** -sets, each is exactly equal to the power set P (X) of X. Consequently, any arbitrary function with any choice of domain and range turns out to be GLC and GLC**-continuous function which is not desirable from analytic point of view.

Keywords: Topological spaces, locally closed sets, glc-set, GLC **-set, GLC-continuity, GLC**-continuity.
AMS Subject Classification (2000): primary 54XX; secondary 54CXX.

1. INTRODUCTION

The idea of locally closed set was introduced by Bourbaki [2] in 1966. (see also [3]). This concept of locally closed set had been used by Ganster and Reilly [4] for defining the generalized version of continuity viz. LC-irresolute, LC-continuity and sub-LC-continuity. Balachandran et al. [1] had extended the definition of locally closed sets and initiated the notion of "Generalized locally closed set", in particular, glc-set, GLC*-set and GLC**-set. Since last few decades many topologist (cf. [4], [5], [6], [7], [8], [9], [10], [11]) are trying to explore the possibility of generalizing the classical phenomenon "continuity" of the function defined in the topological space. Following this trend Balachandran et al. [1] have also defined and explored the idea of GLC-irresolute maps and GLC-continuous maps. Extending the idea of Balachandran et al. [1], Park et al. [8] have defined semi generalized locally closed sets and locally-generalized closed sets along with SGLC-continuous functions and LδGLC-continuous functions respectively. (see also [9], [10], [11]).
Recently, Patil et al. [10] have further extended the concept of glc-sets and introduced the notion of
g*wα-lc sets and g*wα^*-lc sets and g*wα^{**}-lc sets and have applied these concepts to define relevant different types of continuous functions.
In the present paper, authors have established that the respective collection of glc-sets, and the collection of GLC**-sets (cf. [1]) generated by the topology yield precisely the power set P(X) of X. This information leads to the conclusion that the corresponding GLC and GLC** - idea of continuity is not enhancing the class of continuous functions with some relaxed conditions however all functions with arbitrary domain and range turns out to be GLC and GLC**-continuous functions which is inadequate. In view of this observation, all the extensions turned out to be superfluous.

2. PRE-REQUISITES

The following notations have been referred throughout this work:

(X, τ) -	Topological space with topology defined on the set X ($\neq \phi$).
cl(A) -	Closure of A for the subset A of X with respect to (X, τ).
int(A)-	Interior of A for the subset A of X with respect to (X, τ).
P(X) -	Power set of X.

Definition 2.1. A subset B of (X, τ) is called g-closed [12] if $cl(B) \subseteq G$ whenever $B \subseteq G$ for an open set G in a topological space (X, τ). A subset C of (X, τ) is called **g-open** if its complement $X - C$ is **g-closed**.

Example 2.1. Consider a topological space X = {a, b, c, d} with the topology τ = {X, ϕ, {a, b, c}}, F_X = {X, ϕ, {d}}, where F_X is the collection of closed sets in (X, τ). Let A = {a, d} be a subset of X. There is only one open set say U = X containing A. Then it is easy to check that cl{a, d} = X which follows by the definition that cl{a, d} = X = U = X. Hence A = {a, d} is **g-closed**.

Remark 2.1. It is a direct consequence from the definition of g-closed sets that every open set is g-open and every closed set is g-closed but the respective converse is not true in general.

Definition 2.2. Let S be a subset of a topological space (X,τ). S is said to be **generalized locally closed** (**glc-set**) [1] if there exists g-open set G and g-closed set F such that S = G∩F . The collection
of all generalized locally closed set is denoted by **GLC** (cf. [1]).

Example 2.2. Consider a topological space X = {a, b, c, d} with the topology τ = {X, ϕ, {b, c, d},{a, c, d}, {c, d}}, F_X = {τ, X, {a}, {b}, {a, b}}. In view of Definition 2.1, the collection of g-closed sets = {X, ϕ, {a}, {b}, {a, b}, {a, b, c}, {a, b, d}} and the collection of g-open sets = {ϕ, X, {b, c, d}, {a, c, d}, {b, c, d}, {a, c, d}, {c, d}, {d}, {c}}. We now show that A = {b, c} ⊆ X is a glc-set.
Claim: A = {U ∩ V : U is g-open and V is g-closed}.
We now consider U = {b ,c, d} a g-open set and V = {a, b, c} a g-closed set. Then,
U ∩ V = {b, c, d} ∩ {a, b, c} = {b, c} is a glc-set. It may be verified easily that the collection of all glc-sets is exactly equal to P (X).

