



DESIGN OF OPTIMAL FRACTIONAL DELAY-IIR FILTER USING EVOLUTIONARY ALGORITHMS

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Abstract: This paper presents the optimal design of fractional delay-Infinite Impulse Response filter (IIR) using a meta-heuristic approach called Modified Cuckoo Search Algorithm (MCSA). The Fractional Delay (FD) filters are used to give fraction of delay to signal and the FD-IIR filters are being used in various applications of signal processing. Coefficients of optimized Fractional Delay-Infinite Impulse Filter (FD-IIR) is calculated using Modified Cuckoo Search Algorithm (MCSA) to match the response of fractional delay IIR filter with ideal response of the filter. Different heuristic optimization algorithms such as Cat, Bat, Genetic Algorithm (GA), Simulated Annealing (SA), Cuckoo Search Algorithm (CSA), Particle Swarm Optimization (PSO) etc. have been used to design optimal fractional delay-IIR filter. FD-IIR filter design is a multimodal design problem. Hence Meta-heuristic optimization algorithm is used in the paper. The proposed algorithm is a modification of cuckoo search algorithm and hence called Modified Cuckoo Search Algorithm. It is a meta-heuristic optimization technique based on population of birds and cuckoos behavior. It is simple and is a global optimization algorithm. The performance of MCSA is compared with genetic algorithm, particle swarm optimization and cuckoo search algorithm. It is found that MCSA provides better results than GA, PSO and CSA. The fitness function used to evaluate the algorithm is Weighted Least Square function. The simulation results show that proposed algorithm, MCSA has less absolute magnitude error. The statistical data analysis also shows that MCSA has higher value of percentage improvement in magnitude error and has faster convergence rate than GA, PSO and CSA in terms of execution time.

Keywords: FD-IIR filter, Modified Cuckoo Search Algorithm, Weighted Least Square, magnitude response

I. INTRODUCTION

Digital signal processing is very useful in the field of science and technology today. Digital signals are used to handle discrete time signals and filters are one of the important parts of this field. Filters are not only for frequency selective purpose but are also used in many applications that is as simple as from reducing ripples to higher sophisticated circuits like reduction in noise level, enhancement in video signal in astrological and biological systems and equalization in graphics [1]. Delay filters provide a fractional amount of delay to the signals where accuracy of the system is very important. Hence, nowadays, a principle field of fractional delay filters is in signal processing [2]. Two categories of digital filters are recursive and non-recursive. FIR filters are non-recursive filters whose output depends on present and past input values only whereas recursive are IIR filters whose output depend on past input and output values and thus provide better performance than non-recursive filters i.e. FIR filter [1]. Some of the limitations of gradient optimization are: continuous and differentiable fitness function is required and provide local minimum solution as it is centered on the sub-optimal solution [27]. Thus, design of IIR filters using optimization algorithms makes their design easier and simple. These algorithms are differential evolution (DE) [2], simulated annealing (SA) [3], genetic algorithm (GA) [4], ANT colony optimization [5], Gravitation search algorithm (GSA) [1], BAT algorithm [6], Firefly algorithm [7], Particle Swarm Optimization (PSO) [8],[9], CAT swarm optimization [10],[11], Cuckoo Search Algorithm (CSA) [12],[13],[14]. The proposed algorithm in the paper is a modification to cuckoo search algorithm which provide

global minimum solution and has better performance results than CSA. This paper is divided into four sections. First section describe introduction. Then section 2 describes about the FD-IIR filter and the proposed algorithm for the optimal design of filter. In section 3 discusses about the simulation results and statistical data and the comparison between the results is analyzed. Finally, section 4 concludes the paper.

II. METHODS

A. Design of Fractional delay IIR filter problem:

Digital fractional delay filter has ideal frequency response as given below [14]:

$$H_i(w, v) = e^{(-jwv)} \quad (1)$$

where w is digital frequency, $w \in [0, \pi]$ and v is fractional delay, $v \in [0, 1]$.

The discrete signal can be delayed and can be expressed mathematically as

$$y(n) = x(n - Z) \quad (2)$$

where $x(n)$ and $y(n)$ is input signal and output signal respectively and Z is a positive integer. If delay is not integer then it is written as

$$y(n) = x(n - (Z + p)) \quad (3)$$

Where Z is integral part of delay and p is fractional delay and its value ranges from 0 to 1, that is, $p \in [0, 1]$ [15].

