



Geometric Knapsack to Facility Location: A Mapping

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Abstract: Knapsack problems have been extensively studied in operations research for last few decades. We review the method of mapping geometric knapsack problems into facility location problems. Then it is shown that a wide class of problems in geometric optimization and facility location can be represented as geometric knapsack problems.

Keywords: Geometric Knapsack Problem, Facility Location, Covering Location Problem, Enclosing Problem, Maximum Covering Location problem.

I. INTRODUCTION

Classical knapsack problems have been extensively studied in operations research [42] for last few decades. It has also attracted both theorist and practitioners in the field of algorithmic research in computer science communities. This name is derived from the maximization problem of best selection of essentials that can fit into a bag to be carried on a tour. Knapsack problem [42] is in the class of combinatorial optimization problems and one of its classical version can be defined as follows. We are given n items and a knapsack. Each item has weight w_i and W is the knapsack's capacity. If i -th object is placed into the knapsack then a profit p_i is earned. The objective is to fill the knapsack such that maximum profit is earned. Formally, the problem is formulated as:

$$\begin{aligned} & \text{maximize} && \sum_{i=1}^n p_i x_i \\ & \text{subject to} && \sum_{i=1}^n w_i x_i \leq W, \\ & && x_i = 0 \text{ or } 1, 1 \leq i \leq n \end{aligned}$$

This classical version is also called the 0-1 knapsack problem. In 0-1 knapsack problem, each item must either be chosen or not chosen. Moreover, it is not allowed to take a fractional amount of an item or take an item more than once. For the special case $w_i = p_i$, the objective is to find $S' \subseteq S$ from a set S of nonnegative integers such that it adds up to exactly W . This special case of 0-1 knapsack problem is known as Subset Sum problem [42]. Both subset sum problem and 0-1 knapsack problem are NP-hard [42]. In fractional knapsack problem [10], it is allowed to take a fraction of an item to fill the knapsack. Besides above two versions, Martello and Toth [42] also considered bounded and unbounded knapsack problem. The bounded knapsack problem restricts the number x_j of copies of each kind of

item to a maximum integer value b_j . For unbounded knapsack problem, there is no restriction on the number of copies of each kind of item. Since our area of interest is related to 0-1 knapsack problem so from now on, we will not consider fractional knapsack problem. Generally, a classical 0-1 knapsack problem is defined with one knapsack. This version is also known as 0-1 single knapsack problem. Sometimes, it is allowed to have more than one knapsack and problem is referred as 0-1 multiple knapsack problem. In this discussion, the term multiple knapsack problem means 0-1 multiple knapsack problem. Martello and Toth [42] define multiple knapsack problem in the following ways. Given a set of n items and m knapsacks ($m \leq n$) with the information of profit p_j and weight w_j of j -th item, and capacity c_i of i -th knapsack, the objective is to select m disjoint subsets of items so that the total profit of the selected items is maximized, and each subset can be assigned to a different knapsack whose capacity is no less than the total weight of the items in the subset. Formally,

$$\begin{aligned} & \text{maximize} && \sum_{i=1}^m \sum_{j=1}^n p_j x_{ij} \\ & \text{subject to} && \sum_{j=1}^n w_j x_{ij} \leq c_i, \quad i \in M = 1, 2, \dots, m \\ & && \sum_{i=1}^m x_{ij} \leq 1, \quad j \in N = 1, 2, \dots, n \\ & && x_i = 0 \text{ or } 1, \quad i \in M \text{ and } j \in N, \end{aligned}$$

where

$$x_{ij} = \begin{cases} 1 & j\text{-th item is assigned to } i\text{-th knapsack} \\ 0 & \text{otherwise.} \end{cases}$$

For $m = 1$, multiple knapsack problem reduces to 0-1 single knapsack problem. Different versions of a knapsack problem have been studied extensively in last few decades. In this discussion, we concentrate on various geometric

optimization problems related to a classical knapsack problem.

II. CLASSICAL KNAPSACK TO GEOMETRICAL KNAPSACK: A MAPPING

Arkin and et al. [3] mapped classical knapsack problems into a new class of *geometric knapsack problems*. For their [3] purposes, a knapsack is a simple closed curve, its *capacity* is its perimeter or area and item may be point, polygon, line segment etc. In classical knapsack problem, the selection of an item depends upon its weight and capacity. But in geometrical knapsack problem, the selection of an item depends not only upon its weight and capacity but also on the positions of other items. This geometric feature leads to a new direction in solving such problems with the tools in computational geometry.

In this survey, we study on some types of geometric multiple knapsack problem where the knapsack is a rectangular object, its capacity is its size, the item to be placed within knapsack is a point with arbitrary weight, the number of knapsack may be more than one but each of them have same capacity. There are many variations in this problem. For example, the rectangular objects may be axis parallel or are allowed to rotate but must remain parallel to each other or allowed to rotate independently.

