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ATOM-BOND CONNECTIVITY INDEX OF SUBDIVISION GRAPHS OF SOME SPECIAL GRAPHS

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Abstract: Aim of this paper is to investigate the Atom-Bond Connectivity (ABC) index of Banana tree, Centipede graph, Dutch Windmill graph, Fire cracker graph, Friendship graph, Jahangir's graph and Tadpole graph using the concept of sub-division graph.

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Keywords: Atom-Bond Connectivity (ABC) index; sub-division graph; Banana tree; Centipede graph; Dutch Windmill graph; Firecracker graph; Friendship graph; Jahangir's graph; Tadpole graph.

I. INTRODUCTION

Topological indices are numerical parameters of a graph which characterize its topology and are usually graph invariants.Topological indices have a prominent role in chemistry and pharmacology [1]. The Atom-Bond Connectivity (ABC) index has been applied to study the stability of alkanes and the energy of cycloalkanes. Furtula et al., [2] obtained extremely ABC index values for chemical trees and also shown that the star graph $K_{1,n-1}$ has the maximal ABC index values for trees. K. C. Das presents the lower and upper bound on ABC index of trees and characterization of graphs for which these bonds are best fit [3 and 6]. R. Xing et al., found results on ABC index of connected graphs, Trees etc., [7, 8 and 9].

Here we study the Atom-Bond Connectivity (ABC) index of some special graphs [10]: Banana tree, Centipede Graph, Dutch windmill graph, Firecracker Graph, Friendship Graph, Jahangir's Graph, and Tadpole Graph using the concept of sub-division graphs.

Let $G=(V,E)$ be a simple connected graph with vertex set $V(G)=\{v_1,v_2,..........,v_n\}$ and edge set $E(G)$. Let d_i be the degree of vertex v_i, where $i=\{1,2,..........,n\}$. The ABC index, proposed by Ernesto Estrada et al., is defined as follows.

$ABC(G)=\sum\sqrt{\frac{d_i+d_j-2}{d_i.d_j}}$, $(v_i,v_j)\in E(G)$. We suggest the reader to refer [4] for the proof of the above result.

The subdivision graph [5] $S(G)$ is the graph obtained from G by replacing each of its edge by a path of length 2 or equivalently, by inserting an additional vertex into each edge of G.

II. RESULTS

Here we derive an expression for Atom-Bond Connectivity (ABC) index of some special graphs: Banana tree, Centipede Graph, Dutch windmill graph, Firecracker Graph, Friendship Graph, Jahangir's Graph, and Tadpole Graph using the concept of sub-division graphs.

Theorem 2.1. The Atom-Bond Connectivity index for the subdivision graph of Banana tree $B_{n,m}$ is

$$ABC\left[S\left(B_{n,m}\right)\right]=nm\sqrt{2}\,.$$

Proof. Let subdivision graph of Banana tree $S\left(B_{n,m}\right)$ contains one vertex of degree n, n vertices of degree$(m-1)$, n^2 pendants, n vertices of degree 2 and nm vertices of subdivision graph is degree 2.

The edges of subdivision graph of Banana tree $S\left(B_{n,m}\right)$ are formed by the vertices of degrees$(n,2)$, $(m-1,2)$, $(1,2)$ and $(2,2)$.

Each of the above edges gives $\frac{1}{\sqrt{2}}$.

Therefore, the Atom-Bond Connectivity index of $S\left(B_{n,m}\right)$ is given by

$$ABC\left[S\left(B_{n,m}\right)\right]=nm\sqrt{2}\,.$$

Theorem 2.2. The Atom-Bond Connectivity index for the subdivision graph of Centipede graph C_n is

$$ABC\left[S\left(C_n\right)\right]=(2n-1)\sqrt{2}\,.$$

Proof. Let subdivision graph of Centipede graph $S\left(C_n\right)$ contains (n-2) vertices of degree 3, n pendant vertices, 2 vertices of degree 2 and (2n-1) vertices of subdivision graph is of degree 2.

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The edges of subdivision graph, Centipede graph $S(C_n)$ are formed by the vertices of degrees$(3,2)$, $(1,2)$ and $(2,2)$.

Each of the above edges gives $\frac{1}{\sqrt{2}}$.

Therefore, the Atom-Bond Connectivity index of $S(C_n)$ is given by

$$ABC\left[S(C_n)\right]=(2n-1)\sqrt{2}.$$

Theorem 2.3. The Atom-Bond Connectivity index for the subdivision graph of Dutch windmill graph D_n^m is

$$ABC\left[S\left(D_n^m\right)\right]=nm\sqrt{2}.$$

Proof. Let subdivision graph of Dutch windmill graph $S(D_n^m)$ contains one vertex of degree $2m$, $m(n-1)$ vertices of degree 2 and nm vertices of subdivision graph is degree 2.

