



WAVELET BASED SPECTRAL METHOD TO NON-LINEAR BVP IN IMMOBILIZED GLUCOAMYLASE KINETICS

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Abstract; A theoretical model to describe the enzyme reaction, heat and mass transfer effect in the calorimetric system is discussed. The mathematical model is based on non-stationary diffusion equation containing a nonlinear term. This paper presents a wavelet based approximation method to solve the singular nonlinear differential equation that depicts the coupled diffusion equation with nonlinear terms. Approximate analytical expressions for substrate concentration have been addressed for all values of parameters α, β, γ_e . The proposed wavelet results are compared with the ADM, HAM, HPM and other numerical results and found to be in good agreement.

Keywords: Immobilized Enzyme, Wavelets, Non linear BVP.

1. Introduction

Immobilization of an enzyme is the limitation of enzyme mobility, which can be achieved in different approaches. Some examples are physical or chemical immobilization which can occur either on the surface or inside the particle. Recently many scientists are much interested in working on immobilized biocatalysts (IMB)-enzymes because it has been used in wide applications in the medical and industrial applications. One of the major issues in the immobilization is that, identify of correct enzyme parameters. This can be achieved through some experimental analysis. For this purpose many experimental techniques can be used that are more or less laborious and time consuming. Michaelis-Menten modeled a simple mathematical model for identifying the enzyme activity and their kinetics and it has been popularly called as Michaelis-Menten Kinetics [1].

In experimental part, Stefuca et al [10], have described the principles and applications of flow calorimetry (FC) in the investigation of IMB properties. In his experimental set up he has used "auto-calibration" principle based on reaction solution which determines the true reaction rate of biocatalyst reaction. On analytical part, F Malik [4] has developed the mathematical model describing the enzyme reaction, mass transfer and heat effects in the calorimetric system has been thoroughly analyzed. To my knowledge no rigorous analytical solutions of the substrate with steady state conditions for all values of parameters α, β , and γ_e have been reported. Moreover we are the first to use the concept of Legendre wavelets to find the approximate analytical solution for the above model. Our numerical results are compared with Adomain decomposition Method

(ADM) [5], Homotopy Analysis Method (HAM) [2] and Homotopy Perturbation Method (HPM) [3].

2. Mathematical formulation of the problem:

We consider the Substrate concentration gradient on the particle surface given by the following equation [8]

$$\frac{\partial c_{sp}}{\partial t} = D_e \left(\frac{\partial^2 c_{sp}}{\partial r^2} + \frac{2}{r} \frac{\partial c_{sp}}{\partial r} \right) - \frac{V_m c_{sp}}{K_m + c_{sp} + \left(\frac{c_s^2}{K} \right)} \quad (1)$$

The equation must be solved subject to the following initial and boundary conditions:

$$c_{sp} = 0 \quad \text{at } t = 0, \quad 0 \leq r \leq 1 \quad (2)$$

$$\frac{dc_{sp}}{dr} = 0 \quad \text{at } r = 0 \quad (3)$$

$$c_{sp} = c_s \quad \text{at } r = R_p \quad (4)$$

where c_{sp} is the substrate concentration in the particle, c_s is the concentration, D_e is the diffusion coefficient, V_m, K_m, K_i are the kinetic parameters and r is the particle radial coordinate, R_p is the particle radius. We can write the steady state equation as [8]

$$D_e \left(\frac{d^2 c_{sp}}{dr^2} + \frac{2}{r} \frac{dc_{sp}}{dr} \right) - \frac{V_m C_{sp}}{K_m + c_{sp} + \left(\frac{c_{sp}^2}{K_i} \right)} = 0 \quad (5)$$

The system governs the substrate concentration c_{sp} when there is no competitive inhibition in the reaction. The nonlinear ODE in Eq.(5) is made dimensionless by defining the following parameters:

$$x = \frac{r}{R_p}; U = \frac{c_{sp}}{c_s}; \gamma_E = \frac{R_p^2 V_m}{D_e K_m}, \alpha = \frac{c_s}{K_m}, \beta = \frac{c_s^2}{K_i K_n} \quad (6)$$

where γ_E denote the reaction diffusion parameter, x is the dimensionless distance and $U(x)$ is the dimensionless