In other variations, capacity of the knapsack can be defined by the area of the rectangular object instead of its size. This problem is somewhat more difficult as we can generate infinite number of rectangular objects of different sizes having same area. Here, it should be mentioned that positions of the items are fixed and recall that items are points in two dimensional plane with arbitrary weights. Our objective is to pack the knapsacks so that total weight of the points packed is maximized. In other words, find a placement of the knapsacks such that the total weight of the points covered or enclosed by the knapsacks is maximized. Therefore, some covering location problems in facility location [18], can be seen as geometrical multiple knapsack problem.

In the next section, we will briefly overview the existing literature on geometric multiple knapsack problem and some related problems which can be seen as geometric multiple knapsack problem.

A. First Work

Although a lot of works on geometric version of classical knapsack problem were done in last few decades, Arkin and et al. [3] first posed these problems with title geometric knapsack problem.

In particular, they considered the following *fence enclosure* problem. Given a set S of items in the plane, with the i -th item having a given value, v_i , fence enclosure problem wishes to construct one or more fences that encloses some or all of the items obeying capacity constraints and/or costs associated with the fences such that "net profit" is maximized. The net profit is defined [3] to be the sum of the values of the items enclosed *minus* the cost (if any) of the fence used. They [3] solved many different problems in this general class, depending on item type, values of v_i , length of the fence or area enclosed by the fence. For example, item may be point, v_i may be unrestricted in sign and upper

bound $L \leq +\infty$ of the length of fence is available. A pseudopolynomial-time algorithm was proposed to solve fence enclosure problem when each v_i has integral value. At the time of developing this pseudopolynomial-time algorithm, they proposed an efficient solution to find the smallest perimeter polygon that encloses k points from a set S of n points. Their solution requires $O(kn^3)$ time and $O(kn^2)$ space. This solution can also compute smallest area convex polygon enclosing k points within same time and space complexities. These k -enclosure problems can be seen as geometric single knapsack problem. For these k -enclosure problems, knapsack is a convex polygon, its capacity is defined by those k points and item is a point with unit weight. Recall that, in this case, the net profit is k *minus* the cost of enclosure. Therefore, to maximize the net profit, the knapsack of smallest parameter (respectively area) is required.

B. k -enclosure and Geometrical Knapsack: A Mapping

k -enclosure problems of many variations involving a point set $P = \{p_1, p_2, \dots, p_n\}$ have been extensively studied in computational geometry [8]. Efrat and et al. [24, 47] studied the problem of computing smallest k -enclosing circle and k -enclosing homothetic copy of a given convex polygon. Eppstein and Erickson [23] studied a number of extensions including finding subsets of size k from the given set P that minimizes area, perimeter, diameter, and circumradius. Problems of computing k -enclosing rectangles and squares are also studied [1, 19, 17, 23, 57] extensively. In contrast with minimum k -enclosing problem, Eppstein and et al. [25] studied the maximum area or perimeter enclosure whose vertices come from the given set S of n weighted points. For their problem, the enclosure is convex k -gon or empty convex k -gon or convex polygon that contains exactly k points from S . An $O(kn^3)$ time and $O(kn^2)$ space algorithm was proposed to solve these problems. Boyce and et al. [6] considered the problem of locating maximum perimeter/area k -gon whose vertices come from a given set S of n points and proposed an algorithm that runs in $O(kn + n \log n)$ time and $O(n)$ space.

C. Covering Location Problem and Geometrical Knapsack: A Mapping

In particular, for $k = n$, the motives of k -enclosure problem and covering location problem in facility location [18] are same. The set covering location problem (LSCP) [18] is an example of classical covering location problem [18] that can be viewed as enclosure problem. This problem computes the locations of least number of facilities such that each of demand node has at least one facility sited at a node within a specified maximum distance. Problems of computing smallest enclosing circle [51], triangle [12, 38, 50], square and rectangle [58] are well known. In facility location, the problem of finding smallest enclosing circle is

referred as 1-center problem [61]. 1-center problem and its different *constraint* versions are well studied [7, 32, 53, 39]. The problem of finding the smallest enclosing convex polygon is the famous convex hull problem. Another important variations of enclosure problem are Euclidean and Rectilinear p -center and p -piercing problems. Megiddo and Supowit [48] have shown that both the above problems are NP-complete. Each of the problem stated above is an optimization problem that covers a point set P by a geometrical object. All these k -enclosure problems are also known as *single k -enclosure* problem or *full covering* problem.

A lot of work has been done when *two* geometrical objects are used to cover a set P on n points in a plane. Considering geometrical objects as squares, the problem is called *square 2-center* problem [35]. In discrete version of this problem, the centers of the geometrical objects are points of P , whereas for non-discrete case, there is no restriction on the placement of geometrical objects. More results on related problems are available in [35, 36, 37, 4, 54, 55, 34]. Katz and et al. [35] studied discrete square 2-center problem with the area of the larger square is minimum for three cases. First they considered the squares as isothetic and computed them in $O(n \log^2 n)$ and $O(n)$ space. In case, squares are allowed to rotate but remain parallel, their algorithm to compute these two squares runs in $O(n^2 \log^4 n)$ time and uses $O(n^2)$ space. Finally, each square is allowed to rotate independently and proposed an algorithm that runs in $O(n^3 \log^2 n)$ time and $O(n^2)$ space.

