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Thermal Stresses of a Thick Annular Disc due to Heat Generation by Integral Transform Method

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Abstract: In this paper, an attempt has been made to study thermoelastic response of a thick circular plate occupying the space $D: a \le r \le b,\ -h \le z \le h,$ due to heat generation with radiation type boundary conditions. Here we apply integral transform techniques to find the thermoelastic solution.

Keywords: Thermoelastic problem, annular disc, Thermal Stresses, integral transform method

INTRODUCTION

Nowacki [1] discussed the state of stress in thick circular plate due to temperature field. Noda et al. [2] published a book on Thermal Stresses, second edition. Ghume et al. [3] determined Thermoelastic solution of a thin circular plate due to partially distributed heat supply. Khobragade [4] discussed Thermoelastic analysis of a thick circular plate and Khobragade [5] studied Thermal stresses of a thin circular plate with internal heat source. Pathak et al. [6] studied Transient Thermo elastic Problem of a Circular Plate with Heat Generation. Hamna Parveen et al. [7] discussed thermal stresses of a circular disk with internal heat sources. Gahane et al. [8] studied thermal stresses in a thick circular plate with internal heat sources. Navneet Kumar et al. [9] studied Thermal deflection of a thin circular plate with radiation. Hamna Parveen et al. [10] analysed thermal stresses of a thick circular plate due to heat generation. Lamba et al. [11] studied analytical thermal stress analysis in a thin circular plate due to diametrical compression. Varghese et al. [12] found alternative solution of a transient heat conduction in a circular plate with radiation. Dange et al. [13] discussed deflection of isosceles triangular plate under unsteady temperature distribution. Love [14] published a book on Treatise on the Mathematical Theory of Elasticity.

This paper is concerned with transient thermoelastic problem of a thick annular disc occupying the space $D: a \le r \le b,\ -h \le z \le h,$ due to heat generation with radiation type boundary conditions.

STATEMENT OF THE PROBLEM

Consider thick circular plate of thickness $2h$ occupying the space $D: a \le r \le b,\ -h \le z \le h,$ the material is homogenous and isotropic. The differential equation governing the displacement potential function $\phi(r,z,t)$ as Nowacki [1] is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left(\frac{1+\nu}{1-\nu}\right) a_t T \tag{1}$$

where ν and a_t are Poisson's ratio and linear coefficient of thermal expansion of the material of the plate and T is the temperature of the plate satisfying the differential equation as Noda [2] is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \chi(r,z,t) = \frac{1}{k}\frac{\partial T}{\partial t} \tag{2}$$

Subject to initial condition

$$M_t(T,1,0,0) = F(r,z) \quad a \le r \le b, -h \le z \le h. \tag{3}$$

The boundary conditions are

$$\left.\begin{aligned} M_r(T,1,k_1,a) &= 0 \quad -h \le z \le h,\ t>0 \\ M_r(T,1,k_2,b) &= 0 \quad -h \le z \le h,\ t>0 \end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned} M_z(T,1,k_3,h) &= f_1(r,t) \quad a \le r \le b,\ t>0 \\ M_z(T,1,k_4,-h) &= f_2(r,t) \quad a \le r \le b,\ t>0 \end{aligned}\right\} \tag{5}$$

where k is thermal diffusivity of material of the plate.

The displacement function in the cylindrical coordinate system are represented by Love's function as Khobragade [3] are

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 L}{\partial r \partial z} \tag{8}$$

$$u_z = \frac{\partial \phi}{\partial z} + 2(1-\nu)\nabla^2 L - \frac{\partial^2 L}{\partial z^2} \tag{9}$$

The Love's function [14] must satisfy

$$\nabla^2 \nabla^2 L = 0 \tag{10}$$

Where $\nabla^2 = \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{\partial^2}{\partial z^2}$

The component of stresses are represented by the thermoelastic displacement potential ϕ and Love's function L as Noda et al.[2] are

$$\sigma_{rr} = 2G\left\{\left(\frac{\partial^2 \phi}{\partial r^2} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\nu \nabla^2 L - \frac{\partial^2 L}{\partial r^2}\right)\right\} \tag{11}$$

$$\sigma_{\theta\theta} = 2G\left\{\left(\frac{1}{r}\frac{\partial \phi}{\partial r} - \nabla^2 \phi\right) + \frac{\partial}{\partial z}\left(\nu \nabla^2 L - \frac{1}{r}\frac{\partial^2 L}{\partial r^2}\right)\right\} \tag{12}$$

$$\sigma_{zz} = 2G\left\{\left(\frac{\partial^2\phi}{\partial z^2} - \nabla^2\phi\right) + \frac{\partial}{\partial z}\left\{\left((2-\nu)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\}\right\} \tag{13}$$

$$\sigma_{rz} = 2G\left\{\frac{\partial^2\phi}{\partial r\partial z} + \frac{\partial}{\partial r}\left\{\left((1-\nu)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right)\right\}\right\} \tag{14}$$