The integral part of delay can be implemented as chain of Z unit delays but the fractional part of delay is to be subjected to approximation to control the delay value continuously whenever a fraction delay value is needed [16]. There are several techniques used to compute coefficients of fractional delay filter such as Lagrange interpolation [17], Farrow structures [18], minimax design method [19], and least

squares and weighted least squares [15]. To approximate the results of FD filter weighted least square (WLS) method is used as objective function in this paper. WLS is used to reduce the complexity of the design and it also uses different weighing function over the whole frequency band thus meeting the results of the requirement of the system whereas in least square method it assumes same importance all over the frequency range.

Two categories into which Fractional delay filters can be classified is: (i) fixed fractional delay filter in which delay p is fixed. In this the signal is delayed to some pre specified value and, (ii) variable fractional delay filter in which delay is variable [15], that is, in this the desired delay is introduced in the signal as parameter to the filter. The filter design problem is to approximate the desired filter response to the ideal filter response [23],[24],[25].

FD-IIR filter has transfer function as given below:

$$H(z, v) = \frac{\sum_{l=0}^M b_v(l)z^{-l}}{1 + \sum_{l=1}^N a_v(l)z^{-l}} \quad (4)$$

Where M and N are degrees of numerator and denominator, respectively and $N \geq M$ is the filter order. a_v and b_v are the real filter coefficients that need to be optimized [14].

FD-IIR filter has difference equation expressed as below:

$$y(n) + \sum_{l=1}^N a(l)y(n-l) = \sum_{l=0}^M b(l)x(n-l) \quad (5)$$

The frequency response of the filter is given by:

$$H(w, v) = \frac{\sum_{l=0}^M b_v(l)e^{-jwl}}{1 + \sum_{l=1}^N a_v(l)e^{-jwl}} \quad (6)$$

The error to be minimized is given by:

$$e(w, v) = H_i(w, v) - H(w, v) \quad (7)$$

where $H_i(w, v)$ and $H(w, v)$ is the ideal and approximated filter response, respectively. The weighted least square objective function is expressed as:

$$J(w) = \int_0^1 \int_0^\pi F(w, v) |e(w, v)|^2 dw dv \quad (8)$$

Where $F(w)$ is given by:

$$F(w) = \begin{cases} 0.0513 & \text{for } w \in [0, 0.7\pi] \\ 0.2564 & \text{for } w \in (0.7\pi, 0.855\pi] \\ 1 & \text{for } w \in (0.855\pi, \pi] \end{cases} \quad (9)$$

This fitness function is used to minimize the error given in equation (7) using Genetic algorithm, Particle Swarm optimization, Cuckoo Search Algorithm [26] and Modified Cuckoo Search algorithm, thus, computing the optimized filter coefficients and using it for design of FD-IIR filter.

B. Proposed Optimization algorithms:

Different optimization techniques are used for filter design problem. Some of them are classical algorithms which can be used for only simple design problems and some are heuristic and meta-heuristic optimization algorithm. Heuristic algorithms are problem specific that is they perform local search and results obtained are not guaranteed to be global solution. Some of these algorithms are gravitation search algorithm, genetic algorithm, orthogonal GA, simulated annealing, differential evolution and PSO whereas Meta-heuristic algorithms can be defined as one stage higher than heuristic algorithms and they can act as guiding approach for the heuristic optimization algorithms [12],[28]. Some of these are Tabu search [20], ant colony optimization [5], and artificial immune system [21]. These are meta-heuristic algorithms which are based on population, trajectory and stochastic features. Meta-heuristics has two important features: intensification and diversification. Intensification aims to search locally and

more intensively around the current best solution and find the best solution from the search whereas diversification determines the best solution by exploring the search space more efficiently and globally so as to get the best result [22]. Proposed algorithm that is used in this paper is described as below:

Modified Cuckoo Search Algorithm: The proposed algorithm is a modification to the well-developed algorithm that is cuckoo search. Two modifications are done to the present cuckoo search algorithm. One involves change in value of probability of discovering rate of alien eggs and second one is change in the value of step size in levy flight using Mantegna's algorithm [29]. This modification has shown improvement in performance than cuckoo search and also has shown better results when compared with genetic, particle swarm and cuckoo search algorithms. In cuckoo search algorithm the fraction of nest to be abandoned or the discovery rate of alien egg was kept 0.25 but it is found that by changing the value to 0.75 it gives better results over the cuckoo search algorithm in terms of optimized filter coefficients. Also, according to Mantegna's algorithm which has same behavior as Levy flight distribution, the distribution parameter (β) can be set in the range $\beta \in [0.3, 1.99]$. Thus value of β is set as one in the proposed algorithm as opposed to $3/2$ in the cuckoo search algorithm and it is found to give better results in terms of minimum magnitude error and faster execution. According to Mantegna's algorithm [29], the distribution is calculated using equation (10):

