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STRONGLY MULTIPLICATIVE CAYLEY GRAPHS

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Abstract: A graph with $\boldsymbol{n}$ vertices is *stronglymultiplicative* if the vertices of $\boldsymbol{G}$ can be labelled with distinct integers $\mathbf{1,2,3,\ldots,n}$ such that the labels induced on the edge by the product of the end vertices are distinct.We proved that the Cayley graph on cyclic and dihedral groups with specified generating sets admits strongly multiplicative labeling.

Keywords: Cayley graph; cyclic groups; circulant graph; dihedral groups; strongly multiplicative.

I. INTRODUCTION

Our graph terminologies are standard except as indicated; see[1]and [3]. A graph G be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$. We use $|V(G)| = n$ and $|E(G)| = q$. A labeling is assignment of integers to the vertices or edges, or both subject to the certain conditions.

In [2], Beineke and Hegde call a graph with n vertices is strongly multiplicative if the vertices of G can be labeled with distinct integers $1,2,3,\ldots,n$ such that the labels induced on the edges by the product of the end vertices are distinct.

Let a finite nontrivial group with the identity elementτ. LetΩ be a generating set of Γ satisfying $\tau \notin \Omega$ and if $a \in \Omega$ then$a^{-1} \in \Omega$. The Cayley graph corresponding to Γ is the graph $G = (V,E)$, where $V(G) = \Gamma$ and $E(G) = \{(x,xa)\backslash x \in V(G), a \in \Omega\}$and it is denoted by $G = Cay(\Gamma;\Omega)$. A cayley graph$Cay(\mathbb{Z}_n;\Omega)$ on the cyclic group $\mathbb{Z}_n$ is called a *circulant graph*. Note that the Cayley graph Cay $(\mathbb{Z}_{2n},\{1,3,\ldots 2n-1\})$ is isomorphic to the complete bipartite graph $K_{r,r}$. In [2], they proved that $K_{r,r}$admits strongly multiplicative labeling. However, proving the existence of strongly multiplicative labeling for a general circulant graph is still open Let D_{2n} be the *dihedral groups* with identity τ and it is generated by the two elements *r*,*s* with $r^n = s^2 = \tau$ and $rs = sr^{-1}$. From this defined relations, one can take $D_{2n} = \{r,r^2,r^3,\ldots,r^{n-1},r^n = \tau, s, sr, sr^2,\ldots,sr^{n-1}\}$and $G = Cay\ (D_{2n};\Omega)$, where Ω is a generating set of D_{2n} . In this paper,we prove that Cayley graph $G = Cay\ (D_{2n};\Omega)$, constructed on the dihedral group D_{2n} , for n ≤ 3 and a generating set $\Omega = \{r, r^{n-1}, s\}$ admits strongly multiplicative labelings.

II. CAYLEY GRAPH ON CYCLIC GROUP

Wherever In this section, we present the theorems for the existence of stronglymultiplicative labeling on Cayley graph constructed on cyclic group $\mathbb{Z}_n$ with a given specified generating set.

Theorem 1.
Let$n(\geq 6)$ be an integer, then the Cayley graphCay$(\mathbb{Z}_n,\{1,2,n-2,n-1\})$ admits strongly multiplicative labeling

Proof.

Let G =Cay $(\mathbb{Z}_{2n},\{1,2,n\text{-}2,n\text{-}1\})$ be the cayley graph with$n(\geq 6)$. Let the function$f$: V(G) $\longrightarrow$\{1,2,3,…,n\} defined by

$$f(v_i)=\begin{cases} 1 & for\ i = 0 \\ 2i & for\ i = 1,2,3,\ldots,\left\lfloor\frac{n}{2}\right\rfloor \\ 2(n-i)+1 & for\ i = \left\lfloor\frac{n}{2}\right\rfloor+1,\left\lfloor\frac{n}{2}\right\rfloor+2\ldots,n-1 \end{cases}$$

