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SPLIT AND EQUITABLE DOMINATION IN BOOK GRAPH AND STACKED BOOK GRAPH

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***Abstract* :**A subset D of V (G) is called an equitable dominating set of a graph G if for every v ∈ (V − D), there exists a vertex u ∈ D such that uv ∈ E(G) and |deg(u)− deg(v)| ≤ 1. The minimum cardinality of such a dominating set is denoted by $\gamma^{e}(G)$ and is called equitable domination number of G. A dominating set D of graph G is called a split dominating set, if the induced sub graph $< V - D >$is disconnected. In this paper first we introduce split domination in Book graph B_n and stacked book graph $B_{3,n}$. Later we prove the results for the equitable domination in $B_n(V-D)$.

***Keywords* :** Domination, Cartesian Product graph ,Split Domination , Equitable domination , Book Graph , Stacked Book Graph.

I. INTRODUCTION

By a graph G = (V,E) we mean a finite, undirected graph with neither loops nor multiple edges. The order and size of G are denoted by p and q respectively. For graph theoretic terminology we refer to Chartrand and Lesnaik [3] .Graphs have various special patterns like path, cycle, star, complete graph, bipartite graph, complete bipartite graph, regular graph , strongly regular graph etc. For the definitions of all such graphs we refer to Harary [5]. The study of Cross product of graph was initiated by Imrich [9]. For structure and recognition of Cross Product of graph we refer to Imrich[10].

The rigorous study of dominating sets in graph theory began around 1960, even though the subject has historical roots dating back to 1862 when de Jaenisch studied the problems of determining the minimum number of queens which are necessary to cover or dominate a n x n chessboard. In 1958, Berge defined the concept of the domination number of a graph, calling this as “coefficient of External Stability”. In 1962, Ore used the name “dominating set” and “domination number” for the same concept. In 1977 Cockayne and Hedetniemi made an interesting and extensive survey of the results known at that time about dominating sets in graphs. They have used the notation (G) for the domination number of a graph, which has become very popular since then. The survey paper of Cockayane and Hedetniemi has generated lot of interest in the study of domination in graphs. In a span of about twenty years after the survey, more than 1,200 research papers have been published on this topic, and the number of papers continued to be on the increase. Since then a number of graph theorists Konig, Ore, Bauer, Harary, Lasker, Berge, Cockayne, Hedetniemi, Alavi, Allan, Chartrand, Kulli, Sampthkumar, Walikar, Armugam, Acharya, Neeralgi, NagarajaRao, Vangipuram many others have done very interesting and significant work in the domination numbers and the other related topics. Recent book on domination , has stimulated sufficient inspiration leading to the expansive growth of this field of study. It has also put some order into this huge collection of research papers, and organized the study of dominating sets in graphs into meaningful sub areas, placing the study of dominating sets in even broader mathematical and algorithmic contexts. We refer to Haynes [6] & [7] and H. B. Walikar [8].

The split domination in graphs was introduced by Kulli & Janakiram [11]. They defined the split dominating set, the split domination number and obtained several interesting results regarding the split domination number of some standard graphs. They have also obtained relations of split domination number with the other parameters such as domination number, connected domination number, vertex covering number etc., These are all refer to Kulli & Janakiram [11 & 12]

Swaminathan*et al*[13] introduced the concept of equitable domination in graphs, by considering the following real world problems like : in a network nodes with nearly equal capacity may interact with each other in a better way, in this society persons with nearly equal status, tend to be friendly, in an industry, employees with nearly equal powers form association and move closely, equitability among citizens in terms of wealth, health, status etc is the goal of a democratic nation. Book graph and Stacked book graph concept we refer to [4].

In this paper we first introduce split domination in book graph and stack book graph, later we introduce equitable domination in book graph and stack graph.

II. PRELIMINARIES

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Definition 2.1 :

A path graph is a graph whose vertices can be listed in the order v_1, v_2, …, v_n such that the edges are $\{v_i, v_{i+1}\}$where i =1,2,……………...n-1. Equivalently, a path with at least two vertices is connected and has two terminal vertices (vertices that have degree 1),while all others (if any) have degree 2. Path can be denoted by P_n and edges n-1.

Definition 2.2 :

A complete bipartite graph of the form $K_{1,\ n-1}$ is a star graph with n-vertices. A star graph is a complete bipartite graph if a single vertex belongs to one set and all the remaining vertices belong to the other set.