$$s = (\Gamma(1 + \beta) * \sin(\pi * \beta / 2)) / (\Gamma((1 + \beta) / 2) * \beta * 2^{((\beta - 1) / 2)})^{(1 / \beta)} \quad (10)$$

where β is distribution parameter. The step size is calculated using equation (11). Step value is multiplied with factor 0.01 considering $L/100$ to be step size of walk and so that the egg doesn't move out of search space. (m-best) factor remains unchanged when the best solution is obtained; otherwise the difference tells that solution obtained is not best.

$$\text{step size} = 0.01 * \text{step} * (\text{m-best}) \quad (11)$$

where m is given by

$$m = m + \text{step size} * \text{randn}(\text{size}(m)) \quad (12)$$

The algorithm provides efficient result when parameter $\beta = 1$ and it is observed that calculation becomes faster with an integer value. Also the execution time decreases when value of β increases from lower fraction values. Firstly, solution is calculated using random nest, then by using Mantegna's algorithm and both solutions are compared. Out of both functions whichever has lesser value that will be the new solution. The discovery rate of alien eggs is fixed to 0.75 and the best solution is kept. The flowchart for the modified cuckoo search algorithm is shown in Fig. 1.

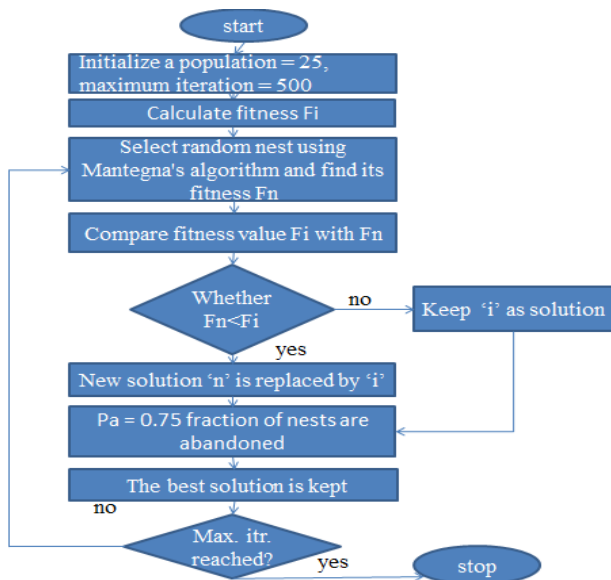


Fig. 1 Flowchart for Modified Cuckoo Search Algorithm

III. RESULTS

A. Study of magnitude feedback of FD-IIR filter

MATLAB is the software used to perform different works like simulations, analysis of data, development of algorithms, etc. All the simulations are experimented on MATLAB 9.0 version on Intel core 2 duo processor with 3 GB RAM. Performance comparison of GA, PSO, CSA and MCSA algorithms of fractional-delay IIR filter for different order is executed. Matlab simulations are performed to design optimized FD-IIR filter for order 2, 3, 4 and 5. The value of delay value is set at $v = 0.5$. Size of Population is $N = 25$, maximum iterations are set = 500 and the lower and upper bound is kept at $[-1, 1]$. The discovery rate of alien eggs is $pa = 0.75$ and distribution parameter $\beta = 1$. Graphs for magnitude response and error response for different orders are represented. Analysis of graphs depicts that MCSA has lesser error in magnitude, higher percentage improvement, and lesser execution time. The response using different algorithm is recorded in Fig. 2 - 5. Numerator and denominator coefficients with optimized value for the design of filter are calculated using GA, PSO, CSA and MCSA are given in Table I.

B. Analogy between different algorithms on the basis of magnitude response and their error:

Magnitude response of different algorithms for different order of filter is analyzed by simulations. The order of filter taken for inspection is 2, 3, 4 and 5. Comparison of Magnitude response of each order for different algorithms and their magnitude error is computed and plotted as shown in Fig. 2- Fig. 5. Analysis of graphs shows that MCSA has low magnitude error as compared to other optimization algorithms.