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WAVELETS AND LEGENDRE WAVELETS [6]

In recent years, wavelets have found their way into many different fields of science and engineering. Wavelets constitute a family of functions constructed from dilation and translation of single function called the mother wavelet. When the dilation parameter a and the translation parameter b vary continuously, we have the following family of continuous wavelets:

$$\psi_{a,b}(t) = |a|^{-1/2} \psi \left(\frac{t-b}{a} \right) \quad a, b \in R, a \neq 0. \quad (10)$$

Legendre wavelets have four arguments: n, k can assume any positive integer, m is the order for Legendre polynomials and t is the normalized time. They are defined on the interval $[0, 1]$ by

$$\psi_{nm}(t) = \begin{cases} \sqrt{(m+1/2)2^{k+1/2}} L_m(2^{k+1}t - (2n+1)) & (1) \\ 0 & (1) \end{cases}$$

where $m = 0, 1, \dots, M$ and $n = 0, 1, \dots, 2^{k-1}$. The coefficient $\sqrt{(m+1/2)2^{k+1/2}}$ is for orthonormality. Here $L_m(t)$ are the well-known Legendre polynomials of order m . Any function $f(t)$ defined over $[0, 1]$ may be expanded in the terms of Legendre wavelets as

$$f(t) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c_{nm} \psi_{nm}(t) \quad (12)$$

where $c_{nm} = \langle f(t), \psi_{nm}(t) \rangle$ can be calculated by standard inner product. If the infinite series in Eq.(12) is truncated, then it can be written as

$$f(t) = \sum_{n=0}^{2^k-1} \sum_{m=0}^{M-1} c_{nm} \psi_{nm}(t) = C^T \Psi(t). \quad (13)$$

where C and $\Psi(t)$ are $2^k(M+1) \times 1$ matrices given by

concentration. Here α and β denote the saturation parameters. The above eq. (5) reduces to the following dimensionless form as

$$\frac{d^2 U}{dx^2} + \frac{2}{x} \frac{dU}{dx} - \frac{\gamma_E U}{1 + \alpha U + \beta U^2} = 0 \quad (7)$$

the corresponding boundary conditions are

$$U = 1 \text{ at } x = 1 \quad (8)$$

$$\frac{dU}{dx} = 0 \text{ at } x = 0$$

$$C = \left[c_{0,0}, c_{0,1}, \dots, c_{0,M}, \dots, c_{2^k-1,0}, c_{2^k-1,1}, \dots, c_{2^k-1,M} \right]$$

$$\Psi(t) = \left[\psi_{0,0}, \psi_{0,1}, \dots, \psi_{0,M}, \dots, \psi_{2^k-1,0}, \psi_{2^k-1,1}, \dots, \psi_{2^k-1,M} \right] \quad (14)$$

3.1 OPERATIONAL MATRIX OF DERIVATIVE AND PRODUCT OPERATION MATRIX [9]

In the following section, we introduce a new Legendre wavelets operational matrix of derivative.

Theorem 1. Let the Legendre wavelets vector defined in Eq.(14). The derivative of the vector can be expressed by

$$\frac{d\Psi(t)}{dt} = D\Psi(t) \quad (15)$$

where D is the $2^k(M+1)$ operational matrix of derivative defined as follows:

$$D = \begin{pmatrix} F & 0 & \dots & 0 \\ 0 & F & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & F \end{pmatrix}, \quad (16)$$

in which F is $(M+1)(M+1)$ matrix and (r,s) th its element is defined as follows:

$$F_{r,s} = \begin{cases} 2^{k+1} \sqrt{(2r-1)(2s-1)} & r = 2, \dots, (M+1), \\ 0 & \text{o } 7) \end{cases} \quad (17)$$

