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A STUDY ON 3-RAINBOW DOMINATION NUMBER OF SOME SPECIAL CLASSES OF GRAPHS

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ABSTRACT: For a given simple, finite, connected and undirected graph G= (V,E) and a set of k-colours numbered 1,2,3,…k, the 3-rainbow domination is defined as a mapping f : V(G)→þ{1,2,3} such that for all v∈V(G) with f(v)=φ$\cup_{u\in N(v)} f(u) = \{1,2,3\}$Such function is called a 3-rainbow domination function (3RDF) and the minimum weight of such function is called the 3-rainbow domination number of G and is denoted by γ_{r3}(G). In this paper, we obtained the 3-rainbow domination number of some special graphs. Here [x]-denote the integral part of x, $\lceil x \rceil$ denote the upper integral part of x and $\lfloor x \rfloor$ denote the lower integral part of x.

Keywords:Domination, Domination number, k-rainbow domination number, 3-rainbow domination, number, wheel graph, Triangular snake graph, Double triangular snake graph ,n-Barbell graph, n-Sunlet graph, n-Centipede graph, Crown graph, Clebsch graph, Interconnection network, Silicate network.

1. INTRODUCTION

In graph theory, varities of domination problems are solved by using k- rainbow domination. The k- rainbow domination was introduced by Bresar, Henning &Rall[2] at first. A subset D of the vertex set V(G) of a graph G is said to be dominating set if every vertex in (V-D) is adjacent to a vertex in D. The Minimum Cardinality of a dominating set D is said to the domination numbers and is denoted by γ(G). The open neighborhood of v is the set N(v)={u∈V(G)/uv∈E(G)} and the closed neighborhood of v is the set N[v]={v}∪ N(v).

Let f:V(G) → $\mathcal{P}$\{1,2,…k\} be a function that assigns to each vertex of G a set of colours chosen from the power set {1,2,…k}. If v∈V(G) and f(v) = ∅ then $\cup_{u\in N(v)} f(u)$ ={1,2,…k}. Therefore the function f is called k-rainbow dominating function (k-RDF) of G. The Weight of the function is defined by $W(f) = \sum_{v\in V(G)} |f(v)|$The minimum weight of a k-RDF is called the k-rainbow domination number of G and it is denoted by γ_{rk}(G). When k=3 we define a mapping f: V(G)→$\mathcal{P}$ {1,2,3} such that for each vertex v∈ $V(G)$ with $f(v) = \emptyset$ we have

$\cup_{u\in N(v)} f(u) = \{1,2,3\}$Such function f is said to be a 3-rainbow dominating function (3RDF) and minimum weight of such function is said to be 3- rainbow domination number of G and it is denoted byγ_{r3}(G).

In this paper, we determined the domination number and 3-rainbow domination number of Wheel graph, Triangular snake graph, Double Triangular snake graph ,n-Barbell graph, n-Sunlet graph, n-Centipede graph, Crown graph, Clebsch graph and Silicate network.

2.PRELIMINARIES:

Definition 2.1A subset D of vertex set V(G) of a graph G is said to be dominating set if every vertex in (V-D) is adjacent to a vertex in D. The minimum cardinality of dominating set is said to be domination number and is denoted by γ(G).

Definition 2.2 Let f: v(G)→□{1,2,3……k} be a function that assigns to each vertex of G a set of colours chosen from the power set of {1,2,3……k}, if v∈V(G)withf(v)=$\varphi$and$\cup_{u\in N(v)} f(u) = \{1,2,..k\}$Then the function f is called k-rainbow dominating function of G and is denoted by k-RDF.

Definition 2.3 The weight of the function is defined as w(f)=$\sum_{v\in V(G)} |f(v)|$. The minimum weight of a k-RDF is called the k-rainbow domination number of G and it is denoted by γ_{rk}(G).

Definition 2.4 When k=3, we define a mapping f:V(G)→□{1,2,3} such that for each vertex v∈V(G) with f(v)= φ we have $\cup_{u\in N(v)} f(u)$={1,2,3} such function f is said to be a 3-rainbow dominating function(3-RDF) and the

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minimum weight of such function is called a 3-rainbow domination number of G and it is denoted by γ_{r3}(G).

Definition 2.5 The Triangular cactus is a connected graph all of whose blocks are triangles. It is obtained from a path P=$v_1,v_2,v_3,\ldots\ldots v_{n+1}$ by joining v_i and v_{i+1} to a new vertex $u_i \forall i = 1,2,3,\ldots,n$. A triangular snake has 2n+1 vertices and 3n edges, where n is the number of blocks in the triangular snake. It is denoted by T_n.