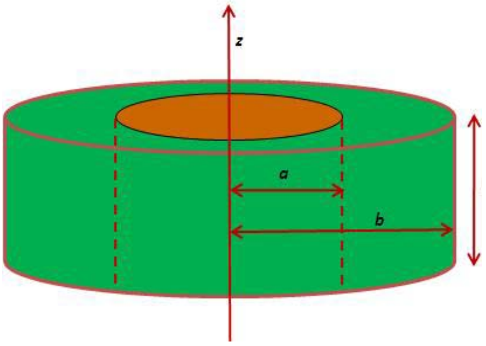

Fig. 1: Shows the geometry of the problem

For traction free surface stress function

$\sigma_{\theta z} = \sigma_{r\theta} = 0$ at $z = \pm h$ for thick annular disc.

Equations (1) to (14) constitute the mathematical formulation of the problem under consideration.

SOLUTION OF THE PROBLEM

Applying Marchi-Zgrablich transform to the equation (2), we get

$$-\mu_m^2\overline{T}(\mu_m,z,t) + \frac{d^2\overline{T}}{dz^2}(\mu_m,z,t) + \overline{\chi}(\mu_m,z,t) = \frac{1}{k}\frac{d\overline{T}}{dt} \tag{15}$$

Again applying Marchi-Fasulo transform to above equation, we obtain

$$\frac{d\overline{T}^*}{dt} + kp^2\overline{T}^* = \Psi \tag{16}$$

Where

$$p^2 = \mu_m^2 + \lambda_n^2 \qquad \& \qquad \Psi = \frac{P_n(h)}{k_3}\overline{f}_1 - \frac{P_n(-h)}{k_4}\overline{f}_2 + \overline{x}^*$$

Equation (16) is a linear equation whose solution is given by

$$\overline{T}^*(m,n,t) = e^{-kp^2t}\int_0^t \Psi e^{kp^2t'}dt' + Ce^{-kp^2t} \tag{17}$$

Thus we have

$$\overline{T}^*(m,n,t) = e^{-kp^2t}\left[\int_0^t \Psi e^{kp^2t'}dt' + \overline{F}^*(m,n)\right] \tag{18}$$

Applying inversion of Marchi-Fasulo transform to the differential equation (18), we get

$$\overline{T}(m,z,t) = \sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n} \times e^{-kp^2t}\left[\int_0^t \Psi e^{kp^2t'}dt' + \overline{F}^*(m,n)\right] \tag{19}$$

Applying inversion of Marchi-Zgrablich transform to the differential equation (19), we get

$$T(r,z,t) = \sum_{m,n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_m r)}{\mu_m}\frac{P_n(z)}{\lambda_n} \times\left[e^{-kp^2t}\int_0^t \Psi e^{kp^2t'}dt' + \overline{F}^*(m,n)\right] \tag{20}$$

This is the desired solution of the given problem.

$$\phi(r,z,t) = \left(\frac{1+\nu}{1+\nu}\right)a_t\sum_{m,n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_m r)}{\mu_m}\frac{P_n(z)}{\lambda_n} \times e^{-kp^2t}\left[\int_0^t \Psi e^{kp^2t'}dt' + \overline{F}^*(m,n)\right]$$

$$L = \sum_{m,n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_m r)}{\mu_m}\frac{P_n(z)}{\lambda_n}$$

DETERMINATION OF DISPLACEMENT FUNCTION

Substituting equations (25) and (26) in equation (6), (7) we get

$$u_r = \left(\frac{1+\nu}{1+\nu}\right)a_t\sum\frac{S'_0(k_1,k_2,\mu_m r)P'_n(z)}{\lambda_n}e^{-kp^2t}\left[\int_0^t \Psi e^{kp^2t'}dt' + \overline{F}^*(m,n)\right] - \sum\frac{S'_0(k_1,k_2,\mu_m r)P'_n(z)}{\lambda_n} \tag{27}$$

