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FIXED POINT RESULTS IN RANDOM UNIFORM SPACE

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Abstract : In this paper we prove some random common fixed point theorems for pair of weakly commutative mapping and semi compatible mappings with rational expression and the notation of E-distance in random uniform space.
MATHEMATICAL CLASSIFICATION(2000) – 49H10, 54H25.

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1 INTRODUCTION AND PRELIMINARIES

Aamri and Moutawakil [1] introduce the concept of A-distance and E-distance in uniform space. Cho. Et. al. [2] introduced the notion of semi compatible maps in a d- topological space and Jungck & Rhoades [3] termed a pair of self maps to be coincidentally commuting or equivalently weak compatible if they commute at their coincidence points. Followed by the latest work of V.B. Dhagat [4] extended the results for weakly compatible. Here we have considered random uniform space with random uniformity. With the help of these A-distance and E-distance we prove common fixed point theorems for weak commutative mappings and semi compatible mappings.

Definition 1.1. Let $(\Omega, \sum)$be a measurable space with a sigma algebra of subsets of Ω and M a non-empty subset of a metric space $X = (X, d)$. Let 2^M be the family of all non-empty subset of M and $C(M)$ the family of non-empty closed subsets of M. A mapping $G: \Omega \to 2^M$ is called measurable if, for each open subset U of M,

$$G^{-1}(U) \in \sum, \text{ where } G^{-1}(U) = \{\omega \in \Omega : G(\omega) \cap U \neq \emptyset\}.$$

Definition 1.2. A mapping $\xi : \Omega \to M$ is called a measurable selector of a measurable mapping $G : \Omega \to 2^M$ if ξ is measurable and $\xi(\omega) \in G(\omega)$ for each $\omega \in \Omega$.

Definition 1.3. A mapping $T : \Omega \times M \to X$ is said to be a random operator if , for each fixed $x \in M$, $T(.,x) : \Omega \to X$ is measurable.

Definition 1.4. A measurable mapping $\xi: \Omega \to M$ is a random fixed point of random operator $T : \Omega \times M \to X$ if $\xi(\omega) \in (\xi, T(\omega))$ for each $\omega \in \Omega$.

Random Uniform space $(X, \vartheta(\omega))$ a non-empty set X endowed of random uniformity $\vartheta(\omega)$, the latter being a special kind of filter on $X \times X$, all whose elements contain the random diagonal $\Delta(\omega) = \{(x(\omega), x(\omega))/x(\omega) \in X\}$, If $V(\omega) \in \vartheta(\omega)$ and $(x(\omega), y(\omega)) \in V(\omega)$, $x(\omega)$ and $y(\omega)$ are said to be $V(\omega)$-close and a sequence $(x^n(\omega))$ in X is a Cauchy sequence for $\vartheta(\omega)$ if for any $V(\omega) \in \vartheta(\omega)$, there exists $N \geq 1$ such that $x^n(\omega)$ and $x^m(\omega)$ are $V(\omega)$-close for $n, m \geq N$. An uniformly $\vartheta(\omega)$ defines a unique topology $T(\vartheta(\omega))$ on $X \times \Omega$ for which the neighborhoods of $x(\omega) \in X \times \Omega$ are the sets $V(x(\omega)) = \{y(\omega) \in X \times \Omega/(x(\omega), y(\omega)) \in V(\omega)\}$ when $V(\omega)$ runs over $\vartheta(\omega)$ for each ω. A uniform space $(X \times \Omega, \vartheta(\omega))$ is said to be Hausdorff if and only if the intersection of all the $V(\omega) \in \vartheta(\omega)$ reduces to the diagonal $\Delta(\omega)$ of $X \times \Omega$ i.e., if $(x(\omega), y(\omega)) \in V(\omega)$ for all $V(\omega) \in \vartheta(\omega)$ for each ω implies $x(\omega) = y(\omega)$. This guarantees the uniqueness of limits of sequences. $V(\omega) \in \vartheta(\omega)$ is said to be symmetrical if $V(\omega) = V^{-1}(\omega) = \{(y(\omega), x(\omega))/(x(\omega), y(\omega)) \in W(\omega)$ then $x(\omega)$ and $y(\omega)$ are both $W(\omega)$ and $V(\omega)$-close, then for our purpose, we assume that each $V(\omega) \in \vartheta(\omega)$ is symmetrical. When topological concepts are mentioned in the context of a uniform space $(X, (\omega), \vartheta(\omega))$, they always refer to the topological space $(X, (\omega), T(\vartheta(\omega)))$.

