Volume 8, No. 6, July 2017 (Special Issue III)
ISSN No. 0976-5697
International Journal of Advanced Research in Computer Science
RESEARCH PAPER
Available Online at www.ijarcs.info

DOMINATING FUNCTIONS OF CORONA PRODUCT GRAPH OF K_n AND P_m

C Shobha Rani
Research Scholar, Department of Mathematics,
Madanapalle Institute of Technology & Science,
Madanapalle-517325, India
E-mail: charapallishobha@gmail.com

B Maheswari
Professor, Department of Applied Mathematics,
Sri Padmavati Mahila Visvavidyalayam,
Tirupati-517502, India[2]
E-mail: maherahul55@gmail.com

S Jeelani Begum
Assistant Professor, Department of Mathematics,
Madanapalle Institute of Technology & Science,
Madanapalle-517325, India
E-mail: sjb.maths@gmail.com

G S S Raju
Professor, Department of Mathematics,
JNTU College of Engineering,
Pulivendula- 516390, India[4]
E-mail: rajugss@yahoo.com

S Gouse Mohiddin
Assistant Professor, Department of Mathematics,
Madanapalle Institute of Technology & Science,
Madanapalle-517325, India
E-mail: gousemaths@gmail.com

Abstract: Let G be a simple graph with vertex set V and edge set E. A subset D of a vertex set V is known as dominating set of G, if for every vertex v in V-D, there exists a vertex u in D such that (u, v) ∈ E. Let $G(V,E)$ be a graph and a function $f : V \rightarrow [0, 1]$ is called a dominating function (DF) of G, if $f[N[v]] = \sum_{u \in N[v]} f(u) \geq 1$, for each $v \in V$. The dominating function f of G is called a minimal dominating function, if for all $g < f$, g is not a dominating function. In this paper we study dominating functions of corona product graph of complete graph K_n with path P_n.

Keywords: Corona product graph, Dominating sets, Dominating functions.

I. INTRODUCTION

Domination theory gain an importance in graph theory which aids to find efficient routes within ad-hoc mobile networks and designing secure systems for electrical grids. The study on theory of product graphs is useful to understand computational complexity in wireless networking.

Frucht and Harary [1] introduced a new product on two graphs G_1 and G_2, called corona product denoted by $G_1 \odot G_2$. Generally Product of graphs occurs in discrete mathematics. Allan and Laskar [2], Cockayne and Hedetniemi [3,4] have studied various domination parameters of graphs. Dominating functions are studied in [5,6,7].

A nonempty subset D of V in a graph G is a dominating set of G, if every vertex in V-D is adjacent to at least one vertex in D. The number of vertices in a minimum dominating set is defined as the domination number of G and is denoted by $\gamma(G)$. If D consists of minimum number of vertices among all dominating sets, then D is called the minimum dominating set(MDS).

II. CORONA PRODUCT OF K_n AND P_m

The corona product of a K_n and P_m is a graph obtained by taking one copy of a n-vertex complete graph K_n and n copies of P_m and then joining the i^{th} vertex of K_n to every vertex of i^{th} copy of P_m and it is denoted by $G = K_n \odot P_m$.

Now some properties of the graph $G = K_n \odot P_m$ is discussed in the following.

Theorem 1: The graph $G = K_n \odot P_m$ is a connected graph.

Proof: Consider the graph $G = K_n \odot P_m$ By the definition of corona product, we know that the i^{th} vertex of K_n is adjacent to each copy of i^{th} copy of P_m in G. That is the vertices in K_n are connected to the vertices of P_m thus it becomes a one component. Hence it follows that G is connected.

Theorem 2: The degree of a vertex v in $G = K_n \odot P_m$ is given by $d(v) = \begin{cases} m+n-1, & if\ v \in K_n \\ 3\ or\ 2, & if\ v \in P_m \end{cases}$

Proof: In the graph G, i^{th} vertex of K_n is joined to m vertices of i^{th} copy of P_m in G. We observe that any vertex

CONFERENCE PAPER
National Conference dated 27-28 July 2017 on
Recent Advances in Graph Theory and its Applications (NCRAGTA2017)
Organized by Dept of Applied Mathematics
Sri Padmawati Mahila Vishvavidyalayam (Women's University) Tirupati, A.P., India

v in K_n is adjacent to (n-1) vertices of K_n .Therefore the degree of a vertex v in K_n is (n+m-1) in G.

