ISSN No. 0976-5697

Volume 8, No. 6, July 2017 (Special Issue III)

International Journal of Advanced Research in Computer Science

RESEARCH PAPER

Available Online at www.ijarcs.info

# THE INVERSE DISTANCE – 2 DOMINATION IN GRAPHS AND ITS APPLICATIONS

A. Lakshmi1st
Department of Mathematics
D.K.M.College for Women(Autonomous)
Vellore, India

K. Ameenal Bibi 2nd
Department of Mathematics
D.K.M.College for Women(Autonomous)
Vellore, India

***Abstract:*** Let G = (V, E) be a simple, finite, connected and undirected graph. Let D⊆G be the non-empty subset of G such that D is the minimum distance - 2 dominating set in the graph G = (V, E). If V-D contains a distance - 2 dominating set D′ of G, then D′ is called an inverse distance - 2 dominating set with respect to D. The inverse distance - 2 domination number $\gamma_{\leq 2}^{-1}$(G) of G is the cardinality of a minimum inverse distance - 2 dominating set of G. In this paper, we defined the notion of an inverse distance - 2 domination number of graphs. We get many bounds on inverse distance - 2 domination number. Exact values of this new parameter are obtained for some standard graphs and also its relationship with other domination parameters was obtained. Nordhaus - Gaddum type results are also obtained for this new parameter.

***Keywords:*** Dominating set, inverse dominating set, distance - 2 dominating set, inverse distance - 2 dominating set, inverse distance - 2 domination number.

## I. INTRODUCTION

All graphs considered here are simple, finite, connected and undirected. Let n and m denote the order and size of a graph G. We use the terminology of [8]. Let **Δ**(G) (**δ**(G)) denote the maximum (minimum) degree of G. The greatest (least) integer less (greater) than or equal to x is denoted by□x□(□x□). The independence number $\beta_0$(G) is the maximum cardinality among the independent set of vertices of G. The girth g(G) of a graph G is the length of the shortest cycle in G. The circumference c(G) is the length of the longest cycle in G.

A non-empty subset D⊆V in a graph G = (V, E) is called a dominating set if every vertex in V-D is adjacent to atleast one vertex in D. The domination number of G is the minimum cardinality of a minimal dominating set and it is denoted by γ(G). A recent survey of γ(G) can be found in [8]. Kulli, V.R. and Sigarkanti,S.C. introduced the concept of inverse domination in graph in 1991 [9].

Let D be the minimum dominating set in a graph G = (V, E). If V-D contains a dominating set D′ of G, then D′ is called an inverse dominating set with respect to D. The inverse domination number $\gamma^{-1}$(G) of G is the minimum cardinality of the minimal inverse dominating set of G.

A non-empty subset D⊆V in a graph G = (V, E) is a distance -2 dominating set if every vertex in V-D is within a distance 2 of atleast one vertex in D. The distance -2 domination number $\gamma_{\leq 2}$ (G) of G equals the minimum cardinality of a minimal distance -2 dominating set in G [8].

**Definition 1.1**

Let D be a minimum distance - 2 dominating set in a graph G = (V, E). If V-D contains a distance - 2 dominating set D′ of G, and then D′ is called an inverse distance - 2 dominating set with respect to D. The inverse distance - 2 domination number $\gamma_{\leq 2}^{-1}$ (G) of G is the cardinality of a minimum inverse distance - 2 dominating set of G.