Definition 2.6 The Double triangular snake D[T_n] consists of two triangular snake that have a common path. It is obtained from a path $v_1,v_2,\ldots\ldots v_n$ by joining v_i and v_{i+1} to the new vertices w_i and u_i for $1\leq i \leq$n.

Definition 2.7 The n-Sunlet graph S_n is a graph with cycle C_n and each vertex of the cycle attached to one pendent vertex. Each n-Sunlet graph consists 2n vertices and 2n edges.

Definition 2.8 The n-centipede graph is a tree on 2n vertices obtained by joining the bottoms of n-copies of the path graph P_2 laid in a row with an edge C_n.

Definition 2.9 The n-barbell graph is the simple graph obtained by connecting two copies of a complete graph K_n by a bridge and it is denoted by B(k_n,k_n). `

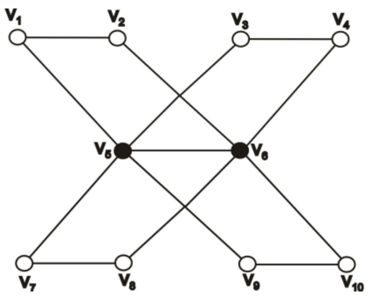
Definition 2.10 The Crown graph S_n^0 for an integer n> 2 is the graph with the vertex set {$u_1,u_2,\ldots.u_n,v_1,v_2,\ldots\ldots v_n$} and edge set {($u_i$,$v_i$):$1\leq i,j \leq n$,i≠j}

Definition 2.11 The Clebsch graph is a strongly regular quintic graph on 16 vertices and 40 edges .It is also known as the Greenwood Gleason graph.

3. ON 3-RAINBOW DOMINATION NUMBER OF CERTAIN GRAPHS

In this paper, we determined the 3-rainbow domination number of Wheel graph, Triangular snake graph, Double triangular snake graph, n-Sunlet graph, n-Barbell graph, n-Centipede graph, Crown graph, Clebsch graph.

Example.3.1



In the above figure,the 3-rainbow domination number is 6.

Theorem :3.1

Let W_n be the wheel graph then γ_{r3} (W_n) =3, ∀ n ≥ 3.

Proof: The wheel graph is obtained by joining cycle graph C_n and complete graph K_1. We prove this theorem by using induction method.

When n=3, the graph W_3 contains 4 vertices and 6 edges. Here all the vertices in the cycle are connected to the hub to form W_3. Let D be the dominating set of W_3 with $|D| = \gamma_{r1}$(W_3)=1and define a function f: V(W_3)→□{1,2,3} such that we assign colour class{1,2,3} to the vertex in the hub and assign empty colour to the remaining vertices in the wheel graph. The minimum sum of numbers of assigned colours overall vertices of W_3 is 3.The 3-rainbow domination number of W_3 is 3.(i.e) γ_{r3} (W_3) = 3.

When n=4 the graph W_4 contains 5 vertices and 8 edges. Here all the vertices in the cycle C_4 is connected to the hub to form W_4. Let D be the dominating set of W_4 with $|D| = \gamma_{r1}$(W_4)=1and define a function f: V(W_4)→□{1,2,3} such that we assign colour class{1,2,3} to the vertex in the hub and assign empty colour to the remaining vertices in the wheel graph. The minimum sum of numbers of assigned colours overall vertices of W_4 is 3.

The 3-rainbow domination number of W_4 is 3.(i.e.) γ_{r3} (W_4) = 3.

By proceeding in this manner we get the general term for n. The graph $W_n$contains n+1 vertices and 2n edges here all the vertices in the cycle C_n is connected to the hub to form W_n. Let D be the dominating set of $W_n$with $|D| = \gamma_{r1}$(W_n)=1anddefine a function f: V(W_n)→□{1,2,3} such that we assign colour class{1,2,3} to the vertex in the hub and assign empty colour to the remaining vertices in the wheel graph. The minimum sum of numbers of assigned colours overall vertices of $W_n$is 3.

The 3-rainbow domination number of W_n is 3.(i.e.) γ_{r3} (W_n) = 3.$\forall n \geq 3$.

Example: 3.2

Consider the wheel graph W_5 with |V|=6 & |E| = 10.