i.e., $d(v)=\{m+n-1, \quad if\ v\in K_n \quad \rightarrow (1)$

And there are m vertices in each copy of P_m , such that each vertex v in P_m is of degree 2, if v is the end vertex in P_m and v in P_m is of degree 3, if v is the not end vertex in P_m . Since this vertex is adjacent to a correspond vertex of K_n in G, it follows that the degree of a vertex $v \in P_m$ in G is either 2 or 3.

i.e., $d(v)=\begin{cases}3, & if\ v\in P_m \text{ and v is not a end vertex,}\\ 2, & if\ v\in P_m \text{ and v is an end vertex.}\end{cases} \rightarrow (2)$

Finally from (1) & (2), we get

$$d(v)=\begin{cases}m+n-1, & if\ v\in K_n\\ 3\ or\ 2, & if\ v\in P_m\end{cases}$$

Theorem 3: The number of vertices and edges in $G=K_n \odot P_m$ is given by

$$|V(G)|=n(m+1) \quad \text{and} \quad |E(G)|=\frac{n}{2}(4m+n-1).$$

Proof: Let us consider the graph $G=K_n \odot P_m$ with the vertex set V. In G, we know that n, m denotes the number of vertices of K_n and the cycle P_m respectively. By the definition, the vertex set of G contains the vertices of K_n and the vertices P_m in n- copies. Hence, it follows that $|V(G)| = n+nm=n(m+1)$.

By the above theorem, the degree of a vertex is given by

$$d(v)=\begin{cases}m+n-1, & if\ v\in K_n\\ 3\ or\ 2, & if\ v\in P_m\end{cases}$$

Hence $|E(G)|=\frac{1}{2}\left(\sum_{v\in K_n}\deg(v)+n\sum_{v\in P_m}\deg(v)\right)$

$$=\frac{1}{2}\left[n(m+n-1)+2n(2)+n(m-2)(3)\right]$$

$$=\frac{1}{2}\left[mn+n^2-n+4n+3mn-6n\right]$$

$$=\frac{1}{2}\left[n^2+4mn-3n\right]$$

$$|E(G)|=\frac{n}{2}\left[4m+n-3\right].$$

II. III. MAIN RESULTS

Here we study on dominating sets and dominating functions of the graph $G=K_n \odot P_m$.

Theorem 4: The minimal dominating set for the graph $G=K_n \odot P_m$ is set of all vertices of K_n .

Proof: Consider $G=K_n \odot P_m$. Let D denote a dominating set of the graph $G=K_n \odot P_m$. Suppose D contains the set of all vertices of K_n . By the definition of the graph $G=K_n \odot P_m$, every vertex in K_n is adjacent to all vertices of each copy of P_m . That is, the vertices in K_n dominates the vertices in each copy of P_m . Thus D becomes a dominating set of $G=K_n \odot P_m$. If possible to remove a vertex in D, that vertex is v_i is the i^{th} vertex in K_n , then the remaining set becomes $D_1=D-\{v_i\}$ is not a dominating set. Because v_i in K_n not dominates the vertices in i^{th} copy of P_m . That means the subset of D is not a dominating set. Hence D becomes a minimal dominating set of $G=K_n \odot P_m$.

Theorem 5: The domination number of the graph $G=K_n \odot P_m$ is n.

Proof: Let D denote a dominating set of G. Suppose D contains the vertices of K_n. By the definition of the graph, every vertex in K_n is adjacent to all vertices of associated copy of P_m . That is the vertices in K_n dominate the vertices in all copies of P_m respectively. Further these vertices being in K_n , they dominate among themselves. Thus becomes a DS of G. Therefore $\gamma(G)=n$.

Theorem 6: Let D be a minimal dominating set (MDS) of $G=K_n \odot P_m$. Let a function $f:V\rightarrow[0,1]$ be defined by

$$f(v)=\begin{cases}1, & if\ v\in D\\ 0, & otherwise\end{cases}$$

Then f becomes a MDF.

Proof: Consider $G=K_n \odot P_m$ be corona product of K_n and P_m .

Let D be a MDS of $G=K_n \odot P_m$. Clearly this set contains all vertices of K_n and this set is also minimal.

Case (1): Let v in K_n be such that $d(v)=(m+n-1)$ in G , then $N[v]$ contains m vertices of P_m and n vertices of K_n in G .

Thus $\sum_{u\in N[v]} f(u)=\left(\underbrace{1+---+1}_{n-times}\right)+\left(\underbrace{0+---+0}_{m-times}\right)=n$

Case (2): Suppose v in P_m then

(i)If $d(v)=2$ in G , then $N[v]$ contains two vertices of P_m and one vertex of K_n in G . Thus $\sum_{u\in N[v]} f(u)=1+0+0=1$

(ii)If $d(v)=3$ in G , then $N[v]$ contains three vertices of P_m and one vertex of K_n in G .Thus $\sum_{u\in N[v]} f(u)=1+0+0=1$

Therefore all the possibilities, we get $\sum_{u\in N[v]} f(u)\geq 1,\ \forall v\in V$

Therefore the function f is a Dominating Function.