$$u_z = \left(\frac{1+\nu}{1+\nu}\right)a_t\sum_{m,n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_m r)P'_n(z)}{\lambda_m\lambda_n}e^{-kp^2t}\left[\int_0^t \Psi e^{kp^2t'}dt' + \overline{F}^*(m,n)\right]$$
$$+2(1-\nu)\sum_{m,n=1}^{\infty}\left[\frac{\mu_m S_0''(k_1,k_2,\mu_m r)P_n(z)}{\lambda_n} + \frac{1}{r}\frac{S_0'(k_1,k_2,\mu_m r)P_n(z)}{\lambda_n}\right.$$
$$\left.\frac{S_0(k_1,k_2,\mu_m r)P_n''(z)}{\mu_m\lambda_n}\right] - \sum_{m,n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_m r)P_n''(z)}{\mu_m\lambda_n} \tag{28}$$

Substituting equations (25) and (26) in equations (9) to (12), we obtain

$$\sigma_{rr} = 2G\left\{\left(\frac{1+\nu}{1-\nu}\right)a_t\sum_{m,n=1}^{\infty}\left[-\frac{1}{r}S_0'(k_1,k_2,\mu_m r)P_n(z) - \frac{S_0(k_1,k_2,\mu_m r)P''_n(z)}{\mu_m}\right]\right.$$
$$\frac{e^{-kp^2t}}{\lambda_n}\left[\int_0^t \Psi e^{kp^2t'}dt' + \overline{F}^*(m,n)\right] +$$
$$\sum_{m,n=1}^{\infty}\left[\frac{\nu\,\mu_m S''_0(k_1,k_2,\mu_m r)P'_n(z)}{\lambda_n} + \frac{1}{r}\frac{S'_0(k_1,k_2,\mu_m r)P'_n(z)}{\lambda_n}\right.$$
$$\left.\left.+\frac{S_0(k_1,k_2,\mu_m r)P_n''(z)}{\mu_m\lambda_n} - \frac{\mu_m S''_0(k_1,k_2,\mu_m r)P'_n(z)}{\lambda_n}\right]\right\} \tag{29}$$

$$\sigma_{\theta\theta} = 2G\left\{\left(\frac{1+\nu}{1+\nu}\right)a_t\sum_m\sum_n\left[\frac{1}{r}S_0'(k_1,k_2,\mu_m r)P_n(z)\right.\right.$$
$$-\mu_m S_0''(k_1,k_2,\mu_m r)P_n(z) - \frac{1}{r}S_0'(k_1,k_2,\mu_m r)P_n(z)$$
$$\left.-\frac{S_0(k_1,k_2,\mu_m r)}{\lambda_n\mu_m}P_n''(z)\right]\Omega$$
$$+(2-\nu)\left[\sum_{m,n=1}^{\infty}\frac{S''_0(k_1,k_2,\mu_m r)P'_n(z)}{\lambda_n} + \sum_{m,n=1}^{\infty}\frac{1}{r}\frac{S'_0(k_1,k_2,\mu_m r)P'_n(z)}{\lambda_n}\right.$$
$$\left.\left.+\frac{S_0(k_1,k_2,\mu_m r)P_n''(z)}{\mu_m} - \frac{S_0(k_1,k_2,\mu_m r)P_n'''(z)}{\lambda_n}\right]\right\}$$

$$\sigma_{zz} = 2G\left\{\left(\frac{1+\nu}{1+\nu}\right)a_t\sum_m\sum_n\left[\frac{S_0(k_1,k_2,\mu_m r)}{\lambda_n\mu_m}P_n''(z)\right.\right.$$
$$-\mu_m S_0''(k_1,k_2,\mu_m r)P_n(z) - \frac{1}{r}S_0'(k_1,k_2,\mu_m r)P_n(z)$$

$$-\frac{S_0}{\mu_m}P_n''\Bigg]\frac{e^{-kp^2t}}{\lambda_n}\left[\int_0^t \Psi\, e^{kp^2t'}dt' + \overline{F}^*(m,n)\right]$$

$$+\nu\left[\sum_{m,n=1}^{\infty}\frac{\mu_m S''_0(k_1,k_2,\mu_m r)P_n'(z)}{\lambda_n}+\sum_{m,n=1}^{\infty}\frac{1}{r}\frac{S'_0(k_1,k_2,\mu_m r)P_n'(z)}{\lambda_n}\right.$$

$$\left.+\frac{S_0(k_1,k_2,\mu_m r)P_n''(z)}{\mu_m}-\frac{1}{r}\frac{\mu_m S''_0(k_1,k_2,\mu_m r)P_n'(z)}{\lambda_n}\right] \tag{30}$$