**Example 1**

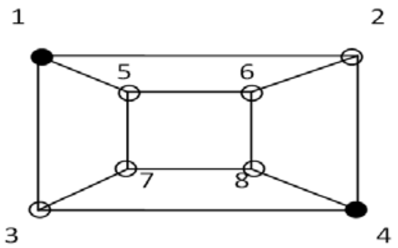


**Figure 1. Example of an inverse distance-2 dominating set of a graph G**

Here D = {2,3}, D′ ={1,4}, $\gamma_{\leq 2}^{-1}$(G) = {2}

2. **Exact values of $\gamma_{\leq 2}^{-1}$(G) for some standard graphs.**

2.1: **Observation:**

1. For any path,$P_n$ for n ≥ 2
$\gamma_{\leq 2}^{-1}(P_n)$ =□n/5□+1
2. For any cycle,$C_n$ for ≥ 3
$\gamma_{\leq 2}^{-1}(C_n)$ =□n/5□
3. For any wheel graph,$W_n$ for n ≥ 3
$\gamma_{\leq 2}^{-1}(W_n) = 1$
4. For any friendship graph $F_n$, for n ≥ 2
$\gamma_{\leq 2}^{-1}(F_n) =1$
5. For any complete graph $K_n$, for n ≥ 2
$\gamma_{\leq 2}^{-1}(K_n) =1$
6. For any star graph $K_{1,m}$, for m ≥1
$\gamma_{\leq 2}^{-1}(K_{1,m}) =1$
7. For any complete bipartite graph $K_{n,m}$, for m ≥ n,
$\gamma_{\leq 2}^{-1}(K_{nm}) = 1$
8. For any Book graph $B_n$, for n ≥3
$\gamma_{\leq 2}^{-1}(B_n) = 1$
9. For any helm graph $H_n$, for n ≥3
$\gamma_{\leq 2}^{-1}(H_n) = 1$
10. For grid graph $P_2$ X $P_j$, for j ≥2
$\gamma_{\leq 2}^{-1}(P_2 X P_j)$ =□2+j /3□

**Observation 2.2**

For any graph G, $\gamma_{\leq 2}^{-1}(G) = \gamma_{\leq 2}(G)$ if and only if G is any one of the following common graphs $C_n$, $W_n$, $K_n$, $B_n$, $F_n$ $K_{1,m}$ and $K_{n,m}$ .

**Note 2.3** For any wheel graph $W_n$,





CONFERENCE PAPER
National Conference dated 27-28 July 2017 on
Recent Advances in Graph Theory and its Applications (NCRAGTA2017)
Organized by Dept of Applied Mathematics
Sri Padmawati Mahila Vishvavidyalayam (Women's University) Tirupati, A.P., India

$\gamma_{\leq 2}^{-1}(W_n) = \gamma_{\leq 2}(W_n) = \gamma(W_n)$

## 3. Bounds on the inverse distance -2 domination number

**Observation 3.1**

If a graph G has no isolated vertices, then

$$\gamma_{\leq 2}^{-1}(G) + \gamma_{\leq 2}^{-1}(G) \leq n$$

Furthermore, equality holds if G = $P_2$, or $K_2$ .

**Proposition 3.2**

For any graph G, $\gamma_{\leq 2}(G) \leq \gamma_{\leq 2}^{-1}(G)$

**Proof**

Every inverse distance - 2 dominating set of G is a distance -2 dominating set of G,

We have , $\gamma_{\leq 2}(G) \leq \gamma_{\leq 2}^{-1}(G)$

**Proposition 3.3**

For any graph G, $\gamma_{\leq 2}^{-1}(G) \leq \gamma^{-1}(G)$

**Proof**

Every inverse dominating set of G is an inverse distance - 2 dominating set of G,

We have $\gamma_{\leq 2}^{-1}(G) \leq \gamma^{-1}(G)$

**Observation 3.4**

For any graph G, $\gamma_{\leq 2}(G) \leq \gamma_{\leq 2}^{-1}(G) \leq \gamma(G) \leq \gamma^{-1}(G)$

**Theorem 3.5**

For any graph G, $1 \leq \gamma_{\leq 2}^{-1}(G) \leq n-1$

**Proof**

Let D be the minimum distance - 2 dominating set in a graph G = (V,E) then, V-D contains a distance 2- dominating set D′ of G then D′ is called an inverse distance 2- dominating set with atleast one vertex. Since the order of the graph is n, the upper bound of the inverse distance - 2 dominating set must have atmost n-1 vertices.