Definition 2.3 : In graph theory, the Cartesian product $G \times H$ of graphs G and H of graphs G and H is a graph such that

- the vertex set of $G \times H$ is the Cartesian product V(G) × V(H); and
- any two vertices (u, u') and (v, v') are adjacent in $G \times H$ if and only if either
    - u = v and u' is adjacent with v' in H. Oo
    - u' = v' and u is adjacent with v in G
    -

Definition 2.4: Book graph is a Cartesian product of a star and single edge, denoted by B_n.

The m-book graph is defined as the graph Cartesian product $S_{m+1} \times P_2$, where S_m is a star graph and P_2 is the path graph on two nodes. The generalization of the book graph to n "stacked" pages is the (m, n)-stacked book graph. We refer to [2]

Definition 2.5:

The stacked book graph of order (m, n) is defined as the graph Cartesian product $S_{m+1} \times P_n$, where S_{m+1} is a star graph and P_n is the path graph on n nodes. It is therefore the graph corresponding to the edges of n copies of an m-page "book" stacked one on top of another and is the generalization of a book graph. we refer to [14]

Definition 2.6: A set D ⊆ V is called a dominating set if every vertex in V- D is adjacent to some vertex of D. Notice that D is a dominating set if and only if N[D] = V . The domination number of G, denoted as γ = γ(G), is the cardinality of a smallest dominating set of V . We call a smallest dominating set a γ-set.

Conjecture 2.7: For any graph G and H ,

1) $\gamma(G \times H) \geq \gamma(G).\gamma(H)$

Proposition 2.8 :

1) For the Path P_n on n vertices, $\gamma(P_n) = \left\lceil \frac{n}{3} \right\rceil$.
2) For the Star S_n on n vertices , $\gamma(S_n) = 1$.

Definition 2.9 : A dominating set D of graph G is called a split dominating set, if the induced sub graph $< V - D >$ is disconnected. The split dominating number γ^S(G) of G is the minimum cardinality of the split dominating set. The minimum cardinality over all split dominating set in a graph G is called split domination number $\gamma^s(G)$ of G.

Proposition 2.10 :

(1) For the paths P_n on n vertices , $\gamma^s(P_n) = \left\lceil \frac{n}{3} \right\rceil$

(2) For the stars S_n on n vertices , $\gamma^s(S_n) = 1$

Result 2.11: For domination of $\boldsymbol{P_n}$ **and** $\boldsymbol{S_n}$

(i) $\gamma(P_n) = \boldsymbol{\gamma^s(P_n)}$

(ii) $\gamma(S_n) = \boldsymbol{\gamma^s(S_n)}$

Definition 2.12 : A subset D of V (G) is called an equitable dominating set of a graph G if for every $v \in (V - D)$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and |deg(u)− deg(v)| ≤ 1. The minimum cardinality of an equitable dominating set of G is called equitable domination number of G and is denoted by γ^e(G).

Proposition 2.13

(i) **For the paths $\boldsymbol{P_n}$ on n vertices ,** $\boldsymbol{\gamma^e(P_n)} = \left\lceil \frac{n}{3} \right\rceil$

Proof : Since the degree of any vertex of P_n is either 1 or 2, any dominating set in P_n is clearly equitable. Hence $\gamma^e(P_n) = \gamma(P_n) = \left\lceil \frac{n}{3} \right\rceil$.

(ii) **Star graph $\boldsymbol{S_n}$ is does not satisfying the equitable domination.**

Proof : Let D be the domination set. The domination number of star S_n graph is one. Let $u \in D$ having $\deg(u) = n + 1$. The vertices$v \in V - D$ have deg(v) = 1. Here equitable domination condition is does not satisfying.

i.e. $|\deg(u) - \deg(v)| \leq 1$ not satisfying.

Hence star graph does not possess equitable dominating set.

III. Split and Equitable Domination number of Book Graph and Stacked Book graph

Theorem 3.1 : For any book graph B_n

$$\gamma(B_n) = 2 \ \ where\ n \geq 3$$

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Proof : Let B_n be a book graph with n vertices $V = \{v_1, v_2, v_3 \dots \dots v_n\}$. The book graph is the cross product of P_2 and S_n . In B_n graph two vertices v_k and v_l are connected to all the vertices, with the neighborhood $N(v_k) = \{v_1', v_2', \dots \dots \dots v_n'\}$ and $N(v_l) = \{v_1'', v_2'', \dots \dots \dots v_n''\}$. Here $N(v_k) \cup N(v_l) = V$ and $N(v_k) \cap N(v_l) = \emptyset$. Therefore the domination number of B_n graph is 2. i.e. $\gamma(B_n) = 2$.