The Assignment of the labeling to the vertices by the above function f ,we obtain the distinct edge labels as follows :

$$f^*(v_i, v_{i+1}) = \begin{cases} 2(i+1) & for\ i = 0 \\ 4i(i+1) & for\ i = 1,2,3,\ldots,\left\lfloor\frac{n}{2}\right\rfloor-1 \\ 2i(2n-2i-1) & for\ i = \left\lfloor\frac{n}{2}\right\rfloor \\ (2n-2i+1)(2n-2i-1) & for\ i = \left\lfloor\frac{n}{2}\right\rfloor+1,\left\lfloor\frac{n}{2}\right\rfloor+2\ldots,n-1 \end{cases}$$

$$f^*(v_i, v_{i+2}) = \begin{cases} 2\ (i+1) & \text{for } i = 0 \\ 4i(i+2) & \text{for } i = 1,2,3,\ldots,\left\lfloor\frac{n}{2}\right\rfloor-2 \\ 2i(2n-2i-3) & \text{for } i = \left\lfloor\frac{n}{2}\right\rfloor-1,\left\lfloor\frac{n}{2}\right\rfloor \\ (2n-2i+1)(2n-2i-3) & \text{for } i = \left\lfloor\frac{n}{2}\right\rfloor+1,\left\lfloor\frac{n}{2}\right\rfloor+2\ldots,n-2 \\ 2(2n-2i+1)(n-i) & \text{for } i = n-1 \end{cases}$$

Clearly, by the function f^* the resulting edge labels edges are independent of i .Thus the edgelabels are distinct.

Theorem .2
Let $n\,(\geq 13)$ be an integer, then the Cayley graph $Cay(\mathbb{Z}_n,\{1,2,3,\ldots,n-3,n-2,n-1\})$ admits strongly multiplicative labeling.

Proof.
Let G =Cay $(\mathbb{Z}_n,\{1,2,3,\ldots,n-3,n-2,n-1\})$be the Cayley graph with$n\ (\geq 13)$. Let the function

$f\ :\ V(G) \longrightarrow \{1,2,3,\ldots,n\}$defined by

$f(v_i)$=

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$$\begin{cases} 1 & for \quad i=0 \\ 2i+1 & for\ i=1,2,3 \\ 2(i-3) & for\ i=4,5,\dots,\left\lfloor\frac{n}{2}\right\rfloor+3 \\ (2n-2i-7) & for\ i= \left\lfloor\frac{n}{2}\right\rfloor+4,\left\lfloor\frac{n}{2}\right\rfloor+5,\dots,n-1 \end{cases}$$

The Assignment of the labeling to the vertices by the above function f ,we obtain the distinct edge labels as follows,

$$f^*(v_i,v_{i+1}) = \begin{cases} 2i+3 & for\ i=0 \\ (2i+1)(2i+3) & for\ i=1,2 \\ 2(2i+1)(i-2) & for\ i=3 \\ 4(i-2)(i-3) & for\ i=4,5,\dots,\left\lfloor\frac{n}{2}\right\rfloor+2 \\ 2(i-3)(2n-2i+5) & for\ i=\left\lfloor\frac{n}{2}\right\rfloor+3 \\ (2n-2i+7)(2n-2i+5) & for\ i=\left\lfloor\frac{n}{2}\right\rfloor+4,\left\lfloor\frac{n}{2}\right\rfloor+5,\dots,n-2 \\ 2n-2i+7 & for i=n-1 \end{cases}$$

$$f^*(v_i,v_{i+2}) = \begin{cases} 2i+5 & for\ i=0 \\ (2i+1)(2i+5) & for\ i=1 \\ 2(2i+1)(i-1) & for\ i=2,3 \\ 4(i-3)(i-1) & for\ i=4,5,\dots,\left\lfloor\frac{n}{2}\right\rfloor+1 \\ 2(i-3)(2n-2i+3) & for\ i=\left\lfloor\frac{n}{2}\right\rfloor+2,\left\lfloor\frac{n}{2}\right\rfloor+3 \\ (2n-2i+7)(2n-2i+3) & for\ i=\left\lfloor\frac{n}{2}\right\rfloor+4,\left\lfloor\frac{n}{2}\right\rfloor+5,\dots,n-3 \\ 2n-2i+7 & for\ i=n-2 \\ (2n-2i+7)(2n-2i+3) & for\ i=n-1 \end{cases}$$