Bespamyatnikh and Segal [9] solved the problem of covering a set S of n points in d -dimensional space, $d \geq 2$, by two axis-parallel boxes such that the measure of the largest box is minimized where measure is a monotone function of the box. They proposed a simple algorithm to find boxes that runs in $O(n \log n + n^{d-1})$ time and $O(n)$ space. Recently, Saha and Das [55] studied the problem of locating two parallel rectangles in arbitrary orientation to cover a set of n points in a two dimensional plane, such that area of the larger rectangle is minimum. They proposed an algorithm to solve the problem that runs in $O(n^3)$ time using $O(n^2)$ space.

If we ignore the cost of enclosing and consider the knapsack as a rectangular object of fixed size or a circular object with fixed radius then our objective is to fill the knapsack in such a way that total weight of the packed items is maximized. In other words, our objective is to locate knapsacks those maximize the sum of the weights of points enclosed. Therefore, maximum covering location problem [52] can also seen as geometrical multiple knapsack problem. For facility location problem having limited number facilities, maximum covering location model may not provide service to each client due to high recurring cost or installation cost that requires for large number of facilities.

D. MCLP and Geometrical Knapsack: A Mapping

Alternatively, limited number of facilities are installed to address maximum demand. In this formulation, the condition for addressing the total demand is relaxed and the objective is

to locate p facilities such that maximum demand can be addressed, for a given covering distance. Maximum Covering Location Problem (MCLP) was originally stated and solved by Churuch and ReVelle, [14]. They proposed three approaches to solve the problem; dubbed greedy adding, greedy adding with substitution and linear programming. Mauricio and Resende [45] also proposed a greedy randomized adaptive search procedure to facilitate maximum clients, though not necessarily optimum. Besides above methods, several heuristic approaches [49, 30, 2] were developed to solve maximum covering location problem.

The formulation for planar maximum covering problems, where facilities can be placed anywhere on the plane, have also been studied by several authors [33, 15, 11, 43, 46, 48, 44]. For the Euclidean distance measure, candidate points would be the points of intersection of circles drawn around the demand points. Similarly, for rectilinear distances, the candidate facility locations would be the points of intersection of diamond shaped boundaries around demand points [11]. In [46], Maherez and et al. developed an algorithm for a facility that is "somewhat desirable" and named it "maximin-minimax" facility location problem. Their method computes the set of intersection points of any two lines forming the equi-rectilinear distances from the demand points. The techniques used in [11, 43, 48, 46] and in the standard location problem models discussed in books on location theory [27, 28, 40] are based on equidistance shapes. Ventura and Dung [59] studied parts inspection with rectangular and square shapes. Their technique used a Euclidean least-square methods to determine the optimal parameters of the straight lines defining the edged of the part being inspected.

A closely related problem is to locate one or more convex objects to maximize the size k ($\leq |P|$) of the subset covered. These so called problems of maximum covering by convex objects (squares, rectangles, parallelograms, convex polygon, circles) have also received attention of many researchers. Korte and Lovasz [41] studied the following problem. Given a set of n points with arbitrary weight and a polygon of fixed size, find a placement of the polygon that maximizes the sum of the weights of enclosed points. They proposed a dynamic programming algorithm that runs in $O(n^5)$ time. Barequet and et al. [5] studied convex polygon translation to maximize point containment and proposed an algorithm to cover maximum number of points (i.e k) from a planar point set P by a given convex polygon with m vertices in $O(nk \log(mk) + m)$ time using $O(m + n)$ space. The work of Barequet and et al. [5] was extended by Dickerson and Scharstein [21] for the case of both translation and rotation of polygon shapes. In both [5] and [21] only one polygon shape was considered for maximum containment of points. The complexity of covering in the plane by squares or rectangles was discussed by Fowler and et al. [26]. Younies and Wesolowsky [60] introduced a zero-one mixed integer formulation for maximum covering problem where point set was covered by inclined parallelograms in a plane. Alternatively, Drezner and Wesolowsky [20] solved for the minimum weight containment in a circle or rectangle. They introduced a mixed integer formulation to solve for the case of axis parallel rectangle. Katz and et al. [37] also studied the

minsum coverage problem to place undesirable facility within an isothetic rectangle of fixed size. They proposed an algorithm that runs in $O(n \log n)$ time and $O(n)$ space. In the context of bichromatic planar point set, Diaz-Banez and et al. [22] proposed algorithms for maximal covering by two disjoint isothetic unit squares and circles in $O(n^2)$ and $O(n^3 \log n)$ time respectively. They later improved the complexities to $O(n \log n)$ and $O(n^{8/3} \log^2 n)$ time respectively [16]. Some more results on related problems are available in Hale and Moberg [31], Serra and Marianov [56], Galvao [29] and Chung [13].

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