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TOTAL DOMINATION NUMBER OF ROOTED PRODUCT GRAPH $P_m \odot C_n$

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***Abstract* :** Total domination number of a rooted product of a path and cycle $P_m \odot C_n$, when root vertex is included in total dominating set is m- times the total domination number of a cycle graph is expressed in terms of total domination number of cycle graph and the path length m.The total domination of a rooted product giving

$$\gamma_t(P_m \odot C_n) = \begin{cases} m\gamma_t(C_n) & if\ n \equiv 0(\ mod4) \\ \gamma_t(P_m) + m\gamma_t(C_{n-1}) & if\ n \equiv 1\ (mod4) \\ m + m\gamma_t(C_{n-2}) & if\ n \equiv 2\ (mod4) \end{cases}$$

***Keywords* :** Total domination number, Domination number, product graphs, Rooted product graphs.

I. INTRODUCTION

Domination in a graph along with its many variations provides an extremely rich area of study. Berge[1] and ore[8] were the first to define dominating sets. Enormous quantities of researchers about domination in graphs have been developed in the past few years. One interesting question in this area is related to the study of total domination related parameters in product graphs.

The domination number of cross product graph or grid graphs $P_m \times P_n$ has been studied extensively since the 1980s. Efforts have been made to obtain lower and upper bounds on $\gamma(P_m \times P_n)$ and there are several studies for small values of (either one or both of) the parameters m,n[2,3,4]

Rooted product is an interesting area of product graphs. Some domination related results for general graphs G$\odot H$ are found in[5]. It is presented in literature that the value of every domination parameter of the rooted product graph depends on the root vertex being included in dominating set or not from the graph H.

In case of rooted product of path and cycle graph, the graph structure is preserved under any choice of vertex being chosen to be a root vertex from cycle graph. Also the total domination number of rooted product $P_m \odot P_n$ is found to be independent of the root vertex chosen from a cycle graph. In this we present exact value of total domination number of rooted product graphs of path and cycle graph$P_m \odot P_n$

II. PRELIMINARIES

2.1 Definition of Domination:

A set D⊆V is called a dominating set if every vertex in V-D is adjacent to some vertex of D. Notice that D is a dominating set iff N$[D] = V$. The domination number of G denoted by $\gamma = \gamma(G)$, is the cardinality of a smallest dominating set of V. we call a smallest dominating set a γ −set.

2.2 Definition of Total Domination:

A set S⊆V is a total dominating set if every vertex in V is adjacent to some vertex of S. Alternatively, we may define a dominating set in D to be a total dominating set if G[D] has no isolated verticies. The total domination number of G, denoted by $\gamma_t = \gamma_t(G)$, is the cardinality of a smallest total dominating set and we denoted as a γ_t - set.

From the definitions of domination number and total domination number is that for any graph G, $\gamma_t \geq \gamma$.

2.3 Definition of Rooted product graph:

The rooted product of a path P_m with a cycle C_n is a graph obtained by taking one copy of m-vertex path graph P_m and m-copies of a cycle graph C_n and then identifying the ith vertex of P_m with the root vertex of ith copy of C_n . This graph is denoted by $P_m \odot C_n$.

2.4 Total domination number of a cycle C_n or a path P_n On n≥ 3 verticies is given by

1) $\gamma_t(C_n) = \gamma_t(\boldsymbol{P_n}) = \frac{n}{2} \quad if\ \ n \equiv 0(\ mod4)$

2) $\gamma_t(C_n) = \gamma_t(\boldsymbol{P_n}) = \frac{n+2}{2} \quad if\ n \equiv 2\ (mod4)$

3) $\gamma_t(C_n) = \gamma_t(\boldsymbol{P_n}) = \frac{n+1}{2}$ otherwise.