$$f^*(v_i,v_{i+3}) =$$

$$\begin{cases} 2i+7 & for\ i=n \\ (2i+1)(2i) & for\ i=1,2,3 \\ 4(i-3)i & for\ i=4,5,\dots,\left\lfloor\frac{n}{2}\right\rfloor \\ 2(i-3)(2n-2i+1) & for\ i=\left\lfloor\frac{n}{2}\right\rfloor+1,\left\lfloor\frac{n}{2}\right\rfloor+2,\left\lfloor\frac{n}{2}\right\rfloor+3 \\ (2n-2i+7)(2n-2i+1) & for\ i=\left\lfloor\frac{n}{2}\right\rfloor+4,\left\lfloor\frac{n}{2}\right\rfloor+5,\dots,n-4 \\ 2n-2i+7 & for\ i=n-3 \\ (2n-2i+7)(2n-2i-7) & for\ i=n-2,n-1 \end{cases}$$

Clearly, by the function f^*,the resulting edge labels edges are independent of i, thus edge labels are distinct.

III. CAYLEY GRAPH ON DIHEDRAL GROUPS

In this section,we present the theorems for the existence of strongly multiplicative labeling on Cayley graph constructed on dihedral group D_{2n} with a given specified generating set.

Theorem .3

Let*n*(≥ 5) be an integer, then the Cayley graph Cay(D_{2n},$\{r,r^{n-1},s\}$) admits strongly multiplicative labeling.

Proof:

Case(i): if n is odd

Let G=Cay(D_{2n},$\{r,r^{n-1},s\}$) be the Cayley graph with odd integern(≥ 5), let V(G)=D_{2n} .Let the function $f : V(G) \longrightarrow \{1,2,3,\dots,2n\}$defined by

$$f\left(r^{i (mod\ n)}\right) = \begin{cases} i+1 & for\ i=0,2,4,\dots,n-1 \\ 2i+n & for\ i=1,3,5,\dots,n-2 \end{cases}$$

$$f\left(s\,r^{(n-i)(mod\ n)}\right) = \{2i+2 \quad for\ i=0,1,2,\dots,n-1$$

The assignment of the labeling to the vertices of G by the above function f ,we obtain the distinct edge labels defined as $f^*(u,v)=f(u)\,f(v)$ we have the following f^* for every $uv \in E(G)$,

$$f^*\left(r^{i(mod\ n)}r^{(i+1)(mod\ n)}\right) = \begin{cases} (i+1)(2i+n+2) & for\ i=0,2,4,\dots,n-1 \\ (2i+n)(i+2) & for\ i=1,3,5,\dots,n-2 \end{cases}$$

$$f^*\left(s\,r^{(n-i)(mod\ n)}\ s\,r^{(n-i+1)(mod\ n)}\right) = \begin{cases} 2(2i+2) & for\ i=n-1 \\ (2i+2)(2i+4) & for\ i=0,1,2,\dots n-2 \end{cases}$$

$$f^*\left(r^{i(mod\ n)}\ sr^{(n-i)(mod\ n)}\right) = \begin{cases} (i+1)(2i+2) & for\ i=0,2,4,\dots,n-1 \\ (2i+n)(2i+2) & for\ i=1,3,5,\dots,n-2 \end{cases}$$

Clearly, by the function $f^*$the edge labels of the edges of G are independent of i.Thus the edge labels are distinct.