Proof. By using shifted Legendre polynomial into $[0, 1]$, the r th element of vector $\Psi(t)$ in Eq.(14) can be written as

$$\Psi_r(t) = \psi_{n,m}(t) = 2^{k+1/2} \sqrt{m + \frac{1}{2}} P_m(2^k t - n) \chi_{[n/2]} \quad (18)$$

where

$$r = n(M+1) + (m+1); m = 0, 1, \dots, M; n = 0, 1, \dots, (2^k - 1); \chi$$

is the characteristic function defined as

$$\chi_{[n/2^k, n+1/2^k]}(t) = \begin{cases} 1, & t \in \left[\frac{n}{2^k}, \frac{n+1}{2^k} \right], \\ 0, & \text{otherwise.} \end{cases}$$

By differentiation with respect to in Eq.(18) we have

$$\frac{d\Psi(t)}{dt} = 2^{k+1/2} \sqrt{m + \frac{1}{2}} 2^k P'_m(2^k t - n) \chi_{[n/2^k, n+1/2^k]} \quad (19)$$

This function is zero outside the interval $[n/2^k, n+1/2^k]$, hence its Legendre wavelets expansion only have those elements of basis Legendre wavelets in that are nonzero in the interval $[n/2^k, n+1/2^k]$, that is $\Psi_i(t)$, $i = n(M+1)+1, n(M+1)+2, \dots, n(M+1)+M+1$.

So its Legendre wavelet expansion has the following form:

$$\frac{d\Psi(t)}{dt} = \sum_{i=n(M+1)+1}^{(n+1)(M+1)} a_i \Psi_i(t)$$

This results that operational matrix is a block matrix as defined in Eq.(16). Moreover we have

$$\frac{d}{dt} P_0(t) = 0$$

this implies that $\frac{d\Psi_r(t)}{dt} = 0$ for

$$r = 1, (M+1)+1, 2(M+1)+1, 3(M+1)+1, \dots, (2^k - 1)(M+1)$$

Consequently, the first row of matrix F defined in Eq.(16) is zero. Now, by substituting $P'_m(2^k t - n)$ from Eq.(9) into Eq.(19) we have

$$\frac{d\Psi(t)}{dt} = 2^{k+1/2} \sqrt{m + \frac{1}{2}} 2^k \sum_{\substack{j=0 \\ j+m \text{ odd}}}^{m-1} 2(2j+1) P_j(2^k t - n) \quad (20)$$

4 APPLICATIONS OF THE OPERATIONAL MATRIX OF DERIVATIVE

4.1 LINEAR DIFFERENTIAL EQUATION

Consider the linear second order differential equation

$$y''(x) + f_1(x)y'(x) + f_2(x)y(x) = g(x) \quad (24)$$

with initial conditions $y(0) = A, y'(0) = B,$ (25)

or boundary conditions $y(0) = A, y'(1) = B.$ (26)

To solve Eq.(24) we approximate $y(x), f_1(x), f_2(x)$ and $g(x)$ by Legendre wavelets as

$$\left. \begin{aligned} y(x) &= C^T \psi(x) \\ f_1(x) &= F_1^T \psi(x) \\ f_2(x) &= F_2^T \psi(x) \\ g(x) &= G^T \psi(x) \end{aligned} \right\} \quad (27)$$

By using Eqs. (22) and (27), we have

$$\begin{aligned} y'(x) &= C^T D \psi(x), \\ y''(x) &= C^T D^2 \psi(x). \end{aligned} \quad (28)$$

Expanding this equation in Legendre wavelets basis we have

$$\begin{aligned} \frac{d\Psi(t)}{dt} &= 2^{\frac{k+1}{2}} \sqrt{m + \frac{1}{2}} 2^k \sum_{\substack{j=0 \\ j+m \text{ odd}}}^{m-1} 2(2j+1) P_j(2^k t - n) \\ &= 2^{\frac{k+1}{2}} \sum_{\substack{s=1 \\ s+r \text{ odd}}}^{r-1} \sqrt{(2r-1)(2s-1)} \Psi_{n(M+1)+s}(t). \end{aligned} \quad (21)$$