We have $1 \leq \gamma_{\leq 2}^{-1}(G) \leq n-1$

**Nordhas - Gaddum Type results**

**Theorem 3.6**

Let G be a graph such that both G and $\overline{G}$ have no isolates. Then,

(i). $2 \leq \gamma_{\leq 2}^{-1}(G) + \gamma_{\leq 2}^{-1}(\overline{G}) \leq 2(n-1)$

(ii). $1 \leq \gamma_{\leq 2}^{-1}(G) . \gamma_{\leq 2}^{-1}(\overline{G}) \leq (n-1)^2$

**Theorem 3.7**

For any star graph $K_{1,m}$, $\gamma_{\leq 2}^{-1}(G) = \lfloor n/(\boldsymbol{\Delta}(K_{1,m})+1) \rfloor$, where n=1+m

**Proof**

Since $\boldsymbol{\Delta}(K_{1,m}) = m$ and $\gamma_{\leq 2}^{-1}(K_{1,m}) = 1$

We have $\gamma_{\leq 2}^{-1}(G) = \lfloor n/ (\boldsymbol{\Delta}(K_{1,m})+1) \rfloor$

**Corollary 3.8**

For any graph G, $\gamma_{\leq 2}^{-1}(G) = \lfloor n/ (\boldsymbol{\Delta}(G)+1) \rfloor$ if and only if G is any one of, $F_n$ , $B_n$ $K_n$ and $W_n$.

The upper bound of $\gamma(G)$ is attained by Berge in [8]

**Theorem 3.9**

For any graph G, $\gamma(G) \leq n-\boldsymbol{\Delta}(G)$ in [8]

**Theorem 3.10**

For any graph G, $\gamma_{\leq 2}^{-1}(G) \leq n-\boldsymbol{\Delta}(G)$.

**Proof**

Since $\gamma_{\leq 2}^{-1}(G) \leq \gamma(G)$ and $\gamma(G) \leq n-\boldsymbol{\Delta}(G)$ in

[By observation 3.4 and Theorem. 3.9]

We have $\gamma_{\leq 2}^{-1}(G) \leq n-\boldsymbol{\Delta}(G)$.

**Theorem 3.11**

For any graph G, $\gamma^{-1}(G) \leq \lfloor n\boldsymbol{\Delta}(G)/(1+\boldsymbol{\Delta}(G)) \rfloor$ in [14]

**Theorem 3.12**

For any graph G, $\gamma_{\leq 2}^{-1}(G) \leq \lfloor n\boldsymbol{\Delta}(G)/(1+\boldsymbol{\Delta}(G)) \rfloor$ Furthermore, the equality holds, if G is $P_2$, and $K_2$.

**Proof**

Since $\gamma_{\leq 2}^{-1}(G) \leq \gamma^{-1}(G)$ and $\gamma^{-1}(G) \leq \lfloor n\boldsymbol{\Delta}(G)/(1+\boldsymbol{\Delta}(G)) \rfloor$ [By observation 3.3 and Theorem 3.11]

We have $\gamma_{\leq 2}^{-1}(G) \leq \lfloor n\boldsymbol{\Delta}(G)/(1+\boldsymbol{\Delta}(G)) \rfloor$

**Theorem 3.13**

If a graph G has no isolated vertices and $\gamma(G) \geq 3$ then,

$\gamma(G) \leq (n+1-\boldsymbol{\delta}(G))/2$ in [8]

**Theorem 3.14**

If a graph G has no isolated vertices and $\gamma(G) \geq 3$ then ,

$\gamma_{\leq 2}^{-1}(G) \leq (n+1-\boldsymbol{\delta}(G))/2$

**Proof**

Since $\gamma_{\leq 2}^{-1}(G) \leq \gamma(G)$ and $\gamma(G) \leq (n+1-\boldsymbol{\delta}(G))/2$

[By observation 3.4 and Theorem 3.13]