Remark 2.2. It is clear that every g-closed set is glc-set and every g-open set is glc-set.

Definition 2.3. Consider a subset S of a topological space. Then S∈ **GLC**** if S∈ G ∩ F for any open set G and a g-closed set F of (X, τ) respectively (cf.[1]).

Definition 2.4. Let (X, τ) and (Y, σ) be two topological spaces. A function f :→ (X, τ) → (Y, σ) is said to be **GLC-continuous** (resp. **GLC**-continuous**) if f^{-1}(V) ∈ GLC (resp. f^{-1}(V) ∈ GLC**) for each V ∈ σ (cf.[1]).

Definition 2.5. A function : (X, τ) → (Y, σ) is said to be **GLC-irresolute** (resp. **GLC**-irresolute**) if $f^{-1}(V)$ ∈ GLC (resp.$f^{-1}(V)$ ∈ GLC) for each V ∈ GLC (resp. V ∈ GLC**) in (Y, σ) (cf.[1]).

3. MAIN RESULT

We are now set to state the main result of this paper.

Theorem 3.1. Let (X, τ) be the topological space and GLC and GLC** be the collection of sets described in the Definition 2.2 and 2.3 respectively. Then

GLC ≅ GLC ≅ P (X)**

where P (X) is the power set of X.

Proof. Let X be any non empty set τ = {ϕ, X, $\{U_\alpha\}_{\alpha \in J}$} be the topology on X. Let A be any non empty proper subset of X. The following cases have been considered:

Case 1. A ⊈ U_α (≠ X) for all $\alpha \in$ J and $U_\alpha \in \tau$ implies A ⊆X only. It is clear that cl(A) ⊆ X. Hence, A is g-closed. Referring Remark 2.2, we conclude that A is glc-set.

Case 2. A⊆ U_α for some $\alpha \in$ J and CA ⊆X but CA ⊈ U_α for each $\alpha \in$ J where C stands for the complement of A in X. It is obvious that cl(CA) ⊆ X. Hence CA is g-closed which implies that A is g-open. In view of Remark 2.2, the set A is glc again.

Case 3. A ⊆ U_α for some $\alpha \in$ J and CA ⊆ U_δ for some $\delta \in$J where U_α, $U_\delta \in \tau$. Let if possible that A is neither g-open nor g-closed.

Claim:

$$A = G_c \cap G_o \tag{3.1}$$

where G_c and G_o are g-closed and g-open sets in (X,τ) respectively. Since, A is not g-closed, there exists at least one index $\beta \in$ J such that

A ⊆ U_β but cl(A) ⊈ U_β (cf. Definition 2.1) Then

Either

(a) $$D(A) \subseteq CU_\beta \tag{3.2}$$

Or

(b) $$S\,(\neq \phi) \subseteq D(A) \text{ and } S \subseteq CU_\beta \text{ such that } A \cup D(A)_{\sim S} \subseteq U_\beta \tag{3.3}$$

Consider the set $A \cup CU_\beta$

Claim: $A \cup CU_\beta$ is g-closed.

There exists a family $\{U_\alpha^*\}_{\alpha \in J}$ of open sets such that

$$A \cup CU_\beta \subseteq U_\alpha^* \quad \text{for } \alpha \in J$$

Since $A \subseteq A \cup CU_\beta$, the collection $\{U_\alpha^*\}_{\alpha \in J}$ is a sub-collection of $\{U_\alpha\}_{\alpha \in J}$ of open sets containing A.

i) Consider $\beta \in$ J such that $A \cup CU_\beta \subseteq U_\beta^*$ and $U_\beta \subseteq U_\beta^*$.