So, if we choose as

$$F_{r,s} = \begin{cases} 2^{\frac{k+1}{2}} \sqrt{(2r-1)(2s-1)} & r = 2, \dots, (M+1), s \\ 0 & \text{oth} \end{cases}$$

then Eq.(15) is held and this leads to the desired result. □

Corollary. By using Eq.(15), the operational matrix for n th derivative can be derived as

$$\frac{d^n \Psi(t)}{dt^n} = D^n \Psi(t) \quad (22)$$

where D^n is the n th power of matrix D . The following property of the product of two Legendre wavelets vector functions is used

$$e^T \Psi \Psi^T = \Psi^T E, \quad (23)$$

where e is a given vector and E is a $(2^k(M+1))(2^k(M+1))$ matrix dependent on vector e , which is called the product operation matrix of Legendre wavelets vector functions.

Employing Eqs. (27) and (28), the residual $R(x)$ for Eq.(24) can be written as

$$R(x) = (C^T D^2 \Psi(x) + F_1^T \Psi(x) \Psi^T(x) D^T C + F_2^T \Psi(x) \Psi^T(x) C - G^T \Psi(x)). \tag{29}$$

By using the product operation matrix of Legendre wavelets, we have

$$R(x) = (\Psi^T(x)(D^2)^T C + \Psi^T(x) \tilde{F}_1 D^T C + \Psi^T(x) \tilde{F}_2 C - \Psi^T(x) G). \tag{30}$$

As in a typical Tau method, we generate $2^k(M+1) - 2$ linear equations by applying

$$\int_0^1 \Psi_j(x) R(x) dx = 0 \quad j = 1, \dots, 2^k(M+1) - 2. \tag{31}$$

Also, by substituting initial conditions (25) in Eqs.(27) and (28), we have

$$\begin{aligned} y(0) &= C^T \Psi(0) = A, \\ y'(0) &= C^T D \Psi(0) = B. \end{aligned} \tag{32}$$

or for boundary value problems, we have

$$\begin{aligned} y(0) &= C^T \Psi(0) = A, \\ y(1) &= C^T D \Psi(1) = B. \end{aligned} \tag{33}$$

Eq.(31)- (33) generate $2^k(M+1)$ set of linear equations. These linear equations can be solved for unknown coefficients of vector C . Consequently, $y(x)$ given in Eq.(27) can be calculated.

NONLINEAR DIFFERENTIAL EQUATION

Consider the nonlinear equation

$$y''(x) = F(x, y'(x), y(x)) \tag{34}$$

with the initial conditions

$$y(0) = A, \quad y'(0) = B. \tag{35}$$

or boundary conditions

$$y(0) = A, \quad y'(1) = B. \tag{36}$$

In order to use shifted Legendre polynomials for this problem, we first approximate $y(x)$ by Legendre wavelets

$$y(x) = C^T \psi(x) \tag{37}$$

By using Eqs.(28) and (34) we have

$$C^T D^2 \Psi(x) = F(x, C^T D \Psi(x), C^T \Psi(x)). \tag{38}$$

Also, initial and boundary value conditions (35) and (36) yield

$$\begin{aligned} y(0) &= C^T \Psi(0) = A, \\ y'(0) &= C^T D \Psi(0) = B. \end{aligned} \tag{39}$$

and

$$\begin{aligned} y(0) &= C^T \Psi(0) = A, \\ y(1) &= C^T D \Psi(1) = B. \end{aligned} \tag{40}$$

To find the solution $y(x)$, we first calculate Eq.(38) at $2^k(M+1) - 2$ points. For a better result, we use the first $2^k(M+1) - 2$ roots of shifted Legendre $P_{2^k(M+1)}(x)$. These equations collectively with Eq.(39) or (40)

generate $2^k(M + 1)$ nonlinear equations which can be solved using Newton’s iterative method. Consequently $y(x)$ given in Eq.(37) can be calculated. The above-mentioned applications for linear and nonlinear differential equations can be easily execute for higher order differential equations.