Case(ii): if n is even

Let G=Cay(D_{2n},$\{r,r^{n-1},s\}$) be the Cayley graph with even integern(≥ 6), let V(G)=D_{2n}. Let the function $f : V(G) \longrightarrow \{1,2,3,\dots,2n\}$defined by

$$f\left(r^{i(mod\ n)}\right) = \begin{cases} i+1 & for\ i=0,2,4,\dots,\frac{n}{2}-1 \\ \frac{3n-2i}{2} & for\ i=\frac{n}{2},\frac{n}{2}+1,\dots,n-1 \end{cases}$$

$$f\left(s\,r^{(n-i)(mod\ n)}\right) = \{n+i+2 \quad for\ i=0,1,2,\dots,n-1$$

The assignment of the labeling to the vertices of G by the above function f ,we obtain the distinct edge labels defined as $f^*(u,v)=f(u)\,f(v)$ we have the following f^* for every $uv \in E(G)$,

$$f^*\left(r^{i(mod\ n)}r^{(i+1)(mod\ n)}\right) = \begin{cases} (i+1)(i+2)\ for\ i=0,1,2,\dots,\frac{n}{2}-2 \\ \frac{(i+1)(3n-2i-2)}{2} & for\ i=\frac{n}{2}-1 \\ \frac{(3n-2i)(3n-2i-2)}{4} & for\ i=\frac{n}{2},\frac{n}{2}+1,\dots,n-2 \\ \frac{3n-2i}{2} & for\ i=n-1 \end{cases}$$

$$f^*\left(s\,r^{(n-i)(mod\ n)}\ s\,r^{(n-i+1)(mod\ n)}\right) = \begin{cases} (n+i+1)(i+2) & for\ i=n-1 \\ (n+i+1)(n+i+2) & for\ i=0,1,2,\dots n-2 \end{cases}$$

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$$f^*\left(r^{i(mod\,n)}\, sr^{(n-i)(mod\,n)}\right) = \begin{cases} (i+1)(n+i+1) & for\ i = 0,1,2,\dots,\frac{n}{2}-1 \\ \frac{(3n-2i)(n+i+1)}{2} & for\ i = \frac{n}{2},\frac{n}{2}+1,\dots,n-1 \end{cases}$$

Clearly, by the function $f^*$the edge labels of the edges of G are independent of i.Thus the edge labels are distinct.

Theorem .4

Let$n(\geq 5)$ be an integer, if n is even then the Cayley graph Cay(D_{2n},$\{ r, r^{n-1}, s, sr\}$) admits strongly multiplicative labeling.

Proof:

Case(i): if n is odd

Let G=Cay(D_{2n},$\{ r, r^{n-1}, s, sr\}$) be the Cayley graph with odd integer$n(\geq 5)$, let V(G)=D_{2n}. Let the function $f : V(G) \longrightarrow \{1,2,3,\dots,2n\}$defined by

$$f\left(r^{i(mod\,n)}\right) = \begin{cases} i+1 & for\ i = 0,2,4,\dots,n-1 \\ 2i+n & for\ i = 1,3,5,\dots,n-2 \end{cases}$$

$$f\left(s\, r^{(n-i)(mod\,n)}\right) = \{2i+2 \quad for\ i = 0,1,2,\dots,n-1$$

The assignment of the labeling to the vertices of G by the above function f ,we obtain the distinct edge labels defined as $f^*(u,v) = f\,(u)\,f(v)$ we have the following f^* for every $uv \in E(G)$,

$$f^*\left(r^{i(mod\,n)} r^{(i+1)(mod\,n)}\right) = \begin{cases} (i+1)(2i+n+2) & for\ i = 0,2,4,\dots,n-1 \\ (2i+n)(i+2) & for\ i = 1,3,5,\dots,n-2 \end{cases}$$

$$f^*\left(s\, r^{(n-i)(mod\,n)}\, s\, r^{(n-i+1)(mod\,n)}\right) = \begin{cases} 2(2i+2) & for\ i = n-1 \\ (2i+2)(2i+4) & for\ i = 0,1,2,\dots n-2 \end{cases}$$

$$f^*\left(r^{i(mod\,n)}\, sr^{(n-i)(mod\,n)}\right) = \begin{cases} (i+1)(2i+2) & for\ i = 0,2,4,\dots,n-1 \\ (2i+n)(2i+2) & for\ i = 1,3,5,\dots,n-2 \end{cases}$$

$$f^*\left(s\, r^{(n-i)(mod\,n)} r^{(i+1)(mod\,n)}\right) = \begin{cases} (2i+2)(2i+n+2) & for\ i = 0,2,4,\dots,n-3 \\ (2i+2)(i+2) & for\ i = 1,3,5,\dots,n-2 \\ (2i+2) & for\ i = n-1 \end{cases}$$

Clearly, by the function $f^*$the edge labels of the edges of G are independent of i.Thus the edge labels are distinct.