- If $U_\beta^* = X$, then $cl(A \cup CU_\beta) \subseteq X$ and $A \cup CU_\beta$ is g-closed.
- If $U_\beta^* \ (\neq \phi, X)$, then

$$\begin{aligned} cl(A \cup CU_\beta) &= cl(A) \cup cl(CU_\beta) \\ &= A \cup D(A)_{\sim S} \cup S \cup CU_\beta \ (\because CU_\beta \text{ is closed}) \\ &= A \cup D(A)_{\sim S} \cup CU_\beta \\ &\subseteq U_\beta^* \ (\because CU_\beta \subseteq U_\beta^* ; U_\beta \subseteq U_\beta^* \text{ and using (3.3)}) \end{aligned}$$

Therefore $A \cup CU_\beta$ is g-closed.

ii) We next consider $\alpha(\neq \beta) \in$ J such that $A \cup CU_\beta \subseteq U_\alpha^*$ for $U_\alpha \subseteq U_\alpha^*$ and $cl(A) \subseteq U_\alpha$

$$\begin{aligned} cl(A \cup CU_\beta) &= cl(A) \cup cl(CU_\beta) \\ &= cl(A) \cup CU_\beta \\ &\subseteq U_\alpha^* \end{aligned}$$

Thus, $A \cup CU_\beta$ is g-closed.

Claim: $A = (A \cup CU_\beta) \cap U_\beta$ where $G_c = A \cup CU_\beta$ is g-closed and $G_o = U_\beta$ is g-open (cf. Remark 2.1).
Consider

$$\begin{aligned} G_c \cap G_o &= (A \cup CU_\beta) \cap U_\beta \\ &= (A \cap U_\beta) \cup (CU_\beta \cap U_\beta) \\ &= A \end{aligned}$$

Hence, (3.1) holds and finally we conclude that A is glc-set. Since, A was arbitrary subset of X, every subset of X is glc-set. Thus, the collection **GLC is precisely equal to P (X)**.

Since $G_o = U_\beta$ (open), it is direct by the Definition 2.3 that

$$\textbf{GLC}^{**} \cong \textbf{P(X)}$$

This completes the proof. □

4. CONCLUSION

- In view of Definitions 2.4 and 2.5, each function f defined from (X,τ) to (Y, σ) turns out to be GLC-continuous (irresolute) and GLC** -continuous (irresolute) which is not acceptable as a generalization of the classical concept of continuity in Topology.
- All generalizations of GLC-set and GLC** -set turned out to be stagnant and finally not desirable.

REFERENCES

[1] K. Balachandran, P. Sundaram, H. Maki, “Generalized locally closed sets and GLC-continuous functions”, Indian J. pure appl. Math., vol. 27(3), pp. 235-244, March 1966.

[2] N. Bourbaki, “General Topology”, Part I., Addition Wesley, Reading Mass, 1966.

[3] A. H. Stone, “Absolutely FG spaces”, Proc. Amer. Math. Soc. vol. 80, pp. 515-520, 1980.

[4] M. Ganster, L. Reilly, M.K. Vamanamurty, “Locally closed sets and LC-continuous functions,” Int. Jl. Math. Soc., vol. 12, pp. 417- 424, 1989.

[5] S.S. Benchalli, P.G. Patil, T.D. Rayanagaudar, “w-closed sets in topological spaces”, The Global. J. Appl. Math. Sci., vol. 2, pp. 53-63, 2009.

[6] S.S. Benchalli, P.G. Patil, P.M. Nalwad,” Generalized w-closed sets in topological spaces”, Journal of New Results in Science, ISSN 1304-7981, Number 7, pp.7-19, 2014.

[7] E. Ekici, “On locally closedness and continuity”, Chaos, Solitons and Fractals, vol. 36, pp. 1244 -1255, 2008.

[8] J.H. Park, J.K. Park, “On semi generalized locally closed sets and SGLC-continuous functions”, Indian J. pure appl. Math., vol. 31(9), pp. 1103-1112, Sept. 2000.

[9] J.H. Park, J.K. Park, “On locally δ-generalized closed sets and LδGLC-continuous functions”,Chaos, Solitons and Fractals, vol. 19(4), pp. 995 -1002, 2004 (Elsevier).

[10] P.G. Patil, S.S. Benchalli, P.S. Mirajkar, “Decomposition of locally closed sets in topological spaces”, Advances in Fuzzy Mathematics (AFM), vol. 12, No. 1, pp. 101-110, 2017.

[11] K. Chandrashekhar Rao, K. Kannan, “Some properties of g-locally closed sets”, Journal of Advanced Research in Pure Mathematics, vol.1, No. 1, pp. 1-9, 2009.

[12] N. Levine, “Generalized closed sets in topology”, Rent. Circ. Mat. Palermo, vol. 19(2), pp. 89-96, 1970.