Example: $\boldsymbol{B_3}$

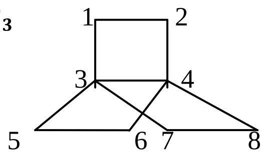
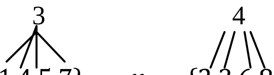


Fig (a)

$$D = \{3,4\}$$



$v_k = \{1,4,5,7\}$ $v_l = \{2,3,6,8\}$

$N(v_k) \cup N(v_l) = \{1,2,3,4,5,6,7,8\} = v$ and $N(v_k) \cap N(v_l) = \emptyset$. **i.e.** $\boldsymbol{\gamma(B_3) = 2}$

Theorem 3.2 : For any book graph B_n

$$\gamma^s(B_n) = 2 \; where \; n \geq 3$$

Proof : From theorem 3.1 $\gamma(B_n) = 2$. Let $D = \{v_k, v_l\}$ be the dominating set of book graph B_n. Consider the induced sub graph $< V - D > \; which$ is disconnected graph. Hence the book graph satisfies split domination condition giving, $\gamma^s(B_n) = 2$ where $n \geq 3$.

Example: B_3

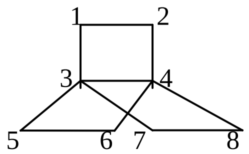


Fig (b)

$\boldsymbol{V - D} =$

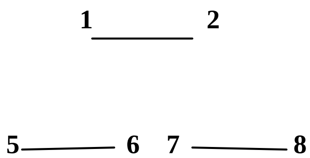


Fig (c)

$V = N(v_k) \cup N(v_l) = \{1,2,3,4,5,6,7,8\}$ and $N(v_K) \cap N(v_l) = \emptyset$.

$V - D = \{1,2,5,6,7,8\}$ is disconnected graph. i.e. $\gamma^s(B_3) = 2$

Proposition 3.3 : Book graph does not satisfy equitable domination condition.

Proof : Let D be the domination of book graph. Let $D \subseteq V$ .if for every vertex $v \in < V - D >$. There exists a vertex $u \in D$ such that $uv \in E[B_n(G)]$. Here $\deg(u) = 2 \; and \; \deg(v) = n + 1$. i.e. $|\deg(u) - \deg(v)| \neq 1 \; or \; 0$. This show that B_n graph is not satisfying the equitable condition.

Example : From fig (a) $D = \{3,4\}$ and $u \in D$, $v \in V - D$ there exits edge uv. $uv \in E(B_2)$. Edge $(3,1) \in E(B_2)$. $\deg(3) = 4 \; and \; \deg(1) = 2$. i.e. $|\deg(3) - \deg\,(1)| = 2$. B_3 is not satisfying equitable condition.

Theorem 3.4 : For any book graph B_n

$$\gamma[B_n(V - D)] = n \; where \; n \geq 3$$

Proof: Let B_n be a book graph with n vertices $V = \{v_1, v_2, v_3 \dots \dots v_n\}$. The book graph is the cross product of P_2 and S_n . In B_n graph two vertices v_k and v_l are connected to all the vertices. with the neighborhood $N(v_k) = \{v_1', v_2', \dots \dots \dots v_n'\}$ and $N(v_l) = \{v_1'', v_2'', \dots \dots \dots v_n''\}$. Here $N(v_k) \cup N(v_l) = V$ and $N(v_k) \cap N(v_l) = \emptyset$. Therefore the domination number of B_n graph is $\gamma(B_n) = 2$. Let D be the domination of book graph B_n. i.e. $D = \{v_k, v_l\}$. Let $D \subseteq V$. Here $< V - D >$is disconnected graph. The component of disconnected $graph$ $< V - D >$ $U_1, U_2, U_3 \dots \dots U_n$. Means n components. Each component having only two vertices. These two vertices are connected each other. i.e. Means path P_2 we are getting. $U_1 = \{a^1, b^1\}$, $U_2 = \{a^2, b^2\}$, ………….. $U_n = \{a^n, b^n\}$. The domination of all the components are unique. i.e. the domination of $\gamma[B_n(V - D)] = n$.