Here the only dominating set is D ={v_6}, So $\gamma(W_5)$=1we assigned the colour set {1,2,3} to the dominating set { v_6}, for each vertex v in (V-D) we have $f\{v\}$=φ and $\bigcup_{u \in N(V)} f(u)$={1,2,3}

Therefore γ_{r3}(W_5)=3

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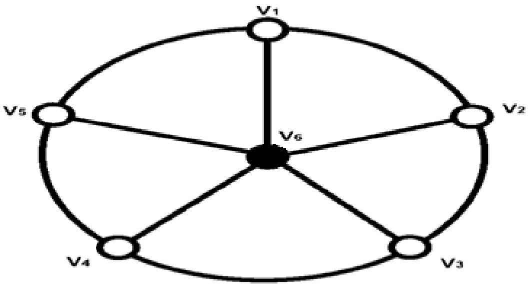

Figure:1 (W_5)

Hence the 3-rainbow domination number of wheel graph of W_5 is 3. i.e.$\gamma_{r3}(W_5) \leq 3$.

Theorem:3.2 Let $T_n$be the Triangular snake graph then

$$\gamma_{r3}(T_n)=\begin{cases} 3\left\lceil\frac{n}{2}\right\rceil ifnisodd \\ 3\left(\frac{n}{2}\right) \text{ if niseven} \end{cases}$$

Proof: Let$T_n$be the Triangular snake graph contains n +1 vertices and 3n edges. It is obtained from a path $P=v_1,v_2,v_3,\ldots\ldots\ldots v_{n+1}$ by joining v_i and v_{i+1} to a new vertex u_i, $\forall\, i = 1$ to n.

Case: 1 when n is odd

Subcase:1.1 when n=1

we have Triangular snake graph T_1 which contains 3 vertices and 3 edges. Let D be the dominating set of $T_1$with $|D| = \gamma_{r1}(T_1)$=1and define a function f:V(T_1)→□{1,2,3}.such that we assign colour class{1,2,3} to the vertex v_1 in D and assign empty colour to the remaining vertices v_2&v_3.The minimum sum of number of assigned colours overall vertices is 1. Clearly f is a 3-rainbow dominating function and $\gamma_{r3}(T_1)$=3.

Subcase: 1.2 when n =3

The Triangular snake graph T_3 contains 7 vertices and 9 edges. Let D be the dominating set of T_3 with $|D| = \gamma_{r1}(T_3)$=2and define a function f:V(T_3)→□{1,2,3} such that we assign colour class{1,2,3} to the vertices in the set D and assign empty colour to the remaining vertices . The minimum sum of numbers of assigned colours overall vertices of T_3 is 2. Clearly f is a 3-rainbow dominating function and $\gamma_{r3}(T_3)$=6.

Repeating in this manner for order n, we get $\gamma_{r3}(T_n)$= $3\left\lceil\frac{n}{2}\right\rceil$ w□en n is odd.

Case: 2 when n is even

Subcase: 2.1 when n=2

The Triangular snake graph T_2 contains 5 vertices and 6 edges. Let D be the dominating set of T_2 with $|D| = \gamma_{r1}(T_2)$=1and define a function f:V(T_2)→□{1,2,3}. Such that we assign colour class{1,2,3} to the vertices in the set S and assign empty colour to the remaining vertices .The minimum sum of number of assigned colours overall vertices is 1. Clearly f is a 3-rainbow domination function and $\gamma_{r3}(T_2)$=3.

Subcase: 2.2 when n =4

The Triangular snake graph T_4 contains 9 vertices and 12 edges. Let D be the dominating set of T_4 with $|D| = \gamma_{r1}(T_4)$=2and define a function f :V(T_4)→□{1,2,3} such that we assign colourclass{1,2,3} to the vertices in the set D and assign empty colour to the remaining vertices .The minimum sum of numbers of assigned colours overall vertices of T_4 is 2 .Clearly f is a 3-rainbow domination function and $\gamma_{r3}(T_4)$=6.

Repeating in this manner for order n, we get $\gamma_{r3}(T_n)=3\left(\frac{n}{2}\right)\forall$ n is even.

This function is a 3-rainbow dominating function of $T_n$and we have

$$\gamma_{r3}(T_n) =\begin{cases} 3\left\lceil\frac{n}{2}\right\rceil ifnisodd \\ 3\left(\frac{n}{2}\right) ifniseven \end{cases}$$

Example: 3.3

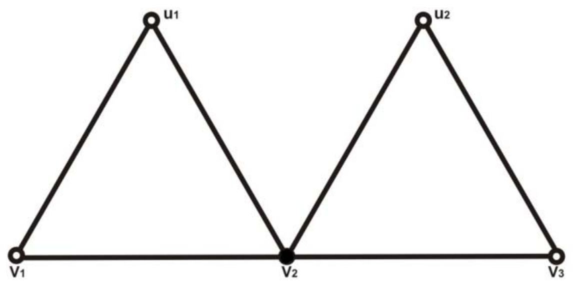

Figure:2-T_2

The 3-rainbow domination of triangular snake graph of T_2 is 3.(i.e.)$\gamma_{r3}(T_2)$=3. We assigned colour set {1,2,3} to the vertex $\{v_2\}$and remaining vertices are assigned empty colour.

