



# ANTI – MAGIC LABELING ON SOME STAR RELATED GRAPHS.

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**Abstract:** Let  $G=(V,E)$  be a simple, finite, undirected and connected graph. A graph  $G=(V,E)$  with order  $p$  and size  $q$  is said to admit anti-magic labelling if there exists a bijection  $f:E(G) \rightarrow \{1,2,\dots,q\}$  such that for each  $u,v \in V(G)$ ,  $\sum f(e)$  are distinct for all  $e=uv \in E(G)$ . In this paper, we have obtained anti- magic labelling on the graphs, obtained by joining apex vertices of some star graphs to a new vertex by assigning both even and odd positive integers to these vertices and edges respectively.

**Keywords:** Star graphs, Edge labelling, vertex labelling, Even Anti – magic labelling, Odd Anti- magic labelling.

## 1.INTRODUCTION

The concept of graph labeling was introduced by Rosa in 1967. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions.

Hartsfield and Ringel introduced the concept of Anti – magic labeling which is an assignment of distinct values to different vertices in a graph in such a way that when taking the sums of the labels, all the sums will be having different constants [4].

### Definition 1.1:

#### Vertex labeling :

Label the vertices of a graph with positive integers. This process is called vertex labeling. Let  $f:V \rightarrow \{1,2,\dots,n\}$ . Under this vertex labeling, the edge weight of an edge  $e=uv$  is defined as  $W(e) = W(uv) = f(u)+f(v)$ .

### Definition 1.2:

#### Edge labeling :

Label the edges of a graph with positive integers. This process is called edge labeling. Let  $f:E \rightarrow \{1,2,\dots,n\}$ . Under this edge labeling, the vertex weight of a vertex  $v \in V(G)$  is defined as the sum of the labels of the edges incident with  $v$  that is  $w(v) = \sum f(uv)$ .

### Definition 1.3:

Consider  $t$  copies of stars namely  $K_{1,n_1}, K_{1,n_2}, \dots, K_{1,n_t}$  then the graph  $G = \langle K_{1,n_1}, K_{1,n_2}, \dots, K_{1,n_t} \rangle$  is the graph obtained by joining apex vertices of each  $K_{1,n_i}$  and  $K_{1,n_{i+1}}$  to a new vertex  $u_i$ , where  $1 \leq i \leq t-1$ .

## II Main Results:

### Theorem 2.1:

The graph  $G$  obtained by joining  $t$  copies of stars  $\langle K_{1,n_1}, K_{1,n_2}, \dots, K_{1,n_t} \rangle$  admits Edge – Even anti – magic labeling.

#### Proof:

Let  $\{v_1, v_2, \dots, v_n\}$  be the vertices and  $\{e_1, e_2, \dots, e_n\}$  be the edges of the star graphs,  $K_{1,n_i}, i=1,2,\dots,t$ . We shall join these graphs  $K_{1,n_i}, K_{1,n_{i+1}}$  and  $K_{1,n_{i+2}}$  by adding a new vertex  $u_i$ ,

where  $1 \leq i \leq t-1$  to their apex vertices. We define the labeling function  $f$  as follows:

$f:E(G) \rightarrow \{2,4,\dots,2q\}$ , where  $q$  is the even number of edges of  $G$ .

$$f(v_{i,0}) = 3q+i, \text{ for } i = 1$$

$$f(v_{i,0}) = 6q+3i, \text{ for } i = 2$$

$$f(v_{i,0}) = 5q+3i, \text{ for } i = 3$$

$$f(e_i) = 2i, \text{ for } i = 1,2,\dots,n$$

$$f(u_i) = 2q+i-1, \text{ for } i=1$$

Thus, the above labeling pattern gives rise to an anti – magic labeling on the given graph  $G$ .