In Fig (c) represent the $B_3\,(V - D)$. The domination set of $B_3\,(V - D)$ is $\{1,5,7\}$.

$$i.e. \gamma[B_3(V - D)] = 3$$

Theorem 3.5: For any book graph B_n

$$\gamma^S[B_n(V - D)] = n \; where \; n \; \geq 3$$

Theorem 3.6 : For any book graph B_n

$$\gamma^e[B_n(V - D)] = n \; where \; n \geq 3$$

Proof : From previous theorem 3.1 $\gamma(B_n) = 2$. Let D be the domination. Let $D \subseteq V$. Here $< V - D >$is disconnected graph. The component of disconnected graphs $< V - D >$ are $U_1, U_2, U_3 \dots \dots U_n$. Means n components. Each components having only two vertices. These two vertices are connected each other. i.e. Means path P_2 we are getting. $U_1 = \{a^1, b^1\}$, $U_2 = \{a^2, b^2\}$, ………….. $U_n = \{a^n, b^n\}$. The domination of all components are one only. I.e. the domination of $\gamma[B_n(V - D)] = n$.

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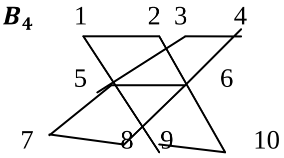
Let W_n be the domination of U_n . if for every vertex $z_n \in (U_n - W_n)$ there exists a vertex $y_n \in W_n$ such that $z_n y_n \in E(U_n - W_n)$. $\deg(y_n) = 1$ *and* $\deg(z_n) = 1$. i.e. $|\deg(y_n) - \deg(z_n)| = 0$. i.e means $< V - D >$is satisfying equitable condition . The equitable Domination

$$\gamma^e[B_n(V-D)] = n \text{ where } n \geq 3$$

Example : $\boldsymbol{B_4}$



D= { 5,6}

Fig (d)

V- D =

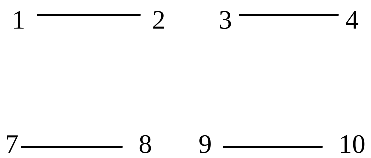


Fig (e)

In Fig (e) represent the $B_4(V-D)$. The domination set of $B_4\ (V-D)$ is $\{1,3,7,9\}$.
$\gamma[B_4(V-D)] = 4$. $x = \{1,2,3,4,7,8,9,10\}$. $y = \{1,3,7,9\}$.
let $z \in y$ and $a \epsilon\, x - y$. There exit the edge za. $za \in E[B_4(V-D)$.
Ex : $(7,8) \in E(B_4\ (V-D))$. $\deg(7) = 1$ *and* $\deg(8) = 1$.
i.e. $|\deg(7) - \deg(8)| = 0$. Equitable domination condition is satisfying.

Result 3.7 : For book graph B_n

(1) $\gamma(B_n) = \gamma^s(B_n)$
(2) $\gamma[B_n(V-D)] = \gamma^s[B_n(V-D)] = \gamma^e[B_n(V - D$

Theorem 3.8: For any stacked book graph $B_{3,n}$

$$\gamma(B_{3,n}) = n \text{ where } n \geq 3$$

Prof: Let $B_{3,n}$ be a stacked book graph with 4n vertices $V = \{v_1, v_2, v_3 \dots\dots v_{4n}\}$. The stacked book graph is the cross product of . $S_{m+1} \times P_n$. The domination of stacked book graph vertices are $\{v_1', v_2', \dots\dots\dots v_n'\}$. $\{v_1', v_2', \dots\dots\dots v_n'\}$ vertices are connected all the vertices. But $N(v_1') \cup N(v_2') \cup N(v_3') \dots\dots \cup N(v_n') = V$ *and* $N(v_1') \cap N(v_2') \cap N(v_3') \dots\dots \cap N(v_n') = \emptyset$. Therefore the domination of stacked book graph is n.

$$\gamma(B_{3,n}) = n \text{ where } n \geq 3$$

Example :

$B_{4,4}$

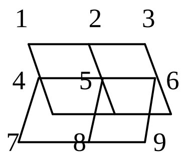
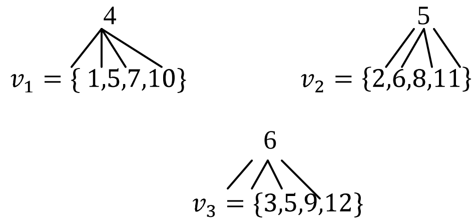


Fig (f)

D = {4,5,6}



$N(v_1) \cup N(v_2) \cup N(v_3) = \{1,2,3,4,5,6,7,8,9,10,11,12\}$
and $N(v_1) \cap N(v_2) \cap N(v_3) = \emptyset$

Therefore $\gamma(B_{3,3}) = 3$.