$$\sigma_{rz} = 2G\left\{\left(\frac{1+\nu}{1+\nu}\right)a_t\sum_m\sum_n\left[\frac{S_0'(k_1,k_2,\mu_m r)}{\lambda_n}P_n'(z)\Omega\right.\right.$$

$$+\left[\sum_{m,n=1}^{\infty}(1-\nu)\frac{\mu_m{}^2 S_0'''(k_1,k_2,\mu_m r)P_n(z)}{\lambda_n}+\sum_{m,n=1}^{\infty}\frac{1}{r}\frac{\mu_m S_0''(k_1,k_2,\mu_m r)P_n(z)}{\lambda_n}\right.$$

$$-\frac{1}{r^2}\frac{S_0'(k_1,k_2,\mu_m r)P_n(z)}{\lambda_n}+\frac{S_0'(k_1,k_2,\mu_m r)P_n''(z)}{\lambda_n}$$

$$\left.\left.\left.-\frac{S_0'(k_1,k_2,\mu_m r)P_n''(z)}{\lambda_n}\right]\right]\right\} \tag{31}$$

where

$$A=\left(\frac{1+\nu}{1-\nu)}\right)\frac{2\alpha_t}{a^2},\ \Omega = e^{-kp^2t}\left[\int_0^t \Psi e^{-kp^2t^1}dt^1+\overline{F}^*(m,n)\right],$$

$$B(t)=\int\Omega\, dt \tag{32}$$

SPECIAL CASE

Set $F(r,z)=z^2\delta(r^2-r_0{}^2)$ (33)

Applying Marchi-Fasulo transform, are obtain

$$\overline{F}(r,n)=\delta(r^2-r_0{}^2)\int_{-h}^{h} z^2 P_n(z)dz$$

$$\overline{F}(r,n)=\delta(r^2-r_0{}^2)\Phi_n\left[\frac{2h^2\sin(a_n h)}{a_n}+\frac{4h\cos(a_n h)}{a_n{}^2}-\frac{4\sin(a_n h)}{a_n{}^3}\right] \tag{34}$$

Where

$P_n(z)=Q_n\cos(a_n z)-W_n\sin(a_n z)$,

$Q_n=a_n(\alpha_1+\alpha_2)\cos(a_n h)+(\beta_1-\beta_2)\sin(a_n h)$

$W_n=(\beta_1-\beta_2)\cos(a_n h)+a_n(\alpha_1-\alpha_2)\sin(a_n h)$

Again on applying Marchi-Zgrablich transform, we obtain

$$\overline{F}^*(m,n)=\Pi_n\left[r_0 S_0(k_1,k_2,\mu_n r_0)\right] \tag{35}$$

Where

$$\Pi_n=\Phi_n\left[\frac{2h^2\sin(a_n h)}{a_n}+\frac{4h\cos(a_n h)}{a_n{}^2}-\frac{4\sin(a_n h)}{a_n{}^3}\right]\quad \text{And}$$

$\Phi_n=a_n(\alpha_1+\alpha_2)\cos(a_n h)+(\beta_1-\beta_2)\sin(a_n h).$

Using equation (27) in equation (17), one obtains

$$T(r,z,t)=\sum_{m,n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_m r)}{\mu_m}\frac{P_n(z)}{\lambda_n}$$

$$\times\left[e^{-kp^2t}\int_0^t\Psi\, e^{kp^2t'}dt'+\Pi_n\left[r_0 S_0(k_1,k_2,\mu_m r_0)\right]\right] \tag{36}$$

NUMERICAL RESULTS

Set

$a=2, k=15.9\times10^6, t=1$ second in equation (36), we get

$$T(r,z,t)=\sum_{m,n=1}^{\infty}\frac{S_0(0.25,0.25,\mu_m r)}{\mu_m}\frac{P_n(z)}{\lambda_n}$$

$$\times\left[e^{-0.86p^2}\int_0^1\Psi\, e^{kp^2t'}dt'+\Pi_n\left[0.75S_0(0.25,0.25,0.75\mu_m)\right]\right] \tag{37}$$

CONCLUSION

In this article, the temperature distribution, displacement and thermal stresses of a thick circular plate are investigated with known boundary conditions. Finite integral transform techniques are used to obtain numerical results. The results are obtained in terms of Bessel's function in the form of infinite series and depicted graphically.

Any particular cases of special interest can be assigned to the parameters and functions in expressions. The results that are obtained can be useful to the design of structure or machines in engineering applications.

$T(r,z,t)/\alpha$

Graph 1

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