Theorem: 3.3 Let D(T_n) be the Double triangular snake graph then $\gamma_{r3}(D(T_n))=\begin{cases} 3\left\lceil\frac{n}{2}\right\rceil \text{ if nisodd} \\ 3\left(\frac{n}{2}\right) \text{ if niseven} \end{cases}$

Proof: Let D(T_n) be the Double Triangular snake graph with 3n+1 vertices and 5n edges. Let$\{v_1,v_2,\ldots\ldots v_{n+1},u_1\ldots\ldots u_n,w_1,w_{2,\ldots\ldots}wn\}$be the vertex set of the Double Triangular snake graph D(T_n).

Case:1 when n is odd

Subcase:1.1when n=1

The Double Triangular snake graph D(T_1)it contains 4 vertices and 5 edges. Let S be the dominating set of D(T_1)with $|D| = \gamma_{r1}$ D(T_1)=1and define a function f:V(D(T_1))→□{1,2,3}.such that we assign colour class{1,2,3} to the vertex v_1 in the set S and assign empty colour to the remaining vertices v_2,v_3,v_4. The minimum sum

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of number of assigned colours overall vertices is 3. Clearly f is a 3-rainbow domination function and $\gamma_{r3}(D(T_1))$ =3.

Subcase:1.2 when n =3

The Double Triangular snake graph $D(T_3)$ contains 7 vertices and 9 edges. Let D be the dominating set of T_3 with $|D| = \gamma_{r1}$ $(D(T_3))$=2and define a function f:V(D(T_3))→□{1,2,3} such that we assign colour class{1,2,3} to the vertices in the set D and assign empty colour to the remaining vertices . The minimum sum of numbers of assigned colours overall vertices of $D(T_3)$ is 6. Clearly f is a 3-rainbow domination function and we have $\gamma_{r3}(D(T_3))$=6.

Repeating this process for n times we get $\gamma_{r3}(D(T_n))=3\left\lceil\frac{n}{2}\right\rceil$ *if* n is odd.

Case:2 when n is even

Subcase:2.1 when n=2

The Double triangular snake graph T_2 contains 7 vertices and 10 edges. Let D be the dominating set of $D(T_2)$with $|D| = \gamma_{r1}D(T_2)$)=1and define a function f:V(D(T_2))→□{1,2,3} such that we assign colour class{1,2,3} to the vertices in the set D and assign empty colour to the remaining vertices .The minimum sum of number of assigned colours overall vertices is 3. Clearly f is a 3-rainbow domination function and $\gamma_{r3}(D(T_2))$=3.

Subcase:2.2 when n =4

The double triangular snake graph $D(T_4)$contains 14 vertices and 20 edges. Let S be the dominating set of T_4 with $|S| = \gamma_{r1}(D(T_4))$=2and define a function f:V(D(T_4))→□{1,2,3} such that we assign colour class{1,2,3} to the vertices in the set S and assign empty colour to the remaining vertices .The minimum sum of numbers of assigned colours overall vertices of T_4 is 6. Clearly f is a 3-rainbow domination function and $\gamma_{r3}(T_4)$=6.

Repeating in this manner we get $\gamma_{r3}(T_n)= 3\left(\frac{n}{2}\right)$ *if* n is even.

This function is a 3-rainbow dominating function of $D(T_n)$ and we have

$$\gamma_{r3}(D(T_n)) = \begin{cases} 3\left\lceil\frac{n}{2}\right\rceil \; if\ nisodd \\ 3\left(\frac{n}{2}\right),\ if\ niseven \end{cases}$$

Example: 3.4

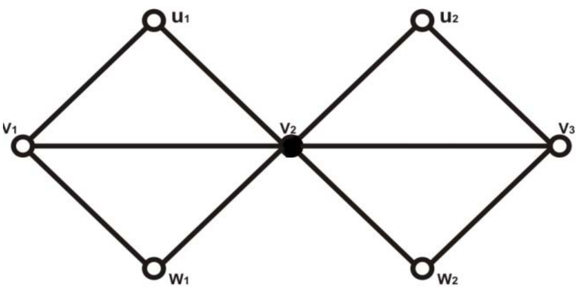

Figure:3-D(T2)

The 3-rainbow domination number of double triangular snake graph T_2 is 3. (i.e.)$\gamma_{r3}(D(T_2))$=3.we assigned colour set {1,2,3} to the vertex $\{v_2\}$and remaining vertices are assigned empty colour.