Now we check for minimality of f , define $g:V\rightarrow[0,1]$ by

$$g(v)=\begin{cases}r, & if\ v=v_k\in D\\ 1, & if\ v\in D-\{v_k\}\\ 0, & \text{otherwise}\end{cases}$$

Where $0<r<1$ .Since, strict inequality holds at the vertex v_k in D , it follows that $g<f$.

Case (1): Let v in K_n be such that $d(v)=(m+n-1)$ in G , then $N[v]$ contains m vertices of P_m and n vertices of K_n in G .

CONFERENCE PAPER
National Conference dated 27-28 July 2017 on
Recent Advances in Graph Theory and its Applications (NCRAGTA2017)
Organized by Dept of Applied Mathematics
Sri Padmawati Mahila Vishvavidyalayam (Women's University) Tirupati, A.P., India

$$If\ v_k \text{ in } N[v] \Rightarrow \sum_{u\in N[v]} g(u) = \left(\underbrace{1+---+1}_{(n-1)-times} + r\right) + \left(\underbrace{0+---+0}_{m-times}\right) = n+r-1$$

$$If\ v_k \text{ not in } N[v] \Rightarrow \sum_{u\in N[v]} g(u) = \left(\underbrace{1+---+1}_{n-times}\right) + \left(\underbrace{0+---+0}_{m-times}\right) = n$$

Case (2): Suppose v in P_m then

(i) If $d(v)=2$ in G, then $N[v]$ contains two vertices of P_m and one vertex of K_n in G.

If v_k in $N[v]$, $then \sum_{u\in N[v]} g(u) = r+0+0 = r<1$

If v_k not in $N[v]$, $then \sum_{u\in N[v]} g(u) = 1+0+0 = 1$

(ii)If $d(v)=3$ in G, then $N[v]$ contains three vertices of P_m and one vertex of K_n in G.

If v_k in $N[v]$, $then \sum_{u\in N[v]} g(u) = r+0+0+0 = r<1$

If v_k not in $N[v]$, $then \sum_{u\in N[v]} g(u) = 1+0+0+0 = 1$

In this case, g is not a dominating function.

Therefore g is not a DF, because

$$\sum_{u\in N[v]} g(u) < 1,\ \ for\ \ some\ \ v\in V$$

Hence f is a minimal dominating function on G.

III. IV. ACKNOWLEDGMENT

The corresponding author acknowledge Department of Science and Technology, Government of India for financial support wide reference no: No.SR/WOS-A/MS-07/2014 (G) under women scientist scheme to carry out this work.

V. CONCLUSION

It is interesting to study the dominating functions of corona product graph of complete graph with a path. This work gives the scope for an extensive study of domination numbers and other dominating functions of this graph.

IV. VI. REFERENCES

[1] R. Frucht, and F. Harary, "On the corona of Two Graphs", Aequationes Mathematicae, Volume 4, Issue 3, pp.322 – 325, 1970.

[2] R.B. Allan, and R.C. Laskar, "On domination, Independent domination numbers of a graph" Discrete Math., 23, pp. 73-76, 1978.

[3] E.J. Cockayne, and S.T.Hedetniemi, "Towards a theory of domination in graphs", Networks, volume 7, pp. 247-261, 1977.

[4] E. J. Cockayne, G. Fricke, S.T. Hedetniemi, and C.M. Mynhardt, "Properties of minimal dominating functions of graphs", ARS. Combinatoria, volume 41, pp. 107-115, 1995.

[5] J.A. Bondy, and U.S.R Murty, "Graph Theory", Springer, 2008.

[6] M. Siva Parvathi, and B. Maheswari, "Minimal Dominating Functions Of Corona Product Graph Of A Path With A Star", IAEME, Volume 5, Issue 1, pp. 01-11, January - April 2014.

[7] T.W. Haynes, S.T. Hedetniemi & J.S. Slater, "Fundamentals of Domination in Graphs", Marcel Decker, Inc. 1998.

CONFERENCE PAPER
National Conference dated 27-28 July 2017 on
Recent Advances in Graph Theory and its Applications (NCRAGTA2017)
Organized by Dept of Applied Mathematics
Sri Padmawati Mahila Vishvavidyalayam (Women's University) Tirupati, A.P., India