3 Main Results

Theorem : For a Rooted product graph $P_m \odot C_n$ where any vertex of C_n is a root , the total domination number is ,

$$\gamma_t(P_m \odot C_n) = \begin{cases} m\gamma_t(C_n) & if\ n \equiv 0(\ mod4) \\ \gamma_t(P_m) + m\gamma_t(C_{n-1}) & if\ n \equiv 1\ (mod4) \\ m + m\gamma_t(C_{n-2}) & if\ n \equiv 2\ (mod4) \end{cases}$$

Proof:

Case (i) : For $n \equiv 0(\ mod4)$

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In Rooted product graph $P_m \odot C_n$, a root vertex of $\boldsymbol{C_n}$ **is identified with one vertex of** P_m in each copy as a root . The rooted product graph has m disjoint cycles $\{C_n^1, C_n^2, C_n^3, \dots\dots\dots\dots\dots C_n^m\}$ with n-verticies each. In Cycle C_n^i having 4k- vertices $\{v_1, v_2, v_3, \dots\dots\dots\dots. v_{4k}\}$, let root vertex be v_2 , divide these 4k- verticies into k-sets of 4-consecutive verticies each. For each of these 4-verticies set Ex:$\{ v_1, v_2, v_3, v_4\}$

middle two verticies :$\{ v_2, v_3\}$ form a minimum total dominating set. This gives total 2k- verticies in minimum total dominating set D_i from k-sets of 4- verticies for C_n^i . The path vertex v_2 is totally dominated as each root vertex is a part of cycle graph and is dominated by vertices of D_i. There fore

$$D_i = \{v_2, v_3, v_5, v_6, \dots\dots\dots\dots. v_{4k-2}, \; v_{4k-1}\},$$

with $|D_i| = 2k = \gamma_t(C_n)$.

All the path verticies are dominated by D_i. Next, cycles rooted at m-path verticies are disjoint, so the minimum total dominating set for the root product graph $P_m \odot C_n$ is disjoint union of dominating sets of cycle C_n^i.

$$\therefore D = D_1 \cup D_2 \cup D_3 \cup \dots\dots\dots\dots. D_m$$

$$|D| = |D_1| + |D_2| + \cdots \dots\dots\dots\dots. |D_m|$$

$$|D| = \sum_{i=1}^{m} \gamma_t(C_n^i) = m\gamma_t(C_n)$$

$$\therefore \gamma_t(P_m \odot C_{4k}) = m\gamma_t(C_{4k})$$

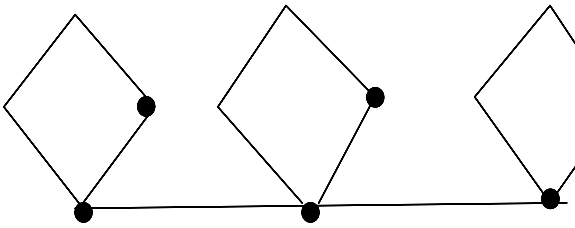

Fig 1 : Total Dominating set for the graph $P_3 \odot C_4$

Case(ii): For $n \equiv 1(\,mod\ 4)$

Let v_0 be the root vertex of C_n identified with path vertex. $C_n - \{v\}$ has 4k vertex. These vertices can be total dominated by 2k vertices by choosing 2 middle vertices in k- sets. Minimum total dominating set of $C_n - \{v\}$ has cardinality 2k. As all the cycle graphs C_n^i are disjoint. Hence the disjoint 2k sets form minimum total dominating set D for $\cup_i\{C_n^i - root\}$ with $m(2k)$ -verticies. These verticies in D will not total dominate the path vertices. For total domination of path vertices, we need to include total domination set vertices for the path that is $\gamma_t(P_m) = \left\lceil \frac{m}{2} \right\rceil$ vertices in to D giving,

$$|D| = m(2k) + \left\lceil \frac{m}{2} \right\rceil$$

$$= m\gamma_t(C_{n-1}) + \gamma_t(P_m)$$

$$\gamma_t(P_m \odot C_{4k+1}) = m\gamma_t(C_{n-1}) + \gamma_t(P_m)$$

$$\gamma_t(P_m \odot C_{4k+1}) = m\gamma_t(C_{n-1}) + \gamma_t(P_m)$$

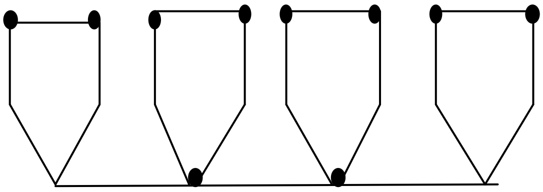

Fig 2 : Total Dominating set for the graph $P_4 \odot C_5$

Case(iii) : For $n \equiv 2\ (mod4)$

Here C_n^i has 4k+2 verticies of which 4k- verticies will be total dominated by 2k verticies as in previous case. For total domination of the remaining 2-verticies in the cycle graph if we include them into total dominating set D_i. But if we select all the vertices of path into a total dominating set they dominate other vertices also. Here as we need minimal total dominating set, when m is smaller than 2n vertices of path into dominating set we get minimal total dominating set for rooted product graph as