Theorem: 3.4 Let S_n be the n-sunlet graph then $\gamma_{r3}(S_n) \leq 3n$, $\forall$ n ≥ 3.

Proof: Let S_n be the n-sunlet graph on 2n vertices which is obtained by attaching n-pendant edges to the cycle C_n. let V={v_1,v_2,v_3……………..v_n} be the set of pendant vertices and dense W={w_1,w_2,……..w_n}be the set of vertices in the cycle C_n.TheSunlet graph S_3 has 6 vertices.

When n =3, let D be the dominating set of S_3 with $|D| = \gamma_{r1}(S_3)$=3and define f:V(S_3)→□{1,2,3} such that we assign colour class{1,2,3}to the vertices in the set D and assign the empty colour to the remaining vertices. The minimum sum of numbers of assigned colours overall vertices of S_3 is 9.This function is a 3-rainbow dominating function of $S_3$and we have $\gamma_{r3}(S_3) \leq 9$.

In general, let D be a dominating set of S_n with $|D| = \gamma_{r1}(S_n)$=nand define f:V(S_n)→□{1,2,3} such that we assign colour class{1,2,3} to the vertices in the set D and assign the empty colour to the remaining vertices. The minimum sum of numbers of assigned colours overall vertices of S_n is 3n. The 3-rainbow dominating function of $S_n$is $\gamma_{r3}(S_n) \leq 3n$. $\forall n \geq 3$.

Example: 3.5

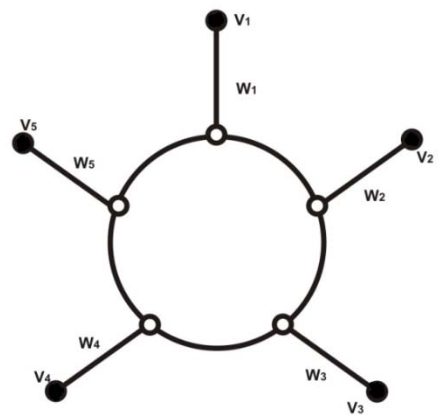

Figure: 4-S_5

The 3-rainbow domination number of n-sunlet graph is 5 (i.e.) $\gamma_{r3}(S_5)$=5.we assigned colour set {1,2,3} to the pendant vertices and remaining vertices are assigned with empty colour

Theorem :3.5

Let G be the n-centipede graph then$\gamma_{r3}(G) \leq 3n$.$\forall n \geq 3$

Proof: The n-centipede graph is a tree on 2n vertices which can be obtained by joining the bottoms of n-copies of the

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path graph p_2 laid in a row with edge. Let the end vertices be the defined by V={v_1,v_2,v_3……………..v_n} and the supporting vertices be W={w_1,w_2,……..w_n}.

When n=3, The n-centipede graph contains 6 vertices and 3 edges .Let D be a dominating set of G with $|D| = \gamma(G)$=3and define f:V(G)→□{1,2,3} such that we assign colour class{1,2,3}to the vertices in the set D and assign the empty colour to the remaining vertices. The minimum sum of numbers of assigned colours overall vertices of G is 9 . This function is a 3-rainbow dominating function of G and we have γ_{r3}(G) ≤ 9.

When G is n-centipede graph having 2n vertices & n edges, then D is a dominating set of G with $|D| = \gamma(G)$=nand define f:V(G)→□{1,2,3} such that we assign colour class{1,2,3}to the vertices in the set D and assign the empty colour to the remaining vertices. The minimum sum of numbers of assigned colours overall vertices of G is 3n. The 3-rainbow dominating function of G is γ_{r3}(G) ≤ 3n. $\forall n \geq 3$.

Example: 3.6

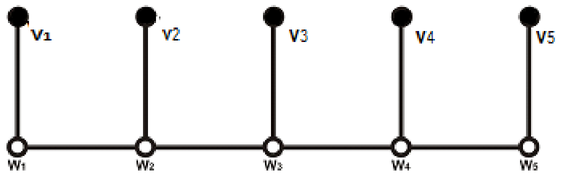

Figure:5-G

The 3-rainbow domination of n-centipede graph is 5 i.e.γ_{r3}(G)=5.we assigned colour set {1,2,3} to the pendant vertices and assigned empty colour to the remaining vertices.