Table I. Optimized Filter Coefficients

Order of filter (N)	Algorithms used	Optimized coefficients of Numerator (b_v)	Optimized coefficients of Denominator (a_v)
2	GA	0.9999 0.9910 -	1.0000

		0.2898	0.9899	-0.2869
	PSO	-0.6293 0.6313	1.0000	0.5991
		0.9801	-0.6162	
	CSA	0.9840 0.4990	1.0000	0.4907
		0.6667	0.6583	
	MCSA	0.9830 0.4989	1.0000	0.4899
		0.6654	0.6578	
3	GA	0.9999 0.3343	1.0000	0.3778
		0.8505	0.7701	-0.9497
		-0.9856		
	PSO	0.5335 0.6690	1.0000	0.7094
		0.8068 0.7941	0.6596	0.4352
	CSA	-0.9999 0.7272	1.0000	-0.7280
		0.9999 1.0000	-0.9999	-0.9993
	MCSA	-0.9997 0.7289	1.0000	-0.7298
		0.9999 1.0000	-1.0000	-0.9994
4	GA	0.2250 0.8635	1.0000	0.6524
		0.6772 0.7843	0.9534	0.5206
		0.9152	0.3475	
	PSO	0.4501 -0.9203	1.0000	0.7810
		-0.5400 -	0.1204	-0.3173
		0.9011 0.3107	0.0138	
	CSA	0.5304 -0.7887	1.0000 -	0.2293
		0.2784 -0.5924	0.8726	0.1728
		-0.8245	-0.4174	
	MCSA	0.5310 -0.7890	1.0000	-0.2301
		.2790 -0.5930 -	0.8731	0.1732
		0.8251	-0.4189	
5	GA	0.7659 0.3099	0.1568	0.8720
		0.4199 0.6389	0.3759	0.4479
		0.7921 0.2618	0.3668	
	PSO	-1.0000 0 0 0	0 0	-
		1.0000 0.9897	1.0000	-0.9965
	CSA	0.1131 0.7849 -	-0.2911	-
		0.2681 0.9999 -	1	-
		0.0960	0.4851	-0.2399
		-0.6888	-0.1442	

	MCSA	0.1120	0.7835	-	1.0000	-
		0.2669	0.9999	-	0.2905	
		0.0945	-0.6875		1.0000	
					-0.4845	
					-0.2377	
				-0.1430		