$$D = \{\bigcup D_i\} \bigcup P_m$$

$$|D| = (\sum |Di|) + |P_m|$$

$$= m(2k) + m$$

$$= m\gamma_t(C_{4k}) + m$$

$$\gamma_t(P_m \odot C_{4k+2}) = m\gamma_t(C_{n-2}) + m$$

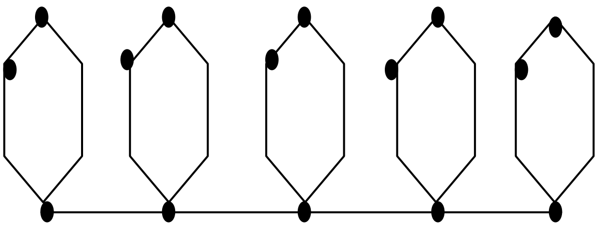

Fig 3 : Total Dominating set for the graph $P_5 \odot C_6$

Case(iv): For $n \equiv 3\ (mod4)$

As in case(iii) $\gamma_t(4k)$ has 2k vertices and 3-vertcies are remaining in each cycle . As in previous case we have to select all the vertices of path into total dominating set they dominate other vertices also. Hence we get same number of total dominating set as in previous case.

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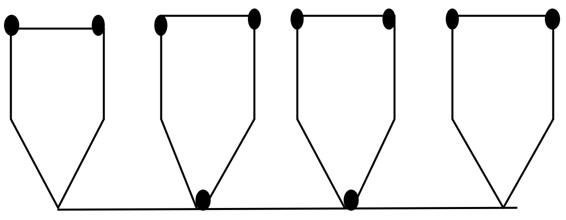

$$\gamma_t(P_m \odot C_{4k+2}) = m\gamma_t(C_{n-2}$$

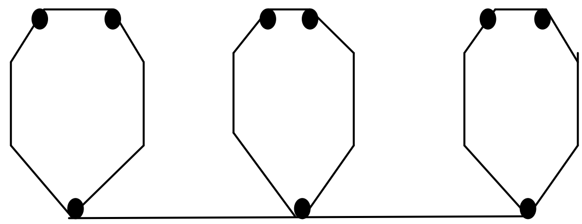

Fig 4 : Total Dominating set for the graph $P_3 \odot C_7$

4. REFERENCES:

1. Berge C. , Graphs and hyper graphs. North –Holland , Amsterdam. 1973.
2. Bresar B., Klavzar S , .Rall D F, Dominating direct product graphs, Discrete Mathematics 307(2007) pp 1636 -1642..
3. Chang T Y, Clark W E, and Hare E O, Domination numbers of complete Grid graphs, Ars combinatorica ,60(2001),307-311.
4. Cockayne , Hare , Hedeteniemi, and T.V. Wimer, Bounds for the domination number of grid graphs, Congr, Number.47(1985),217-228
5. Dorota Kuziak, Magdalena lemanska & Ismael G . Yew, on Domination related parameters in rooted product graphs.
6. Godsil C D, Mckay B D , A New graph product and its spectrum , Bulletin of the Australasian Mathematical society 18(1) (1978) 21-28.
7. Haynes, Hedetniemi, Slater, Fundamentals of Domination in Graphs , Marcel Dekker Inc. New York, 1998.
8. Ore O, Theory of Graphs, Amer. Maths. Soc. Colloq. Pub., 38(1962).
9. Rashmi S B and Indrani Pramod kelkar Domination number of Rooted product graph $P_m \odot C_n$ Journal of Computer and Mathematical sciences vol.7(9),469-471,sept.2016.

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