Definition 1.5. Let $(X \times \Omega, \vartheta(\omega))$ be a random uniform space. A function $p_\omega: (X \times \Omega) \times (X \times \Omega) \to R^+$ is said to be an A-distance if for any $V(\omega) \in \vartheta(\omega)$ there exists $\delta_\omega > 0$ such that if $p(z(\omega), x(\omega) \leq \delta_\omega$ and $p(z(\omega), y(\omega) \leq \delta_\omega$ for some $z(\omega) \in X \times \Omega$, then $(x(\omega), y(\omega)) \in V(\omega)$.

Definition 1.6. Let $(X \times \Omega, \vartheta(\omega))$ be a random uniform space. A function $p_\omega: (X \times \Omega) \times (X \times \Omega) \to R^+$ is said to be an $E - distance$ if

(p_1) p_ω is an $A - distance$,

(p_2) $p_\omega(x(\omega), y(\omega)) \leq p_\omega(x(\omega), z(\omega)) + p_\omega(z(\omega), y(\omega)), \forall\, x(\omega), y(\omega), z(\omega) \in X \times \Omega$

Definition 1.7. Let Let $(X \times \Omega, \vartheta(\omega))$ be random uniform space and p_ω be $A - distance$ on $X \times \Omega$.

(I) X is S^* complete if for every p_ω-Cauchy sequence $\{x_n(\omega)\}$, there exists $x(\omega)$ in $X \times \Omega$ such that

$$\lim_{n\to\infty} p_\omega(x_n(\omega)x(\omega)) = 0.$$

(II) X is p_ω-Cauchy complete if for every p_ω-Cauchy sequence $\{x_n(\omega)\}$, there exists $x(\omega)$ in X such that $\lim_{n\to\infty} x_n(\omega) = x(\omega)$ with respect to $T(\vartheta(\omega))$.

(III) $f: X \to X$ is $p - continuous$ if

$$\lim_{n\to\infty} p_\omega(x_n, x) = 0 \Longrightarrow \lim_{n\to\infty} p_\omega(f\left(x_n(\omega), f(x(\omega))\right) = 0.$$

(IV) $f: X \to X$ is $T(\vartheta(\omega))$-continuous if $\lim_{n\to\infty} x_n(\omega) = x(\omega)$ with respect to $T(\vartheta(\omega))$ implies $\lim_{n\to\infty} f(x_n(\omega)) = f(x(\omega))$.with respect to $T(\vartheta(\omega))$.

(V) $X \times \Omega$ is said to be p-bounded if

$\delta p_\omega(X \times \Omega) = \sup\{p_\omega(x(\omega), y(\omega))/x(\omega), y(\omega) \in X \times \Omega\} < \infty$.

Definition 1.8: Let $(X \times \Omega, \vartheta(\omega))$ be random uniform space and p_ω be an $E - distance$ on X. Two self maps S and T on are said to be semi compatible if

$$\lim_{n\to\infty} p_\omega(STx_n(\omega), Tx(\omega)) = 0,$$

Whenever

$\lim_{n\to\infty} Sx_n(\omega) = \lim_{n\to\infty} Tx_n(\omega) = x(\omega)$.

Definition 1.9: Let $(X \times \Omega, \vartheta(\omega))$ be random uniform space and p_ω be an $E - distance$ on X. Two self maps S and T on are said to be weak commutative if