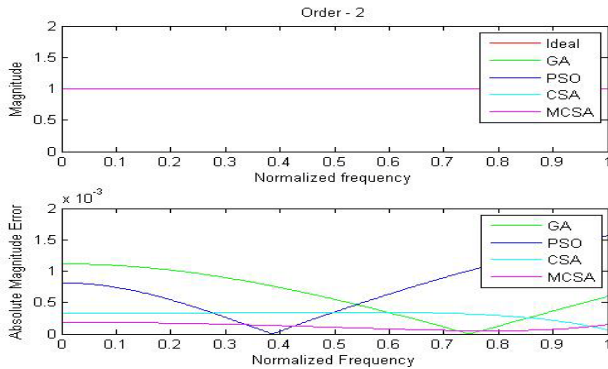


Fig 2: Magnitude response and error of GA, PSO, CSA and MCSA for 2nd order

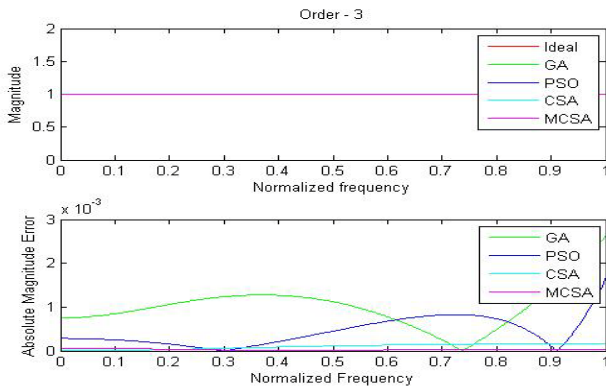


Fig 3: Magnitude response and error of GA, PSO, CSA and MCSA for 3rd order

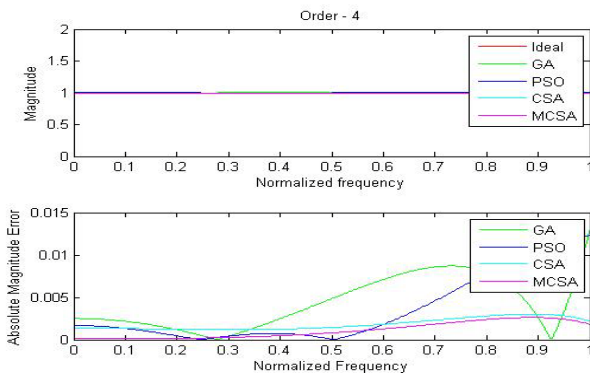


Fig 4: Magnitude response and error of GA, PSO, CSA and MCSA for 4th order

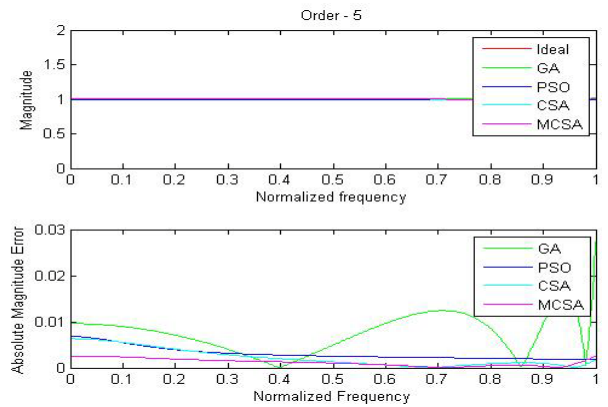


Fig 5: Magnitude response and error of GA, PSO, CSA and MCSA for 5th order

Table II. Analogy of absolute magnitude error

Order of filter	GA	PSO	CSA	MCSA
2	0.6876	.3574	.0986	.0125
3	0.3687	.1353	.0391	.0110
4	.1155	.0576	.0155	.0045
5	.0817	.0496	.0146	.0039

Absolute magnitude error achieved using GA, PSO, CSA and MCSA for 2nd, 3rd, 4th, 5th order of filter is shown on Table II. Absolute magnitude error obtained by applying MCSA for 2nd, 3rd, 4th, 5th order is 0.0125, 0.0110, 0.0045, and 0.0039, respectively. Thus, result shows MCSA has lesser magnitude error among all the orders as compared to GA, PSO and CSA.

C. Analysis of Statistical data

Analysis of Statistical data is done using different filter order and different optimization techniques to get the best optimal solution for filter design. Table III-X shows comparative study of different algorithms in terms of features like average value, standard deviation, and variance, error of magnitude with maximum and minimum value. From Table III it is seen that for 2nd order values of mean, standard deviation and variance that is, 0.0012, 6.0114×10^{-7} and 7.7533×10^{-4} , respectively are obtained using MCSA. Table IV shows maximum, minimum and average value of magnitude error for 2nd order filter which are 0.0051, 4.3213×10^{-6} and 0.0012, respectively computed using MCSA. On similar lines the statistical data and qualitative data for 3rd, 4th and 5th order is analyzed and recorded in Table V-X. It is found that MCSA has lower magnitude error value as compared to other algorithms

Table III. Analytical data for 2nd order

Algorithm used	Mean	Variance	Standard Deviation
GA	0.0075	1.7102×10^{-5}	0.0041
PSO	0.0031	1.8483×10^{-6}	0.0014

CSA	0.0022	3.5871×10^{-6}	0.0019
MCSA	0.0012	6.0114×10^{-7}	7.7533×10^{-4}

Table IV. Subjectively analyzed data for 2nd order

Algorithm used	Maximum magnitude error	Minimum magnitude error	Average value
GA	0.0273	2.8736×10^{-5}	0.0075
PSO	0.0068	1.828×10^{-5}	0.0031
CSA	0.0074	8.8102×10^{-6}	0.0022
MCSA	0.0051	4.3213×10^{-6}	0.0012

Table V. Analytical data for 3rd order

Algorithm used	Mean	Variance	Standard Deviation
GA	0.0043	8.9234×10^{-6}	0.0030
PSO	0.0035	1.5687×10^{-5}	0.0040
CSA	0.0018	4.3928×10^{-7}	6.6278×10^{-4}
MCSA	0.0011	8.4484×10^{-7}	9.1915×10^{-4}

Table VI. Subjectively analyzed data for 3rd order

Algorithm used	Maximum magnitude error	Minimum magnitude error	Average value
GA	0.0129	6.8252×10^{-6}	0.0043
PSO	0.0123	4.8603×10^{-6}	0.0035
CSA	0.0030	9.9845×10^{-7}	0.0018
MCSA	0.0026	1.4313×10^{-7}	0.0011