We have $\gamma_{\leq 2}^{-1}(G) \leq (n+1-\boldsymbol{\delta}(G))/2$

**Theorem 3.15**

If a graph G has $\boldsymbol{\delta}(G) \geq 2$ and $g(G) \geq 5$, then

$\gamma(G) \leq \lfloor (n-\lfloor g(G)/3 \rfloor)/2 \rfloor$ in [8]

**Theorem 3.16**

If a graph G has $\boldsymbol{\delta}(G) \geq 2$ and $g(G) \geq 5$, then

$$\gamma_{\leq 2}^{-1}(G) \leq \lfloor (n-\lfloor g(G)/3 \rfloor)/2 \rfloor$$

**Proof**

Since $\gamma_{\leq 2}^{-1}(G) \leq \gamma(G)$ and $\gamma(G) \leq \lfloor (n-\lfloor g(G)/3 \rfloor)/2 \rfloor$

[By observation 3.4 and Theorem 3.15]

We have $\gamma_{\leq 2}^{-1}(G) \leq \lfloor (n-\lfloor g(G)/3 \rfloor)/2 \rfloor$

**Theorem 3.17**

For any graph G without isolated vertices $\gamma^{-1}(G) \leq \beta_0(G)$ in [14]

**Theorem 3.18**

For any graph G without isolated vertices $\gamma_{\leq 2}^{-1}(G) \leq \beta_0(G)$

**Proof**

Since $\gamma_{\leq 2}^{-1}(G) \leq \gamma^{-1}(G)$ and $\gamma^{-1}(G) \leq \beta_0(G)$ [By the observation 3.3 and Theorem 3.17]

We have $\gamma_{\leq 2}^{-1}(G) \leq \beta_0(G)$

**Theorem 3.19**

In path $P_n$, $\gamma_{\leq 2}^{-1}(P_n) \leq \gamma(P_n)$

**Proof**

Let D be a minimum distance - 2 dominating set of $P_n$ . Let D′ be a minimum distance - 2 dominating set of V-D. Every vertex in V- D′ has a distance at the most 2. Let $D_1$ be the dominating set of $P_n$ then every vertex in V- $D_1$ has a distance one, obviously $D_1$ has more vertices then D′ . Hence the theorem follows.

**Theorem 3.20**

For any graph G, $\gamma_{\leq 2}^{-1}(G) = p - \boldsymbol{\Delta}(G)$ if and only if G is a star graph $K_{1,m}$, for m>1, where p(=1+m) is number of vertices.

**Theorem 3.21**

For any graph G, which is not a tree then $\gamma_{\leq 2}^{-1}(G) \leq c(G)$ where c(G) is the circumference of a graph G.

**Note 3.22**

For any graph G, which is not a tree then $\gamma_{\leq 2}^{-1}(G) \leq g(G)$ where g(G) is the girth of a graph G.

**Theorem 3.23**

If H is a connected spanning subgraph of G, then

$\gamma_{\leq 2}^{-1}(G) \leq \gamma_{\leq 2}^{-1}(H)$

**Theorem 3.24**

Let G be a Hamiltonian graph on n vertices, then

$\gamma_{\leq 2}^{-1}(G) \leq \lfloor n/5 \rfloor$

**Proof**

Let G be Hamiltonian. Then G contains a spanning cycle say $C_n$ . By the observation 2,

We have $\gamma_{\leq 2}^{-1}(C_n) = \gamma_{\leq 2}(C_n) = \lfloor n/5 \rfloor$

We have $\gamma_{\leq 2}^{-1}(G) \leq \gamma_{\leq 2}(C_n) = \lfloor n/5 \rfloor$





CONFERENCE PAPER
National Conference dated 27-28 July 2017 on
Recent Advances in Graph Theory and its Applications (NCRAGTA2017)
Organized by Dept of Applied Mathematics
Sri Padmawati Mahila Vishvavidyalayam (Women's University) Tirupati, A.P., India

**Theorem 3.25**

For any graph G with no isolated vertices, $\gamma^{-1}(G) \le \varepsilon f(G)$ where $\varepsilon f(G)$ denotes the maximum number of pendent edges in a spanning forest of G in [14].