5 Proposed Method

In this section we solve the concentration substrate of dimensionless form using the Legendre Wavelet Method proposed in previous section for the case of $K = 0$ and $M = 2$: Consider the Eq. (7) with various limiting cases.

Case (1) [Substrate concentration $U(x)$ with normal solution for small values of γ_E with $\alpha = 0.1, \beta = 0.001$]
 On simplification of Eq. (7) gives

$$[1 + \alpha U + \beta U^2]U''(x) + \frac{2}{x}(1 + \alpha U + \beta U^2)U'(x) - \gamma_E U(x) = 0 \tag{41}$$

where $U = C^T \psi(x), U' = C^T D\psi(x), U'' = C^T D^2\psi(x)$ (4 2

$$C^T = [c_{00} \quad c_{01} \quad c_{02}]^T,$$

$$\Psi(x) = \begin{pmatrix} 1 \\ \sqrt{3}(2x-1) \\ \sqrt{5}(1-6x+6x^2) \end{pmatrix}, \quad D^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 12\sqrt{5} & 0 & 0 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} 0 & 0 & 0 \\ 2\sqrt{3} & 0 & 0 \\ 0 & 2\sqrt{15} & 0 \end{pmatrix}$$

We calculate collocation point at the first root of $P_3(x)$ at $x_0 = \frac{1}{2}$.

Further, by using the boundary conditions (8) and (9), we have $P_3(x)$ as

$$2\sqrt{3}c_{11} - 6\sqrt{5}c_{12} = 0 \tag{43}$$

$$2\sqrt{3}c_{11} + 6\sqrt{5}c_{12} = 800 \tag{44}$$

On solving the above algebraic system (42)-(44), we obtain the substrate concentration for the various values and the result is shown in Table 1.

Table-1 Comparison of dimensionless substrate concentration $U(x)$ with normal solution for small values of γ_E with $\alpha = 0.1, \beta = 0.001$

Dimensionless substrate concentration $U(x)$ when $\alpha = 0.1, \beta = 0.001$														
X	$\gamma_{E=0.1}$							$\gamma_{E=0.5}$						
	LWM	ADM	Err.	HAM	Err	HPM	Err	LWM	ADM	Err	HAM	Err	HPM	Err
0	0.9837	0.9837	0	0.9879	0.43	0.9837	0	0.9220	0.9223	0.03	0.9405	2.01	0.9224	0.04
0.2	0.9844	0.9844	0	0.9884	0.41	0.9844	0	0.9251	0.9254	0.03	0.9429	1.92	0.9225	0.04
0.4	0.9864	0.9864	0	0.9899	0.36	0.9864	0	0.9345	0.9347	0.02	0.9502	1.68	0.9348	0.03

0.6	0.9897	0.9897	0	0.9924	0.27	0.9897	0	0.9502	0.9504	0.02	0.9623	1.27	0.9504	0.02
0.8	0.9943	0.9943	0	0.9958	0.16	0.9943	0	0.9725	0.9726	0.01	0.9793	0.70	0.9726	0.01
1	1.000	1.000	0	1.000	0	1.000	0	1.000	1.000	0	1.000	0	1.000	0

Table-2a Comparison of dimensionless substrate concentration $U(x)$ with normal solution for small values of γ_E with $\alpha = 0.01, \beta = 0.1$

Dimensionless substrate concentration $U(x)$ when $\alpha = 0.01, \beta = 0.1$														
X	$\gamma_E = 0.02$							$\gamma_E = 0.5$						
	LWM	ADM	Err.	HAM	Err	HPM	Err	LWM	ADM	Err	HAM	Err	HPM	Err
0	0.9970	0.9970	0	0.9977	0.07	0.9970	0.0	0.9281	0.9281	0	0.9424	1.54	0.9307	0.28
0.2	0.9971	0.9971	0	0.9977	0.06	0.9972	0.01	0.9309	0.9309	0	0.9447	1.48	0.9334	0.27
0.4	0.9975	0.9975	0	0.9980	0.05	0.9975	0	0.9394	0.9394	0	0.9516	1.30	0.9715	0.23
0.6	0.9981	0.9981	0	0.9985	0.04	0.9981	0	0.9537	0.9537	0	0.9630	0.98	0.9552	0.16
0.8	0.9989	0.9989	0	0.9992	0.03	0.9989	0	0.9738	0.9738	0	0.9792	0.56	0.9746	0.08
1	1.0000	1.0000	0	1.0000	0	1.000	0	1.0000	1.0000	0	1.0000	1	1.000	0