$$\lim_{n\to\infty} p_\omega\big(STx(\omega), TSx(\omega)\big) = 0,$$

Whenever

$$\lim_{n\to\infty} Sx_n(\omega) = \lim_{n\to\infty} Tx_n(\omega) = x(\omega).$$

Lemma 1.1: Let $(X \times \Omega, \vartheta(\omega))$ be a random Hausdorff uniform space and p be an $A - distance$ on X. Let $\{x_n(\omega)\}, \{y_n(\omega)\}$ be arbitrary sequence in $X \times \Omega$ and $\{\alpha(\omega)_n\}$ and $\{\beta(\omega)_n\}$ be sequence in R^+ and converging to 0.
Then for $x(\omega), y(\omega), z(\omega) \in X \times \Omega$, the following holds:
$(\boldsymbol{a})$ If $p_\omega(x(\omega)_n, y(\omega)) \le \alpha(\omega)_n$ and $p_\omega(x(\omega)_n, z(\omega)) \le \beta(\omega)_n$ for all $n \in N$, then $y(\omega) = z(\omega)$, In particular, if $p_\omega\big(x(\omega), y(\omega)\big) = 0$ and $p_\omega\big(x(\omega), z(\omega)\big) = 0$,then $y(\omega) = z(\omega)$.
$(\boldsymbol{b})$ If $p_\omega(x(\omega)_n, y(\omega)_n) \le \alpha(\omega)_n$ and $p_\omega(x_n(\omega), zv) \le \beta(\omega)_n$ for all $n \in N$, then $\{y(\omega)_n\}$ converges to $z(\omega)$.
$(\boldsymbol{c})$ If $p_\omega(x(\omega)_n, x(\omega)_m) \le \alpha(\omega)_n$ for all $m > n$, then $\{x(\omega)_n\}$ is a Cauchy sequence in $(X \times \Omega, \vartheta(\omega))$.
We also involve a non-decreasing function Ψ on R^+ such that
$\Psi(t) < t$, for $t > 0$ and $\Psi^n(t) = 0$ as $n \to \infty$.

2. Main Results

Theorem 2.1: Let $(X \times \Omega, \vartheta(\omega))$ be a random Hausdorff space and p_ω be an $E - distance$ on $p - cauchy$ complete space X. Let A, B, S and T be self mapping of $X \times \Omega$ satisfying that
(2.1.1) $A(X \times \Omega) \subseteq T(X \times \Omega)$ and $B(X \times \Omega) \subseteq S(X \times \Omega)$;
(2.1.2) The pair (A, S) is semi compatible and (B, T) is weak commutative;
(2.1.3) One of A, B, S and T iscontinuous;
(2.1.4) $p_\omega(A(x(\omega)), B(y(\omega))) \le \Psi[\max\{p_\omega(S(x(\omega)), T(y(\omega))),$
$p_\omega\big(S(x(\omega)), A(x(\omega))\big) + p_\omega(T(y(\omega)), A(x(\omega))),$
$p_\omega\big(T(y(\omega)), B(y(\omega))\big) + p_\omega\big(T(y(\omega)), A(x(\omega))\big), p_\omega(T(y(\omega)), A(x(\omega)))\}].$
Then A, B, S and T have unique common fixed point.