Table VII. Analytical data for 4th order

Algorithm m used	Mean	Variance	Standard Deviation
GA	0.0010	2.3513×10^{-7}	4.8490×10^{-4}
PSO	4.2741×10^{-4}	9.4355×10^{-8}	3.0717×10^{-4}
CSA	9.8188×10^{-5}	3.2429×10^{-9}	5.6946×10^{-5}
MCSA	2.9219×10^{-5}	2.2040×10^{-10}	1.4846×10^{-5}

Table VIII. Subjectively analyzed data for 4th order

Algorithm used	Maximum magnitude error	Minimum magnitude error	Average value
GA	0.0026	9.4858×10^{-6}	0.0010
PSO	0.0016	2.6810×10^{-6}	4.2741×10^{-4}
CSA	1.6282×10^{-4}	8.3674×10^{-7}	9.8188×10^{-5}
MCSA	5.7830×10^{-5}	5.2855×10^{-9}	2.9219×10^{-5}

Table IX. Analytical data for 5th order

Algorithm used	Mean	Variance	Standard Deviation
GA	6.0593×10^{-4}	1.2742×10^{-7}	3.5696×10^{-4}
PSO	7.1630×10^{-4}	1.7727×10^{-7}	4.2103×10^{-4}
CSA	3.0826×10^{-4}	3.7728×10^{-9}	6.1423×10^{-5}
MCSA	1.1724×10^{-4}	2.2646×10^{-9}	4.7588×10^{-5}

Table X. Subjectively analyzed data for 5th order

Algorithm used	Maximum magnitude error	Minimum magnitude error	Average value
GA	0.0011	2.3016×10^{-6}	6.0593×10^{-4}
PSO	0.0016	1.7861×10^{-6}	7.1630×10^{-4}
CSA	3.4797×10^{-4}	6.6511×10^{-7}	3.0826×10^{-4}
MCSA	1.8625×10^{-4}	5.0922×10^{-9}	1.1724×10^{-4}

Fig 6 shows comparison of absolute magnitude error among GA, PSO, CSA and MCSA for 2nd, 3rd, 4th and 5th order of filter.

D. Improvement in Percentage of magnitude error of MCSA

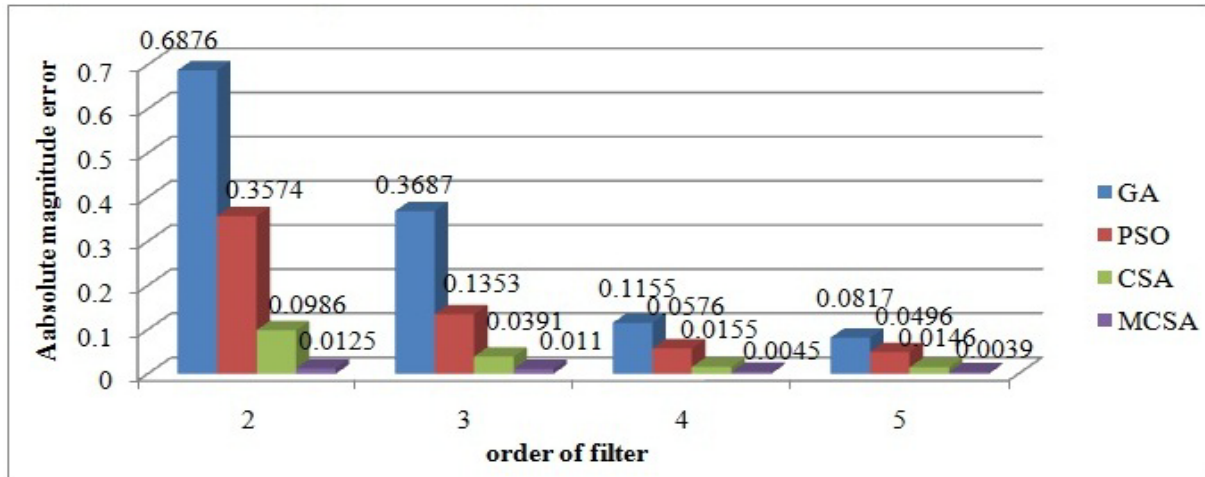


Fig 6: Absolute magnitude error of GA, PSO, CSA and MCSA for filter order 2nd, 3rd, 4th and 5th.