### Illustration 2.2:

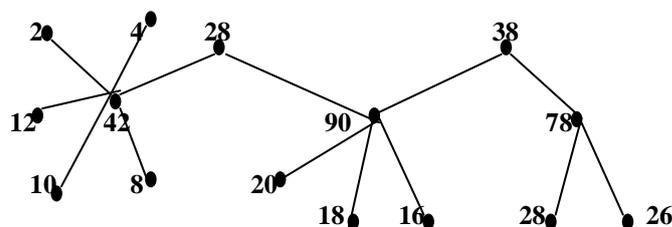


Figure 1: Edge- Even Anti- magic labeling on  $\langle K_{1,2}, K_{1,3}, K_{1,5} \rangle$

### Theorem 2.3:

The graph  $G$  obtained by joining  $t$  copies of stars  $\langle K_{1,n_1}, K_{1,n_2}, \dots, K_{1,n_t} \rangle$  admits Edge – odd anti – magic labeling.

#### Proof:

Let  $\{v_1, v_2, \dots, v_n\}$  be the vertices and  $\{e_1, e_2, \dots, e_n\}$  be the edges of the star graphs,  $K_{1,n_i}, i=1,2,\dots,t$ . We shall join these graphs  $K_{1,n_i}, K_{1,n_{i+1}}, K_{1,n_{i+2}}, K_{1,n_{i+3}}$  by adding a new vertex  $u_i$ , where  $1 \leq i \leq t-1$  to their apex vertices. We define the labeling function  $f$  as follows:

$$f:E(G) \rightarrow \{1,3,\dots,q\},$$

where  $q$  is the odd number of edges of  $G$ .

$$f(v_{i,0}) = 2q+i=7, \text{ for } i = 1$$

$$f(v_{i,0}) = 6q-3i, \text{ for } i = 2$$

$$f(v_{i,0}) = 7q+3i-1, \text{ for } i = 3$$

$$f(v_{i,0}) = 5q+3i, \text{ for } i = 4$$

$$f(e_i) = i, i+1, i+2, i+3, \dots, i+q.$$

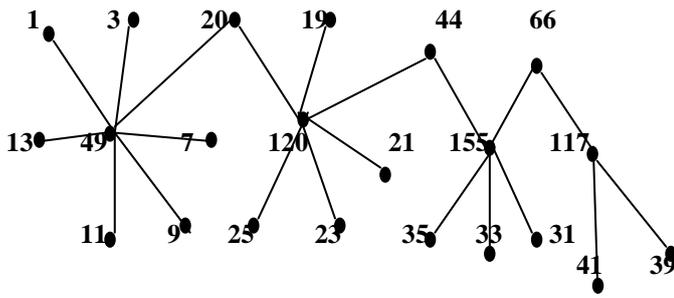
$$f(u_i) = q-i, \text{ for } i=1$$

$$f(u_i) = 2q+i, \text{ for } i=2$$

$$f(u_i) = 3q+i, \text{ for } i=3.$$

Thus, the above labeling pattern gives rise to an anti – magic labeling on the given graph  $G$ .

### Illustration 2.4:



**Figure 2: Edge- Odd Anti- magic labeling on  $\langle K_{1,2}, K_{1,3}, K_{1,4}, K_{1,6} \rangle$**

**Theorem 2.5:**

The graph G obtained by joining t copies of stars  $\langle K_{1,n_1}, K_{1,n_2}, \dots, K_{1,n_t} \rangle$  admits Vertex – even anti – magic labeling.

**Proof:**

Let  $\{v_1, v_2, \dots, v_n\}$  be the vertices and  $\{e_1, e_2, \dots, e_n\}$  be the edges of the star graphs,  $K_{1,n_i}, i= 1, 2, \dots, t$ . We shall join these graphs  $K_{1,n_i}$  and  $K_{1,n_{i+1}}$  by adding a new vertex  $u_i$ , where  $1 \leq i \leq t-1$  to their apex vertices. We define the labeling function f as follows:

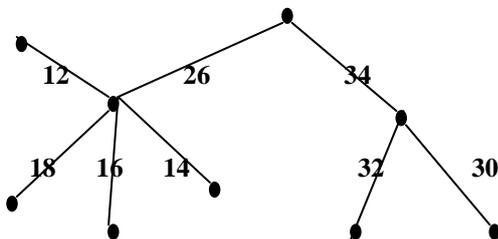
$$f:V(G) \rightarrow \{2, 4, \dots, 2q\},$$

where q is the even number of edges of G.