The edges of subdivision graph of Dutch windmill graph $S(D_n^m)$ are formed by the vertices of degrees $(2m,2)$ and $(2,2)$.

Each of the above edges gives $\frac{1}{\sqrt{2}}$.

Therefore, the Atom-Bond Connectivity index of $S(D_n^m)$ is given by

$$ABC\left[S\left(D_n^m\right)\right]=nm\sqrt{2}.$$

Theorem 2.4. The Atom-Bond Connectivity index for the subdivision graph of Firecracker graph $F_{n,m}$ is

$$ABC\left[S\left(F_{n,m}\right)\right]=(nm-1)\sqrt{2}.$$

Proof. Let subdivision graph of Firecracker graph $S\left(F_{n,m}\right)$ contains $(n-1)$vertices of degree 3, n vertices of degree$(m-1)$, 2 vertices of degree 2, $n(m-2)$ pendants and $2nm$ vertices of subdivision graph is of degree 2.

The edges of subdivision graph of Firecracker graph $S\left(F_{n,m}\right)$ are formed by the vertices of degrees$(2,2)$, $(m-1,2)$, $(1,2)$ and $(3,2)$.

Each of the above edges gives $\frac{1}{\sqrt{2}}$.

Therefore, the Atom-Bond Connectivity index of $S\left(F_{n,m}\right)$ is given by

$$ABC\left[S\left(F_{n,m}\right)\right]=(nm-1)\sqrt{2}.$$

Theorem 2.5 The Atom-Bond Connectivity index for the subdivision graph of Friendship graph F_n is

$$ABC\left[S(F_n)\right]=3n\sqrt{2}.$$

Proof. Let subdivision graph of Friendship graph $S(F_n)$ contains one vertex of degree $2n$, $2n$ vertices of degree 2, and $3n$ vertices of subdivision graph are of degree 2.

The edges of subdivision graph of Friendship graph $S(F_n)$ are formed by the vertices of degrees$(2n,2)$ and $(2,2)$.

Each of the above edges gives $\frac{1}{\sqrt{2}}$.

Therefore, the Atom-Bond Connectivity index of $S(F_n)$ is given by

$$ABC\left[S(F_n)\right]=3n\sqrt{2}.$$

Theorem 2.6. The Atom-Bond Connectivity index for the subdivision graph of Jahangir's graph $J_{n,m}$ is

$$ABC\left[S\left(J_{n,m}\right)\right]=m(n+1)\sqrt{2}.$$

Proof. Let subdivision graph of Jahangir's graph $S\left(J_{n,m}\right)$ contains one vertex of degree m, m vertices of degree 2, m vertices of degree 3 and 3m vertices of subdivision graph is degree 2.

The edges of subdivision graph of Banana tree $S\left(J_{n,m}\right)$ are formed by the vertices of degrees$(m,2)$, $(3,2)$ and $(2,2)$.

Each of the above edges gives $\frac{1}{\sqrt{2}}$.

Therefore, the Atom-Bond Connectivity index of $S\left(J_{n,m}\right)$ is given by

$$ABC\left[S\left(J_{n,m}\right)\right]=m(n+1)\sqrt{2}.$$

Theorem 2.7. The Atom-Bond Connectivity index for the subdivision graph of Tadpole graph $T_{n,m}$ is

$$ABC\left[S\left(T_{n,m}\right)\right]=(n+m)\sqrt{2}.$$

Proof. Let subdivision graph of Tadpole graph $S\left(T_{n,m}\right)$ contains a cycle graph C_n and path graph P_m. Cycle graph C_n contains one vertex of degree 3, $(n-1)$ vertices of degree 2 and n vertices of subdivision graph is degree 2.
Path graph P_m contains $(m-1)$ vertices of degree 2, one pendant vertex and m vertices of subdivision graph is degree 2.

The edges of subdivision graph of cycle graph $S(C_n)$ are formed by the vertices of degrees $(3,2)$ and $(2,2)$.

The edges of subdivision graph of path graph $S(P_m)$ are formed by the vertices of degrees$(3,2)$, $(2,2)$ and $(1,2)$.

Each of the above edges gives $\frac{1}{\sqrt{2}}$.

Therefore, the Atom-Bond Connectivity index of $S\left(T_{n,m}\right)$ is given by

$$ABC\left[S\left(T_{n,m}\right)\right]=(n+m)\sqrt{2}.$$

III. CONCLUSION

The problem of finding the general formula for ABC index of subdivision graphs of some special graphs Banana tree, Centipede Graphs, Dutch windmill graph, Firecrackers Graph, Friendship Graph, Jahangir's Graph and Tadpole Graph are solved here analytically using the concept of subdivision graph without using computer software tools.

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