Theorem 3.6

Let B(k_n,k_n) be the n-barbell graph then γ_{r3}(B(k_n,k_n))=6,∀ n≥3.

Proof: Let the B(k_n,k_n) be the n-barbell graph which is obtained by connecting two copies of a complete graph K_n by a bridge. Let V={v_1,v_2,v_3……………..v_n} be the vertex set of copy A and W={w_1,w_2,……..w_n} be the vertex set copy B.

When n=3, The n-barbell graph B(k_3,k_3) is obtained by connecting two copies of a complete graph K_3 by a bridge let D be a dominating set of B(k_3,k_3) with $|D| = \gamma_{r1}$(B(k_3,k_3))=2and define a function f:V(B(k_3,k_3))→□{1,2,3}Such that we assign colour class{1,2,3} to the vertices in the set D and assign the empty colour to the remaining vertices. The minimum sum of numbers of assigned colours overall vertices of B(k_3,k_3) is 6.This function is a 3-rainbow dominating function of B(k_3,k_3) and we have γ_{r3} (B(k_3,k_3))=6.

When n=4 let D be a dominating set of B(k_4,k_4) with $|D| = \gamma_{r1}$(B(k_4,k_4)) =2and define a function f:V(B(k_4,k_4))→□{1,2,3} Such that we assign colour class{1,2,3}to the vertices in the set D and assign the empty colour to the remaining vertices. The minimum sum of numbers of assigned colours overall vertices of B(k_4,k_4) is 6.This function is a 3-rainbow dominating function of B(k_4,k_4) and we have γ_{r3} (B(k_4,k_4))=6.

By proceeding in this manner for order n,

Let D be a dominating set of B(k_n,k_n) with $|D| = \gamma_{r1}$ B(k_n,k_n)) =2and define a function f:V(B(k_n,k_n))→□{1,2,3}Such that we assign colour class{1,2,3}to the vertices in the set D and assign the empty colour to the remaining vertices. The minimum sum of numbers of assigned colours overall vertices of B(k_n,k_n) is 6.This function is a 3-rainbow dominating function of B(k_n,k_n) and we have γ_{r3} (B(k_n,k_n))=6.$\forall n \geq 3$

Example: 3.7

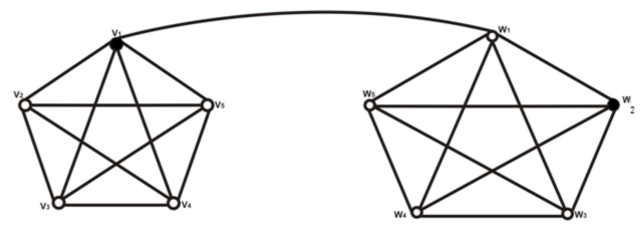

Figure:6-B(k_5,k_5)

The 3-rainbow domination of barbell graph is i.e. γ_{r3}(B(k_5,k_5))=6,we assign colour set {1,2,3} to vertex set $\{v_1, w_2\}$and assign empty colour to the remaining vertices.

Theorem: 3.7 Let S_n^0 be the crown graph then $\gamma_{r3}(S_n^0)$ =6 ∀ n ≥3.

Proof: Consider the crown graph S_n^0 with the vertex set {u_1,u_2,….u_n,v_1,v_2,……v_n} and edge set {(u_i,v_j):1≤ $i, j \leq n$,i≠j},to prove this theorem we use induction method.When n=3 the crown graph contains 6 vertices and 9 edges. Let D be a dominating set of S_3^0 with $|D| = \gamma_{r1}(S_3^0)$ =2and define f:V(S_3^0)→□{1,2,3} such that we assigned colour class{1,2,3}to the vertices in the set D and assigned the empty colour to the remaining vertices. The minimum sum of numbers of assigned colours overall vertices of S_3^0 is 6 . This function is a 3-rainbow dominating function of $S_3^0$and we have $\gamma_{r3}(S_3^0)$ =6.When n=4 the graph contains 8 vertices and 12 edges. Let D be a dominating set of S_4^0 with $|D| = \gamma_{r1}(S_4^0)$ =2and define f:V(S_4^0)→□{1,2,3} such that we assign colour class{1,2,3}to the vertices in the set D and assigned the empty colour to the remaining vertices. The minimum sum of numbers of assigned colours overall

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vertices of S_4^0 is 6 . This function is a 3-rainbow dominating function of $S_4^0$and we have $\gamma_{r3}(S_4^0)$ =6.