- $f(v_{i,0}) = q+2i$  , for  $i = 1$
- $f(v_{i,0}) = q+4i+2$  , for  $i = 2$
- $f(v_{1,j}) = q+2j+2$  , for  $j = 1, 2, \dots, t$
- $f(u_i) = q+6i+2i$  , for  $i=1$
- $f(v_{2,j}) = 3q+2j+4$  , for  $j=1, 2, \dots, t$

Thus ,the above labeling pattern gives rise to an anti – magic labeling on the given graph G.

**Illustration 2.6:**



**Figure 3: Vertex- Even Anti- magic labeling on  $\langle K_{1,2}, K_{1,4} \rangle$**

**Theorem 2.7:**

The graph G obtained by joining t copies of stars  $\langle K_{1,n_1}, K_{1,n_2}, \dots, K_{1,n_t} \rangle$  admits Vertex – odd anti – magic labeling.

**Proof:**

Let  $\{v_1, v_2, \dots, v_n\}$  be the vertices and  $\{e_1, e_2, \dots, e_n\}$  be the edges of the star graphs,  $K_{1,n_i}, i= 1, 2, \dots, t$ . We shall join these graphs  $K_{1,n_i}, K_{1,n_{i+1}}$  and  $K_{1,n_{i+2}}$  by adding a new vertex  $u_i$ , where  $1 \leq i \leq t-1$  to their apex vertices. We define the labeling function f as follows:

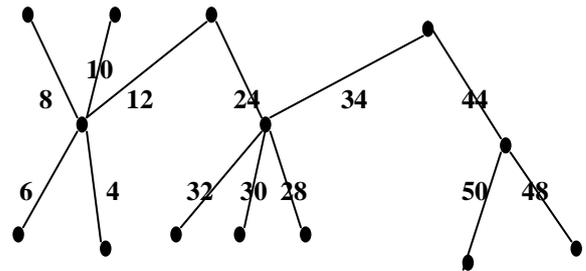
$$f:V(G) \rightarrow \{1, 3, \dots, q\},$$

where q is the odd number of edges of G.

- $f(v_{i,0}) = i$  , for  $i = 1$
- $f(v_{i,0}) = q+i-2$  , for  $i = 2$
- $f(v_{i,0}) = 2q-i$  , for  $i = 3$
- $f(v_{1,j}) = q+2j-1$  , for  $i = 1, 2, \dots, t$
- $f(u_i) = q-2i$  , for  $i=1$
- $f(u_i) = 2q-2i-1$  , for  $i=2$
- $f(v_{2,j}) = 2q-2i$  , for  $j=1$
- $f(v_{2,j}) = 2q+i, 2q+i+1, 2q+i+2, \dots, 2q+i+t$ .
- $f(v_{3,j}) = 3q+5i$  , for  $j=1$
- $f(v_{3,j}) = 4q-2i$  , for  $j=2$
- $f(v_{3,j}) = 4q-i+1$  , for  $j=3$ .

Thus, the above labeling pattern gives rise to an anti – magic labeling on the given graph G.

**Illustration 2.8:**



**Figure 4: Vertex- Odd Anti- magic labeling on  $\langle K_{1,2}, K_{1,3}, K_{1,4} \rangle$**

**CONCLUSION:**

In this paper, We have presented anti- magic labeling on some star related graphs by assigning even and odd positive integers for both the vertices and edges respectively. Here, we obtained anti- magic labeling on 2 copies, 3 copies and 4 copies of star related graphs. Similar results for finite number of copies of star related graphs are under investigation.

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