**Theorem 3.26**

For any graph G with no isolated vertices, $\gamma_{\le 2}^{-1}(G) \le \varepsilon f(G)$ where $\varepsilon f(G)$ denotes the maximum number of pendent edges in a spanning forest of G.

**Proof**

Since $\gamma_{\le 2}^{-1}(G) \le \gamma^{-1}(G)$ and, $\gamma^{-1}(G) \le \varepsilon f(G)$ [By Theorem 3.3 and Theorem 3.25]

We have $\gamma_{\le 2}^{-1}(G) \le \varepsilon f(G)$.

## 4. Applications of Inverse distance - 2 dominating sets

In Graph Theory both dominating sets and their inverse dominating sets have important roles to play. In an information retrieval system, we always have a set of primary nodes to pass on the information. In case, the system fails, we have another set of secondary nodes, to do the job in the complement. Thus, the dominating sets, and the elements in the inverse dominating sets can stand together to facilitate the communication process.

### Radio stations 4.1

Suppose that we have a collection of small villages in a remote part of the world. We would like to locate radio stations in some of these villages so that messages can be broadcasted to all the villages in the region. But since the installations of radio stations are costly, we want to locate as few as possible which can cover all other villages. Let each village be represented by a vertex. An edge between two villages is labeled with the distance, say in kilometers. The distance between the two villages is shown in fig.2. Let us assume that a radio station has a broadcast range of hundred kilometers. In this case we seek a distance - 2 dominating set among all the vertices within the distance of 100 kilometers. The set of vertex {marked in dark with name B} forms a distance - 2 dominating set and the set of vertex {marked in dark with name I} forms an inverse distance - 2 dominating set.

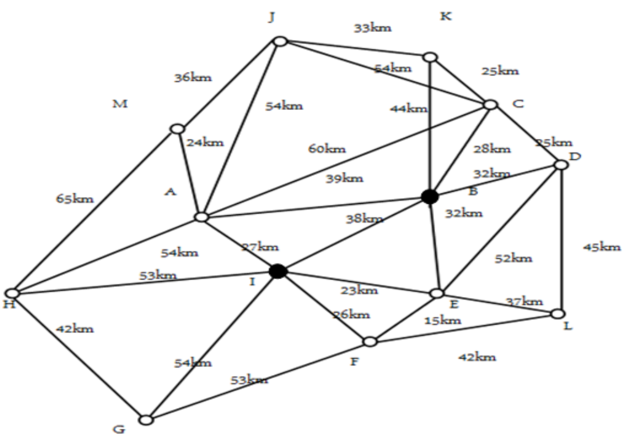


**Figure 2.**

### Computer Communication Networks 4.2

The distance - 2 dominating set plays an important role in computer and communication networks to route the information between the nodes. We consider a computer network modeled by a hypercube. The vertices of the hypercube represents computer and edges represent direct communication link between two computer. So, in this model we have 16 computers or processors and each processor can pass information to the processor to which it is directly connected. Our problem is to collect information from all processors and we would like to do it relatively often and relatively fast. So, we identify a small set of processors called collecting processors and ask each processor to send its information to one of a small set of collecting processors. We assume that at most a two - unit delay between the time a processor sends its information and the time it arrives at a nearest collector is allowed. In this case we have to find a distance - 2 dominating set of all processors. The set of vertices {marked in dark without name} forms a distance - 2 dominating set and the set of vertices{x, y} forms an inverse distance - 2 dominating set in the hypercube network in fig.3.