By proceeding this way for order n ,the crown graph contains 2n vertices and 3n edges. Let D be a dominating set of S_n^0 with $|D| = \gamma_{r1}(S_n^0)$ =2and define f:V(S_n^0)→□{1,2,3} such that we assigned colour class{1,2,3}to the vertices in the set D and assigned the empty colour to the remaining vertices. The minimum sum of numbers of assigned colours overall vertices of S_n^0 is 6. This function is a 3-rainbow dominating function of $S_n^0$and we have $\gamma_{r3}(S_n^0)$ =6. Hence proved.

Example: 3.8

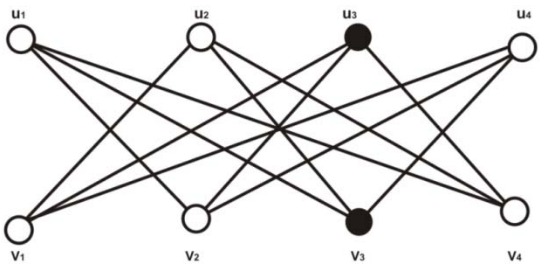


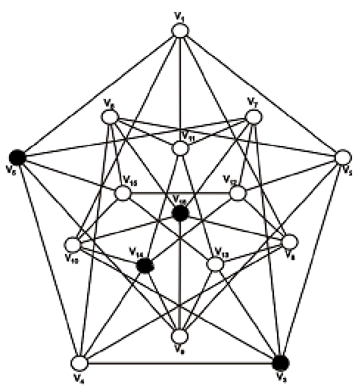
Figure:7- S_4^0

The 3-rainbow domination number of crown graph is 6 (i.e)$\gamma_{r3}(S_4^0)$ =6.And we assigned colour set {1,2,3} to the vertices{u_3, v_3}and remaining vertices are assigned empty colour.

Theorem: 3.8 Let G be the Clebsch graph then γ_{r3}(G) =12.

Proof: Let G be the Clebsch graph which is the strongly regular Quintic graph on 16 vertices and 40 edges. Let D be the dominating set of G with $|D|$=γ_{r1}(G)=4and define a function f:V(G)→□{1,2,3} such that we assigned colour class{1,2,3} to the vertices in the set D and assigned empty colour to the remaining vertices. The minimum sum of numbers of assigned colours overall vertices of G is 12. This function is a 3-rainbow dominating function of Gand we have γ_{r3}(G) =12.

Example: 3.9



Clebsch Graph

Figure:8

The 3-rainbow domination number of clebsch graph is 12. (i.e)γ_{r3}(G)=12.And we assign colour set {1,2,3} to the vertices v_3, v_5, v_{14}, v_{16} and remaining vertices are assign empty colour.

4. THE 3-RAINBOW DOMINATION NUMBER OF SILICATE NETWORK:

4.1SILICATE NETWORK: In this we determined the 3-rainbowdomination number of a silicate network. Interconnection network is used for changing data between two processors in a multistage network. It is placed between various devices in the multiprocessor network. It is central role in determining the overall performance in the system. Interconnection network like customary network system consisting of vertices and edges. Interconnection plays major role in multimedia, mass communication etc. There are many types of interconnection networks among these we have chosen the silicate network to determine the 3-rainbow domination number. Origin of silicate is from rock forming and synthetic minerals. The basic unit is S_iO_4. It is Tetrahedron shape, we consider the silicate sheet as a fixed interconnection parallel architecture and is said to be the silicate network. In chemistry S_iO_4 –tetrahedron represents oxygen ions in outer points and the center points represents the silicon ion. In graph theory outer vertices are represented as oxygen vertices and center vertices are represented as silicon vertices. This structures is used in various places mainly by determining X-ray diffraction. The ability to conduct electricity, produce a high frequency vibration and provide thermal insulation are some of the unique properties. Hence silicon is the perfect material to make microchips which runs every computers and cell phones and gaming devices. Silicate network (SL(n)) of dimension n has $15n^2$+3n vertices and 36 n^2 edges. The diameter of SL(n) is 4n. The 3- degree oxygen vertices of silicate network is said to be boundary vertices[4].