Theorem 3.9: For any stacked book graph $B_{3,n}$

$$\gamma^s(B_{3,n}) = n \text{ where } n \geq 3$$

Proof: From theorem 3.8 $\gamma(B_{3,n}) = n$. Let D be the dominating set of stacked book graph $B_{3,n}$ with $D = \{v_1', v_2', \dots\dots\dots v_n'\}$. Let $D \subseteq V$. The vertices of D are connected each other and the induced sub graph $< V - D >$ is disconnected graph. This show that the book graph is satisfying split domination condition. Hence the split domination

number stacked book graph is n.

$$\gamma^s(B_{3,n}) = n \text{ where } n \geq 3$$

$D = \{4,5,6\}$ and $< V - D > = \{1,2,3,7,8,9,10,11,12\}$.

Proposition 3.10 : Stacked book graph does not satisfy the equitable domination condition.

Proof : Let D be the domination of book graph. Let $D \subseteq V$. $u \in D$ and $\in < V - D >$. There exits edge $uv \in E[B_{3,n}(G)]$. Here $\deg(u) = 4$ *or* 5 *and* $\deg(v) = 2$. i.e. $|\deg(u) - \deg(v)| \neq 1$ *or* 0. This show that $B_{3,n}$ graph is does not satisfying the equitable domination condition.

Result 3.11 : For stacked book graph $B_{3,n}$

1) $\gamma(B_{3,n}) = \gamma^s(B_{3,n})$

IV Conclusion:

The book graph is the cross product *of* $S_{m+1,} \times P_2$. In this paper we found that the domination of book graph is $\gamma(B_n) = n$. And the paper show that the book graph B_n satisfied the split domination condition. The same B_n graph does not satisfied equitable domination condition. Also the paper explain, the introduce to find the b $\gamma[B_n(V-D)] = n$. The book graph $B_n(V-D)$ is satisfied split domination condition and equitable domination condition.

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The stacked book graph is Cartesian product of $S_{m+1,} \times P_n$. In this paper we use only stacked book graph $B_{3,n}$. The procedure involves to find $\gamma(B_{3,n}) = n$. Also the show that the stacked book graph satisfied Split domination. The same $B_{3,n}$ does not satisfied the equitable domination condition. Finally the paper explain to find domination number, split domination number and equitable domination number of book graph and stacked book graph. **V**

V Results:

Graph	Domination number	Split domination number	Equitable domination number
B_n $where$ $n \geq 3$	2	2	Not satisfied
$Bn(V - D)$ $where$ $n \geq 3$	n	n	n
$B_{3,n}$ $where$ $n \geq 3$	n	n	Not satisfied

VI REFERENCE :

1) B. Bollob´as and E. J. Cockayne, Graph-theoretic parameter concerning domination independence, and irredundace, J. Graph Theory 3 (1979), 241-249.
2) "Book Graph." From MathWORLD-A Wolfram Web Resource. http://mathworld.wolfarm.com/Book graph.html
3) G. Chartrand and L. Lesnaik, Graphs and Digraphs, Chapman and Hall.CRC, 4th edition, 2005
4) Gallian, J. A. "Dynamic Survey DS6: Graph Labeling." Electronic J. Combinatorics, DS6, 1-58, Jan. 3, 2007. http://www.combinatorics.org/Surveys/ds6.pdf.
5) Harary, Frank(1969) Graph Theory, Reading. MA: Addison- Wesley.
6) Haynes, T. W.; Hedetniemi, S. T.; and Slater, P. J. Domination in Graphs-Advanced Topics. New York : Dekker, 1998.
7) Haynes, T. W.; Hedetniemi, S. T.; and Slater, P. J. Fundamentals Of Domination in Graphs. New York: Dekker, 1998.
8) H. B. Walikar, B. D. Acharya and E. Sampathkumar, Recent Developments in the theory of domination in graphs, Mehta Research institute, Alahabad, MRI lecture Notes in Math.1 (1979).
9) Imrich, Wilfried; Klavžar, Sandi; Rall, Douglas F. (2008), Graph and their Cartesian product, A. K Peters, ISBN 1-56881-429-1.
10) Imrich, Wilfried; Klavžar, Sandi (2000), Product Graphs: Structure and Recognition, Wiley, ISBN 0-471-37039-8.
11) Kulli, V.R. and Janakiram, B., The split domination number of a graph, Graph Theory Notes of New York, XXXII (1997) 16–19.
12) Kulli, V.R. and Janakiram, B., The non spilt domination number of a graph, Indian J. Pure., Appl. Math., 31(5) (2000) 545–550.
13) V. Swaminathan and K. M. Dharmalingam, Degree Equitable Domination on Graphs, Kragujevak Journal of Mathematics, 35(1), 191-197 (2011).
14) Weisstein, Eric W. "Stacked Book Graph." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/StackedBookGraph.html.

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