Proof: Let $x_0 \in X$ be any point, As $A(X \times \Omega) \subseteq T(X \times \Omega)$ and $B(X \times \Omega) \subseteq S(X \times \Omega)$, there exists $x_1(\omega)$ and $x_2(\omega)$ in $X \times \Omega$ such that,
$A(x(\omega)_0 = T(x(\omega)_1)$ and $B(x(\omega)_1) = S(x(\omega)_2)$.
In general we can construct sequence $\{y(\omega)_n\}$ in $X \times \Omega$ such that for $n = 0,1,2, \ldots.$
$y(\omega)_{2n+1} = A(x(\omega)_{2n}) = T(x(\omega)_{2n+1})$ and $y(\omega)_{2n+2} = B(x(\omega)_{2n+1} = S(x(\omega)_{2n+2}),$
Now by (2.1.4) $p(y(\omega)_{2n+1}, y(\omega)_{2n+2}) = p(A(x(\omega)_{2n})B(x(\omega)_{2n+1}))$
$\le \Psi[\max\{p_\omega(S(x(\omega)_{2n}), T(x(\omega)_{2n+1})),$
$p_\omega\big(S(x(\omega)_{2n}), A(x(\omega)_{2n})\big) + p_\omega(T(x(\omega)_{2n+1}), A(x(\omega)_{2n})),$
$p_\omega\big(T(x(\omega)_{2n+1}), B(x(\omega)_{2n+1})\big) + p_\omega(T(x(\omega)_{2n+1}), A(x(\omega)_{2n})),$
$p_\omega(T(x(\omega)_{2n+1}), A(x(\omega)_{2n}))\}]$
$= \Psi[\max\{p_\omega(y(\omega)_{2n}, (\omega)_{2n+1}), p_\omega(y(\omega)_{2n}, (\omega)_{2n+1}) + p_\omega(y(\omega)_{2n+1}, (\omega)_{2n+1}),$
$p_\omega(y(\omega)_{2n+1}, (\omega)_{2n+2}) + p_\omega(y(\omega)_{2n+1}, y(\omega)_{2n+1}),$
$p_\omega(y(\omega)_{2n+1}, (\omega)_{2n+1})\}]$
$\le \Psi[\max\{p_\omega(y(\omega)_{2n}, (\omega)_{2n+1}), p_\omega(y(\omega)_{2n}, (\omega)_{2n+1}), p_\omega(y(\omega)_{2n+1}, (\omega)_{2n+2}), 0\}]$
$p_\omega(y(\omega)_{2n+1}, (\omega)_{2n+2}) \le \Psi[p_\omega(y(\omega)_{2n}, (\omega)_{2n+1})].$
Similarly
$p_\omega(y(\omega)_n, (\omega)_{n+1}) \le \Psi[p_\omega(y(\omega)_{n-1}, (\omega)_{n+1})].$
$\Rightarrow p_\omega(y(\omega)_n, (\omega)_{n+p}) \le \Psi^n[\delta_p(X)]$
Where $\delta_{p\omega}(X) = \sup\{p_\omega(x(\omega), y(\omega))/x(\omega), y(\omega) \in X \times \Omega\}$.
Then by Ψ_2 and lemma 1.1(c) we have $\{y(\omega)_n\}$ is Cauchy sequence in $X \times \Omega$ and $X \times \Omega$ is S^* complete therefore $\lim_{n\to\infty} p_\omega\big(y(\omega)_n, z(\omega)\big) =$ *0.* By lemma *1.1(b)* there exists a sequences $\alpha\omega n$ and $\beta\omega n$ which converging to *0.*
There the subsequences $A(x(\omega)_{2n}) \to T(x(\omega)_{2n+1})$, $B(x(\omega)_{2n+1})$ and $S(x(\omega)_{2n+2})$ also converge to $z(\omega)$.

Case I. Let S is continuous then $SA(x(\omega)_{2n}) \to Sz(\omega)$ and $SS(x(\omega)_{2n})) \to Sz(\omega)$. By the semi compatibility of the pair (A, S) gives $AS(x(\omega)_{2n}) \to Sz(\omega)$ as $n \to \infty$.
By (6.3.1.4)
$p_\omega(AS(x(\omega)_{2n}), B(x(\omega)_{2n+1})) \le \Psi[\max\{p_\omega(SS(x(\omega)_{2n}), T(x(\omega)_{2n+1})),$
$p_\omega\big(SS(x(\omega)_{2n}), AS(x(\omega)_{2n})\big) + p_\omega(T(x(\omega)_{2n+1}), AS(x(\omega)_{2n})),$
$p_\omega(T(x(\omega)_{2n+1}), B(x(\omega)_{2n+1}) + p_\omega(T(x(\omega)_{2n+1}), AS(x(\omega)_{2n})),$
$p_\omega(T(x(\omega)_{2n+1}), AS(x(\omega)_{2n}))\}]$
Letting $n \to \infty$, we get

$$p_\omega(S(z(\omega)), z(\omega)) \le \Psi[\max\Big\{S\big(z(\omega)\big), z(\omega)\Big),$$
$$p_\omega\big(z(\omega), z(\omega)\big) + p_\omega\Big(z(\omega), S\big(z(\omega)\big)\Big),$$
$$p_\omega(z(\omega), S\big(z(\omega)\big))\}] < p_\omega(S(z(\omega)), z(\omega)).$$
$$\Rightarrow p_\omega\Big(S\big(z(\omega)\big), z(\omega)\Big) = 0.$$