Example: 4.1

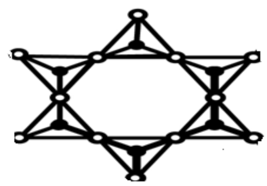


Figure:9 SL(1)

Preliminary result:

Preposition: 4.1 For n>0, the domination number of silicate network of dimension n is $3n^2$. (i.e) $\gamma(SL(n) = 3n^2$

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Proof: Let SL(n) be the silicate network with vertices $15n^2+3n$ and edges $36n^2$.Here D is the minimum dominating set of SL(n).where D contains $3n^2$ vertices which is adjacent to the remaining vertices in silicate network. Let us assume the contrary that D is not a minimum dominating set of SL(n) .let Đbe the minimum dominating set where Đ=V -D. Let u be any vertex in D (ie)u∈D**.**By the minimality condition of dominating set, we know that for all u∈S, N(u)∩S={v}.But N(u)∩S′=φ for all u∈S Therefore S′ is not minimum dominating set.And we conclude that D is the only minimum dominating set of SL(n) then the domination number is $3n^2$.(i.e)$\gamma(SL(n)) = 3n^2$

Preposition: 4.2[3].If G be a graph, then for any k≥2, min{|v(G)|,γ(G)+K-2}≤ γ_{rk} (G)≤ kγ(G) **.**The following theorem gives an upper bound for the 3- rainbow domination number of SL(n).

Theorem:4.3 Let SL(n) be the silicate network of dimension n. The 3-rainbow domination number of silicate network is $9n^2$ (i.e)$\gamma_{r3}(SL(n))\leq 9n^2$ ∀ n ≥ 1.

Proof: Let SL(n) be a silicate network. Let us consider the silicate network for dimension one, the domination number for SL(1) is 3 with the vertices 18 and edges 36 , we define a mapping $f: v(SL(1)) \rightarrow \mathcal{P}\{1,2,3\}$ such that we shall assigned colour class {1,2,3} to the vertices {v_4, v_7, v_9, v_{12} ,v_{15}, v_{18}} and the remaining vertices are assigned with empty colour . The minimum sum of numbers of assigned colours overall vertices of SL(1)is 6. Thus the 3-rainbow domination number of SL(1) is 9.For the silicate network of dimension two, the domination number for SL(2) is 12 with the vertices 66 and edges 144 ,we define a mapping $f: v(SL(2)) \rightarrow \mathcal{P}\{1,2,3\}$ such that we shall assigned colour class {1,2,3} to the vertices v_3, v_4, v_{11}, v_{12} ,v_{13}, v_{17}, v_{18}, v_{19}, v_{28}, v_{29}, v_{30}, v_{31}, v_{36}, v_{37}, v_{38}, v_{39}, v_{48}, v_{49} , $v_{50,}$ v_{54} $v_{,55,}v_{56,}v_{63,}$ v_{64} and the remaining vertices are assigned with empty colour .Thus the 3-rainbow domination number of SL(2) is 24.Similarly the silicate network of dimension three, the domination number for SL(3) is 27 with vertices 144 and edges 324 thus the 3-rainbow domination number of SL(3) is 54.Repeating this process for dimension n, the silicate network of dimension n we define a mapping $f: v(SL(n)) \rightarrow \mathcal{P}\{1,2,3\}$ such that for each vertex v∈ SL(n) with f(v)=∅. We have, $\bigcup_{u\in N(v)} f(u) = \{1,2,3\}$The domination number for SL(n) is $3n^2$ with vertices $15n^2+3n$ and edges $36n^2$ and therefore the 3-rainbow domination number of SL(n) is $\gamma_{r3}(SL(n))\leq 9n^2$ ∀ n≥ 1.

Example:4.2

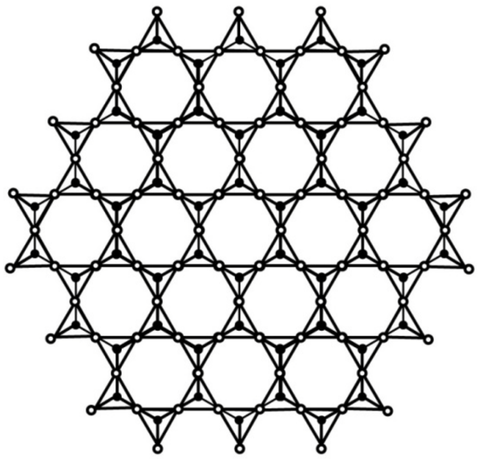

Figure:10- γ_{r3}(SL(3))=54

5. CONCLUSION

In this paper we established bounds for 3-rainbow domination number of Wheel graph, Triangular snake graph, Double triangular snake graph ,n-Barbell graph, n-Sunlet graph, n-Centipede graph, Crown graph, Clebsch graph and Silicate network. This work could be further extended to other classes of graphs also.

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