Table-2b Comparison of dimensionless substrate concentration $U(x)$ with normal solution for small values of γ_E with $\alpha = 0.01, \beta = 0.1$

Dimensionless substrate concentration $U(x)$ when $\alpha = 0.01, \beta = 0.1$														
X	$\gamma_E = 1.0$							$\gamma_E = 2.5$						
	LWM	ADM	Err.	HAM	Err	HPM	Err	LWM	ADM	Err	HAM	Err	HPM	Err
0	0.8628	0.8626	0	0.8625	0.07	0.8711	0.0	0.6981	0.7046	0	0.7346	0.3	0.7345	0.7
0.2	0.8683	0.8679	0	0.8915	0.06	0.8759	0.01	0.7102	0.7151	0	0.7447	0.5	0.7587	0.5
0.4	0.8847	0.88839	0	0.9049	0.05	0.8906	0	0.7464	0.7473	0	0.7754	0.2	0.7836	0.5
0.6	0.9122	0.9108	0	0.9273	0.04	0.9156	0	0.8068	0.803	0	0.8274	0.2	0.8284	0.8
0.8	0.9506	0.9493	0	0.9589	0.03	0.9517	0	0.8913	0.8858	0	0.9017	0.14	0.8983	0.7
1	1.0000	1.0000	0	1.0000	0	1.0000	0	1.0000	1.0000	0	1.0000	0	1.0000	0

Table-3 Comparison of dimensionless substrate concentration $U(x)$ with normal solution for small values of γ_E with $\alpha = 0.2, \beta = 0.5$

Dimensionless substrate concentration $U(x)$ when $\alpha = 0.2, \beta = 0.5$														
X	$\gamma_E = 0.1$							$\gamma = 0.5$						
	LWM	ADM	Err.	HAM	Err	HPM	Err	LWM	ADM	Err	HAM	Err	HPM	Err
0	0.9902	0.9902	0	0.9906	0.04	0.9952	0.51	0.9515	0.9515	0	0.9538	0.24	0.9799	2.99
0.2	0.9906	0.9906	0	0.9909	0.03	0.9954	0.49	0.9535	0.9535	0	0.9556	0.23	0.9806	2.86
0.4	0.9918	0.9918	0	0.9921	0.03	0.9960	0.43	0.9593	0.9593	0	0.9611	0.20	0.9828	2.46
0.6	0.9937	0.9937	0	0.9940	0.03	0.9969	0.32	0.9690	0.9690	0	0.9703	0.15	0.9866	1.83
0.8	0.9965	0.9965	0	0.9966	0.01	0.9983	0.18	0.9826	0.9826	0	0.9832	0.07	0.9923	1.00
1	1.0000	1.0000	0	1.0000	0	1.0000	0.0	1.0000	1.0000	0	1.000	0	1.000	0

Table-4a Comparison of dimensionless substrate concentration $U(x)$ with normal solution for small values of γ_E with $\alpha = 0.05, \beta = 0.0001$