Again, put $x(\omega) = z(\omega)$ and $y(\omega) = x(\omega)_{2n+1}$ in (2.1.4)

$$p_\omega(A(z(\omega)), B(x(\omega)_{2n+1})) \le \Psi[\max\{p_\omega\Big(S\big(z(\omega)\big), T(x(\omega)_{2n+1})\Big),$$
$$p_\omega\Big(S\big(z(\omega)\big), A\big(z(\omega)\big)\Big) + p_\omega\Big(T(x(\omega)_{2n+1}), A\big(z(\omega)\big)\Big),$$
$$p_\omega\big(T(x(\omega)_{2n+1}), B(x(\omega)_{2n+1})\big) + p_\omega\Big(T(x(\omega)_{2n+1}), A\big(z(\omega)\big)\Big),$$
$$p_\omega\Big(T(x(\omega)_{2n+1}), A\big(z(\omega)\big)\Big)\}]$$

$$p_\omega(A(z(\omega)), z(\omega)) \le \Psi[\max\{p_\omega\left(S(z(\omega)), z(\omega)\right), p_\omega\left(S(z(\omega)), A(z(\omega))\right)$$
$$+p_\omega\left(z(\omega), A(z(\omega))\right),$$
$$p_\omega(z(\omega), z(\omega)) + p_\omega\left(z(\omega), A(z(\omega))\right), p_\omega(z(\omega), A(z(\omega)))\}]$$

$$p_\omega(A(z(\omega)), z(\omega)) \le \Psi[\max\{p_\omega(z(\omega), z(\omega)), p_\omega\left(S(z(\omega)), z(\omega)\right)$$
$$+p_\omega\left(z(\omega), A(z(\omega))\right),$$
$$p_\omega(z(\omega), z(\omega)), p_\omega\left(z(\omega), A(z(\omega))\right)\}]$$
$$< p_\omega(A(z(\omega)), z(\omega))$$
$$\Rightarrow p_\omega\left(A(z(\omega)), z(\omega)\right) = 0.$$

Now $p_\omega(S(z(\omega), z(\omega))) = 0$ and $p_\omega\left(A(z(\omega), z(\omega))\right) = 0$.
Hence $Sz(\omega) = Az(\omega)$.

$$p_\omega(z(\omega), z(\omega)) = p_\omega(z(\omega), Az(\omega)) + p_\omega(Az(\omega), z(\omega)).$$

$\Rightarrow p_\omega(z(\omega), z(\omega)) = 0$ and $p_\omega(S\left(z(z(\omega)), z(\omega)\right) = 0$.
Hence $z(\omega) = Sz(\omega) = Az(\omega)$.

Case II. Since $A(X \times \Omega) \subseteq T(X \times \Omega)$, therefore there exists $u(\omega)$ in $X \times \Omega$ such that $Az(\omega) = Tu(\omega)$.
Put $x(\omega) = x(\omega)_{2n}$ and $x(\omega) = u(\omega)$ in (2.1.4)

$$p_\omega(A(x(\omega)_{2n}), B(u(\omega))) \le \Psi[\max\{p_\omega\left(S(x(\omega)_{2n}), T(u(\omega))\right),$$
$$p_\omega(S(x(\omega)_{2n}), A(x(\omega)_{2n})) + p_\omega\left(T(u(\omega)), A(x(\omega)_{2n})\right),$$
$$p_\omega\left(T(u(\omega)), B(u(\omega))\right) + p_\omega\left(T(u(\omega)), A(x(\omega)_{2n})\right),$$
$$p_\omega\left(T(u(\omega)), A(x(\omega)_{2n})\right)\}]$$
$$p_\omega(z(\omega), B(u(\omega))) \le \Psi[p_\omega(z(\omega), z(\omega)), p_\omega(z(\omega), z(\omega)) + p_\omega(z(\omega), z(\omega)),$$
$$p_\omega\left(z(\omega), B(u(\omega))\right) + p_\omega(z(\omega), z(\omega)),$$
$$p_\omega(z(\omega), z(\omega))\}]$$
$$< p_\omega(z(\omega), B(u(\omega)))$$

$\Rightarrow p_\omega\left(z(\omega), B(u(\omega))\right) = 0$. and so $A(z(\omega)), z(\omega)) = 0$
i.e. $p\left(T(u(\omega)), z(\omega)\right) = 0 \Rightarrow Tu(\omega) = Bu(\omega)$.
$BTu(\omega) = TBu(\omega) \Rightarrow Bz(\omega) = Tz(\omega)$.