Table XI shows the comparison of improvement in percentage of magnitude error. It is seen that proposed algorithm shows 96.50%, 91.86%, 92.18%, 92.13% percentage improvement as compared to PSO for order of

filter 2nd, 3rd, 4th and 5th, respectively and 87.32%, 71.86%, 70.96%, 73.28% as compared to CSA for 2nd, 3rd, 4th, 5th order, respectively.

Table XI. Improvement in Percentage of magnitude error

Order of filter	PSO compared to GA	CSA compared to PSO	MCSA compared to PSO	MCSA compared to CSA
2	69.84	72.41	96.50	87.32
3	63.30	71.10	91.80	71.86
4	50.12	73.09	92.18	70.96
5	39.29	70.56	92.13	73.28

Fig. 7 shows improvement in percentage in magnitude error displaying comparison of PSO with GA, CSA with PSO, MCSA with PSO and MCSA with CSA using different filter

orders. The minimum value of absolute magnitude error using GA, PSO, CSA and MCSA for orders 2nd, 3rd, 4th and 5th of the filter is shown in Fig. 8.

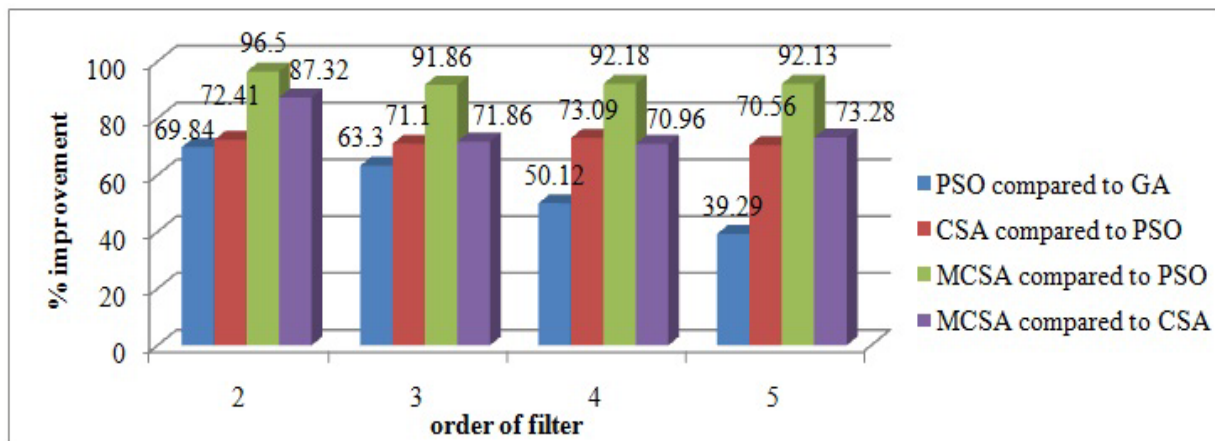


Fig 7: Improvement in Percentage in magnitude error of different algorithms with different filter order

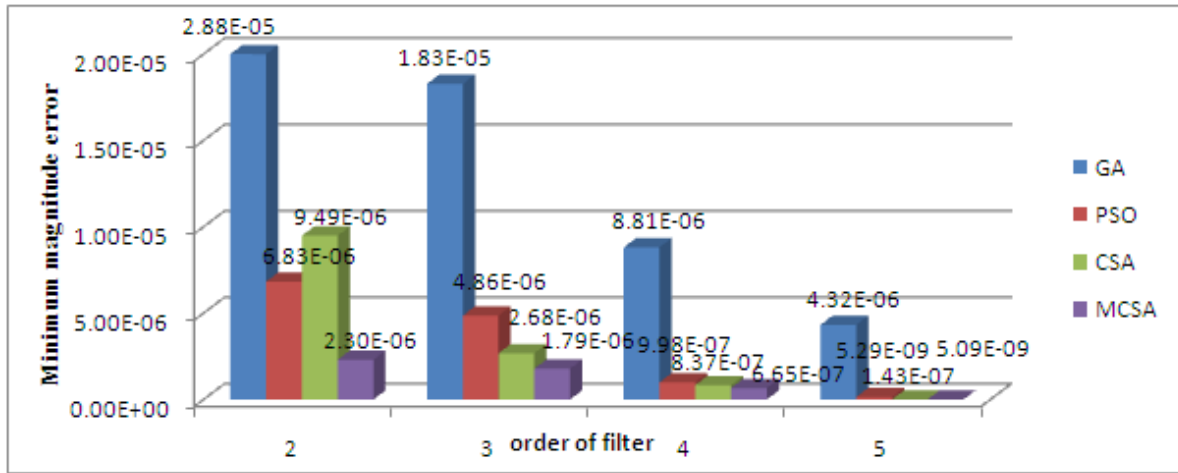


Fig 8: Minimum magnitude error using GA, PSO, CSA and MCSA for different order of filter.