Dimensionless substrate concentration $U(x)$ when $\alpha = 0.05, \beta = 0.0001$														
X	$\gamma_E = 0.01$							$\gamma = 0.1$						
	LWM	ADM	Err.	HAM	Err	HPM	Err	LWM	ADM	Err	HAM	Err	HPM	Err
0	0.9984	0.9984	0	0.9988	0.04	0.9984	0	0.9843	0.9843	0	0.9981	0.38	0.9844	0.01
0.2	0.9985	0.9985	0	0.9989	0.04	0.9985	0	0.9849	0.9849	0	0.9886	0.37	0.9850	0.01
0.4	0.9987	0.9987	0	0.9990	0.03	0.9987	0	0.9868	0.9868	0	0.9900	0.32	0.9869	0.01
0.6	0.9990	0.9990	0	0.9992	0.02	0.9990	0	0.9899	0.9899	0	0.9924	0.25	0.9900	0.01
0.8	0.9994	0.9994	0	0.9996	0.02	0.9994	0	0.9943	0.9943	0	0.9957	0.14	0.9944	0.01
1	1.0000	1.0000	0	1.0000	0	1.0000	0	1.0000	1.0000	0	1.0000	0	1.0000	0

Table-4b Comparison of dimensionless substrate concentration $U(x)$ with normal solution for small values of γ_E with $\alpha = 0.05, \beta = 0.0001$

Dimensionless substrate concentration $U(x)$ when $\alpha = 0.05, \beta = 0.0001$														
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X	$\gamma_E = 0.6$							$\gamma = 1$						
	LWM	ADM	Err.	HAM	Err	HPM	Err	LWM	ADM	Err	HAM	Err	HPM	Err
0	0.9105	0.9108	0.03	0.9298	2.12	0.912	0.17	0.8567	0.8581	0.16	0.8848	3.28	0.8611	0.51
0.2	0.9140	0.9143	0.03	0.9326	2.04	0.9154	0.15	0.8622	0.8635	0.15	0.8893	3.14	0.8664	0.49
0.4	0.9245	0.9247	0.02	0.9409	1.77	0.9257	0.13	0.8788	0.8798	0.11	0.9030	2.75	0.8822	0.39
0.6	0.9422	0.9423	0.01	0.9549	1.35	0.9430	0.09	0.9068	0.9075	0.08	0.9528	2.09	0.9092	0.27
0.8	0.9672	0.9673	0.01	0.9746	0.77	0.9676	0.04	0.9469	0.9472	0.03	0.9591	1.18	0.9481	0.13
1	1.000	1.0000	0	1.0000	0	1.000	0	1.000	1.000	0	1.0000	0	1.000	0

6 CONCLUSION

A nonlinear ordinary differential equation of Michaelis-Mentins kinetics has been solved with the help of Legendre wavelets. Our results coincides with the solutions of other existing method. Moreover, the process time is very less compared to the HAM, HPM and ADM. These analytical results are useful for design and optimization of immobilized enzyme kinetics.

7 REFERENCES

- [1] Bailey, J.E. Ollis, D.F “Biochemical Engineering Fundamentals” 2 Ed. McGraw-Hill, New York, 1986.
- [2] He J.H. “A coupling method of homotopy technique and a perturbation technique for non-linear problems”. Int .J Non. Linear Mech., 35, 37-43, 2000.
- [3] Liao, S.J. “ On the homotopy analysis method for nonlinear problems” ,App. Math. Comp., 147, 499-513, 2004.
- [4] Malik, F., Stefuca, V. and Bales, V. “ Investigation of kinetics of immobilized liver esterase by flow calorimetry”, J. Mol. Cat.B: Enzymatic, 29, 81-87, 2004.
- [5] Majid Wazwaz, A. and Gorguis, A. “The decomposition method applied to systems of partial differential equations”, App. Math. Comp, 149, 807-814 , 2004.
- [6] Mohammadi F, Hosseini MM , “A new operational matrix for solving of singular BVPS Journal of the Franklin Institute” 348(8):1787-1796, 2011.
- [8] Rajendran L , Saravanakumar K, “Analytical expression of transient and steady-state catalytic current of mediated bioelectrocatalysis” “Electrochimica Acta”,147, 678-687, 2014.
- [9] Razzaghi M, Yousefi S. “Legendre wavelets operational matrix” Int. J. Sys. Sci, 32(4),495-50, .2001.
- [10] Stefuca, V. Vikartovska-Welwardova, V. and Gemenier , P.” Flow microcalorimeter auto-calibration for the analysis of immobilized enzyme kinetics” Analytica Chimica Acta, 355, 63,1999.