Case III. Put $x(\omega) = z(\omega)$ and $y(\omega) = z(\omega)$. In (2.1.4), we get

$$p_\omega(A(z(\omega)), B(z(\omega))) \le \Psi[\max\{p_\omega(S\left(z(\omega)), T(z(\omega))\right), p_\omega(S\left(z(\omega)), A(z(\omega))\right),$$
$$p_\omega(T\left(z(\omega)), B(z(\omega))\right), p_\omega\left(T\left(z(\omega)), A(z(\omega))\right)\right\}]$$
$$p_\omega(A(z(\omega)), B(z(\omega))) \le \Psi[\max\{p_\omega\left(A(z(\omega)), T(z(\omega))\right), p_\omega(z(\omega), z(\omega)),$$
$$p_\omega(T\left(z(\omega)), B(z(\omega))\right), p_\omega(T(z(\omega)), z(\omega))\}]$$
$$p_\omega(z(\omega), B(z(\omega))) \le \Psi[\max\{p_\omega(z(\omega), B(z(\omega))), p_\omega(z(\omega), z(\omega))$$
$$+p_\omega\left(B(z(\omega)), z(\omega)\right), p_\omega(T\left(z(\omega)), B(z(\omega))\right) + p_\omega\left(B(z(\omega)), z(\omega)\right) p_\omega\left(B(z(\omega)), z(\omega)\right)\}]$$
$$< (z(\omega), B(z(\omega)))$$

$\Rightarrow p_\omega\left(z(\omega), B(z(\omega))\right) = 0$ and already $p_\omega(z(\omega), z(\omega)) = 0$.
$\Rightarrow Bz(\omega) = z(\omega)$. Hence $Az(\omega) = Bz(\omega) = Sz(\omega) = Tz(\omega)$.

Uniqueness: Let $z_1(\omega)$ and $z_1(\omega)$ are two common fixed point of A, S, B and T in $X \times \Omega$. Then $Az_1(\omega) = Bz_1(\omega) = Sz_1(\omega) = Tz_1(\omega)$ and $Az_1(\omega) = Bz_1(\omega) = Sz_1(\omega) = Tz_1(\omega)$.
Now by (2.1.4)

$$p_\omega(A(z(\omega)_1), B(z(\omega)_2)) \le \Psi[\max\{p_\omega(S(z(\omega)_1), T(z(\omega)_2)),$$
$$p_\omega(S(z(\omega)_1), A(z(\omega)_1)) + p_\omega(S(z(\omega)_2), A(z(\omega)_1)),$$
$$p_\omega(T(z(\omega)_2), B(z(\omega)_2)) + p_\omega(S(z(\omega)_2), A(z(\omega)_1)),$$
$$p_\omega(T(z(\omega)_2), B(z(\omega)_1))\}]$$
$$p_\omega(z(\omega)_1, z(\omega)_2) \le \Psi[\max\{p_\omega(z(\omega)_1, z(\omega)_2),$$
$$p_\omega(z(\omega)_1, z(\omega)_1) + p_\omega(z(\omega)_2, z(\omega)_1),$$
$$p_\omega(z(\omega)_2, z(\omega)_2) + p_\omega(z(\omega)_2, z(\omega)_1), p_\omega(z(\omega)_2, z(\omega)_1)\}]$$
$$< p_\omega(z(\omega)_1, z(\omega)_1)$$
$$p_\omega(z(\omega)_1, z(\omega)_1) = 0.$$

Again $\quad p_\omega(z(\omega)_1, z(\omega)_1) \le p_\omega(z(\omega)_1, z(\omega)_2) + p_\omega(z(\omega)_2, z(\omega)_1)$
$\Rightarrow p_\omega(z(\omega)_1, z(\omega)_1) = 0$.
Hence $z(\omega)_1 = z(\omega)_2$.

Theorem 2.2 : Let Let $(X \times \Omega, \vartheta(\omega))$ be a random Hausdorff uniform space and p_ω be an $E - distance$ on p_ω-cauchy complete space X. Let A, B, S and T be self mapping of $X \times \Omega$ satisfying that
(I) $A(X \times \Omega) \subseteq T(X \times \Omega)$ and $B(X \times \Omega) \subseteq S(X \times \Omega)$;
(II) The pair (A, S) is semi compatible and (B, T) is weak commutative;

(III) One of A, B, S and T iscontinuous;

(IV) $p_\omega(A(x(\omega)), B(y(\omega))) \le \Psi[\max\{p_\omega(S(x(\omega)), T(y(\omega))),$

$p_\omega\left(S(x(\omega)), A(x(\omega))\right) + p_\omega(T(y(\omega)), A(x(\omega))),$

$p_\omega\left(T(y(\omega)), B(y(\omega))\right) + p_\omega\left(T(y(\omega)), A(x(\omega))\right),$

$$1/2\{p_\omega\left(S(x(\omega)), B(y(\omega))\right) + p_\omega\left(T(y(\omega)), A(x(\omega))\right)\}\}].$$

Then A, B, S and T have unique common fixed point.