Table XII outlines the percentage improvement in minimum magnitude error using MCSA when compared to PSO is 76.36%, 25.35%, 99.80%, 99.71% and 50.95%, 85.66%, 99.36% and 99.23% when MCSA is compared to CSA.

E. Comparison of execution time of GA, PSO, CSA and MCSA for different orders of filter

The simulations are performed on MATLAB. The main factors that affect the performance of the algorithm are maximum number of iterations, population size and order of the filter defined in the algorithms. Fig 9 shows comparison of execution

Table XII. Improvement in Percentage in Minimum magnitude error

Order of filter	PSO compared to GA	CSA compared to PSO	MCSA compared to PSO	MCSA compared to CSA
2	36.38	51.80	76.36	50.95
3	20.78	79.45	25.35	85.66
4	71.73	68.79	99.80	99.36
5	22.39	62.76	99.71	99.23

time required to get the minimum fitness value for the order 2nd, 3rd, 4th, 5th of FD-IIR filter using GA, PSO, CSA and MCSA. It is revealed that MCSA takes lesser execution time

than GA, PSO and CSA. Thus it can be seen that the results of MCSA are computed quickly than GA, PSO and CSA.

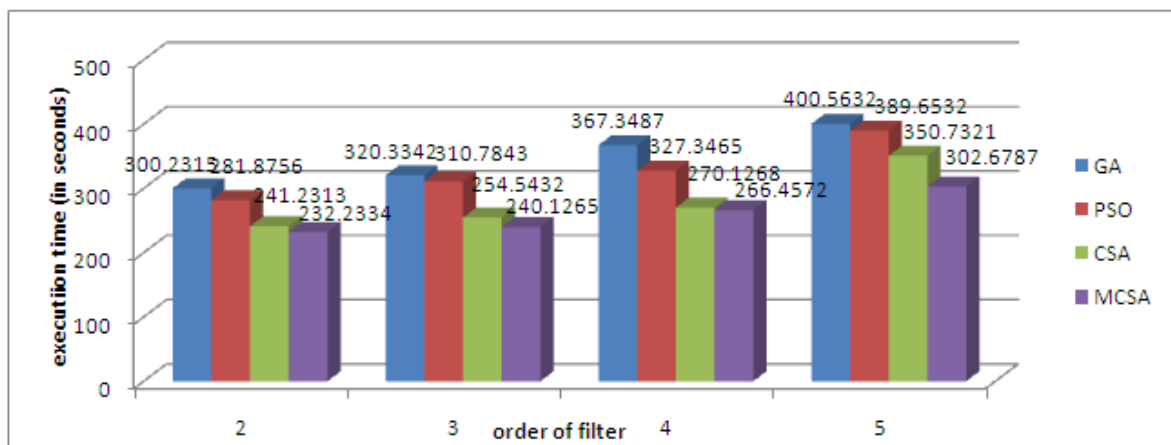


Fig 9: Execution time of GA, PSO, CSA and MCSA for 2nd, 3rd, 4th and 5th order of filter

IV. CONCLUSION

In this paper modification to the well-developed cuckoo search algorithm is applied to get the optimal filter coefficient. A number of optimization algorithms are available to find the optimal solution of fractional delay IIR filter design problem. But as the FD-IIR filter design problem is a multimodal design problem only a few of them can provide the best global solution. The magnitude response of FD-IIR filter approximates the ideal response and it is analyzed that Modified Cuckoo Search Algorithm is superior to cuckoo search algorithm, genetic algorithm and particle swarm optimization algorithm. The filter coefficients are obtained for the filter order 2nd, 3rd, 4th, 5th using GA, PSO, CSA and MCSA and it is found that MCSA provides optimal solution. As the filter design is a multimodal optimization problem so MCSA is suitable for the problem and better results are achieved using this algorithm. The Modified cuckoo search algorithm gives global optimal solution and is simpler and robust. Also the performance of Modified Cuckoo Search Algorithm becomes more effective when Mantegna's algorithm is used which has similar behavior as Levy flights in cuckoo search algorithm rather than using Gaussian or uniform distribution for each solution. Hence, Modified Cuckoo Search algorithm is proved to have given better results among all other algorithms.

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