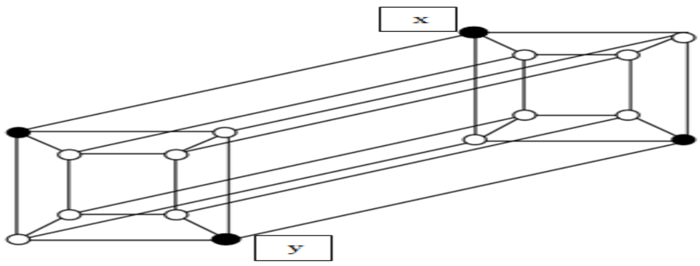


**Figure 3.**

## Conclusion

In this paper, we defined the notion of inverse distance – 2 domination in graphs. We get many bounds on inverse distance - 2 domination number. Exact values of this new parameter are obtained for some standard graphs and also its relationship with other domination parameters was obtained. Nordhaus - Gaddum type results are also obtained for this new parameter. Also we have discussed the applications of Inverse distance - 2 dominating sets in computer and communication networks. The future work covers the proposed algorithm to the Ad – Hoc Wireless networks in which each mobile node has different transmission radii.

## II. ACKNOWLEDGMENT

The authors are highly thankful to everyone who contributed to the success of this work.

## III. REFERENCES

[1] Ameenal Bibi, K., and Selvakumar, R., (2010).The Inverse strong non-split r-domination number of a graph- International Journal of Engineering, Science and Technology,vol.2, No.1, pp. 127-133.

[2] Ameenal Bibi, K., and Selvakumar, R., (2010).The Inverse split and non-split domination numbers in graph - International Journal of Computer Applications (0975-8887) Volume 8 – N0.7, October 2010.

[3] Cockayne, E.J., Dawes, R.M., and Hedetniemi, S.T., (1980). Total domination in graphs. Networks, vol.10.pp.211-219.

[4] Cockayne, E.J., Hedetniemi, S.T., Towards a Theory of Domination in Graphs, Networks, 7:247-261.

[5] Domke G.S., Dunbar J.E. and Markus, L.R., (2007). The Inverse domination number of a graph, Feb' (2007).

[6] Fraisse, P., A note on distance dominating cycles. Discrete Math. 71 (1988), 89-92.

[7] Haynes, T.W., Hedetniemi, .S.T., and Slater, P.J., 1998. "Domination in Graphs: Advanced Topics", Marcel Dekker R.





CONFERENCE PAPER
National Conference dated 27-28 July 2017 on
Recent Advances in Graph Theory and its Applications (NCRAGTA2017)
Organized by Dept of Applied Mathematics
Sri Padmawati Mahila Vishvavidyalayam (Women's University) Tirupati, A.P., India

Nicole, “Title of paper with only first word capitalized,” J. Name Stand. Abbrev., in press.

[8] Haynes, T.W., Hedetniemi S.T., and Slater P.J., (1998). “Fundamentals of domination in graphs”, Marcel Dekker Inc. New York, U.S.A.

[9] Kulli, V.R., and Sigarkanti, S.C., (1991).Inverse dominating in graphs. National Academy Science Letters, 15.

[10] Kulli, V.R., (2012). “Advances in Domination Theory”, Vishwa International Publications, Gulbarga, India. Yorozu, M. Hirano, K.

[11] Lakshmi, A., and Ameenal Bibi, K., (2015). The Inverse Accurate domination in Graphs – Secreat Heart Journal of Science and Humanities, special vol. 6(2)-2015. pp. 144-155.

[12] Nordhaus, E.A., and Gaddam, J.W., (1956). On complementary graphs. Amer. Math. Monthly, Vol.63.pp.175-177.

[13] Ore, O., 1962. Theory of Graphs. American Mathematical Society colloq. Publ., Providence, R1, 38.

[14] Tamizh Chelvam, T., Asir, T., Grace Prema,G.S. “Inverse Domination in Graphs”, Lambert Academic Publishing, 2013.





CONFERENCE PAPER
National Conference dated 27-28 July 2017 on
Recent Advances in Graph Theory and its Applications (NCRAGTA2017)
Organized by Dept of Applied Mathematics
Sri Padmawati Mahila Vishvavidyalayam (Women’s University) Tirupati, A.P., India