Case(ii): if n is even

Let G=Cay(D_{2n},$\{ r, r^{n-1}, s, sr\}$) be the Cayley graph with even integer$n(\geq 6)$, let V(G)=D_{2n}. Let the function $f : V(G) \longrightarrow \{1,2,3,\dots,2n\}$defined by

$$f\left(r^{i(mod\,n)}\right) = \begin{cases} i+1 & for\ i = 0,2,4,\dots,\frac{n}{2}-1 \\ \frac{3n-2i}{2} & for\ i = \frac{n}{2},\frac{n}{2}+1,\dots,n-1 \end{cases}$$

$$f\left(s\, r^{(n-i)(mod\,n)}\right) = \{n+i+2 \quad for\ i = 0,1,2,\dots,n-1$$

The assignment of the labeling to the vertices of G by the above function f ,we obtain the distinct edge labels defined as $f^*(u,v) = f\,(u)\,f(v)$ we have the following f^* for every $uv \in E(G)$,

$$f^*\left(r^{i(mod\,n)} r^{(i+1)(mod\,n)}\right) = \begin{cases} (i+1)(i+2) & for\ i = 0,1,2,\dots,\frac{n}{2}-2 \\ \frac{(i+1)(3n-2i-2)}{2} & for\ i = \frac{n}{2}-1 \\ \frac{(3n-2i)(3n-2i-2)}{4} & for\ i = \frac{n}{2},\frac{n}{2}+1,\dots,n-2 \\ \frac{3n-2i}{2} & for\ i = n-1 \end{cases}$$

$$f^*\left(s\, r^{(n-i)(mod\,n)}\, s\, r^{(n-i+1)(mod\,n)}\right) = \begin{cases} (n+i+1)(i+2) & for\ i = n-1 \\ (n+i+1)(n+i+2) & for\ i = 0,1,2,\dots n-2 \end{cases}$$

$$f^*\left(r^{i(mod\,n)}\, sr^{(n-i)(mod\,n)}\right) = \begin{cases} (i+1)(n+i+1) & for\ i = 0,1,2,\dots,\frac{n}{2}-1 \\ \frac{(3n-2i)(n+i+1)}{2} & for\ i = \frac{n}{2},\frac{n}{2}+1,\dots,n-1 \end{cases}$$

$$f^*\left(s\, r^{(n-i)(mod\,n)} r^{(i+1)(mod\,n)}\right) = \begin{cases} (n+i+1)(i+2) & for\ i = 0,1,2,\dots,\frac{n}{2}-2 \\ \frac{(n+i+1)(3n-2i-2)}{2} & for\ i = \frac{n}{2}-1,\frac{n}{2},\dots,n-2 \\ (n+i+1) & for\ i = n-1 \end{cases}$$

Clearly, by the function $f^*$the edge labels of the edges of G are independent of i.Thus the edge labels are distinct.

IV. Concluding Remarks

In this paper, we proved that the Cayley graphs with particular generating sets admit strongly multiplicative labeling. However, there remain proving the existence of strongly multiplicative labeling for the Cayley graphs on arbitrary group with given generating sets are still open.

V. References

[1] J. A. Gallian, “A dynamic survey of graph labeling,” Electr. J. Combin. 18 , 2016 (DS6).

[2] L. W. Beineke and S. M. Hedge, “strongly multiplicative graphs,” *Discuss.Math. Graph Theory*, 21(2001), pp. 63-75.

[3] D. W. West, it Introduction to graph theory (Prentice-Hall Inc., 2000)

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