Proof: Let $x(\omega)_0 \in X \times \Omega$ be any point. As $A(X \times \Omega) \subseteq T(X \times \Omega)$ and $B(X \times \Omega) \subseteq S(X \times \Omega)$, There exists x_1 and $x(\omega)_2$ in $X \times \Omega$ such that $A(x(\omega)_0) = T(x(\omega)_1$ and $B(x(\omega)_1 = S(x(\omega)_2)$.

In general we can construct sequence $\{y(\omega)_n\}$ in $X \times \Omega$ such that for $n = 0,1,2, \dots \dots$

$y(\omega)_{n+1} = A(x(\omega)_{2n}) = T(x(\omega)_{2n+1})$ and $y(\omega)_{2n+2} = B(x(\omega)_{2n+1}) = S(x(\omega)_{2n+2}),$

Now by (2.1.4)

$$p(y(\omega)_{2n+1}, y(\omega)_{2n+2}) = pA(x(\omega)_{2n})B(x(\omega)_{2n+1}))$$
$$\le \Psi[\max\{p_\omega\left(S(x(\omega)_{2n}), T(x(\omega)_{2n+1})\right),$$
$$p_\omega\left(S(x(\omega)_{2n}), A(x(\omega)_{2n})\right) + p_\omega\left(T(x(\omega)_{2n+1}), A(x(\omega)_{2n})\right),$$
$$p_\omega\left(T(x(\omega)_{2n+1}), B(x(\omega)_{2n+1})\right) + p_\omega\left(T(x(\omega)_{2n+1}), A(x(\omega)_{2n})\right),$$
$$1/2\{p_\omega\left(S(x(\omega)_{2n}), B(x(\omega)_{2n+1})\right) + p_\omega\left(T(x(\omega)_{2n+1}), A(x(\omega)_{2n})\right)\}\}]$$
$$= \Psi[\max\{p_\omega(y(\omega)_{2n}, y(\omega)_{2n+1}), p_\omega(y(\omega)_{2n}, y(\omega)_{2n+1}) + p_\omega(y(\omega)_{2n+1}, y(\omega)_{2n+1}),$$
$$p_\omega(y(\omega)_{2n+1}, y(\omega)_{2n+2}) + p_\omega(y(\omega)_{2n+1}, y(\omega)_{2n+1}),$$
$$1/2\{p_\omega(y(\omega)_{2n}, y(\omega)_{2n+2}) + p_\omega(y(\omega)_{2n+1}, y(\omega)_{2n+1})\}\}]$$

Since p_ω be an $E - distance$ therefore by p_2 of $def.$ 1.2, we have

$$p_\omega(y(\omega)_{2n}, y(\omega)_{2n+2}) \le p_\omega(y(\omega)_{2n}, y(\omega)_{2n+1}) + p_\omega(y(\omega)_{2n+1}, y(\omega)_{2n+2}).$$

Hence $p_\omega(y(\omega)_{2n+1}, y(\omega)_{2n+2}) = p_\omega(A(x(\omega)_{2n})B(y(\omega)_{2n+1}))$

$$\le \Psi[\max\{p_\omega(y(\omega)_{2n}, y(\omega)_{2n+1}), p_\omega(y(\omega)_{2n}, y(\omega)_{2n+1}) + p_\omega(y(\omega)_{2n+1}, y(\omega)_{2n+1}),$$
$$p_\omega(y(\omega)_{2n+1}, y(\omega)_{2n+2}) + p_\omega(y(\omega)_{2n+1}, y(\omega)_{2n+1}),$$
$$1/2\{p_\omega(y(\omega)_{2n}, y(\omega)_{2n+1}) + p_\omega(y(\omega)_{2n+1}, y(\omega)_{2n+2}) + p_\omega(y(\omega)_{2n+1}, y(\omega)_{2n+1})\}]$$
$$p_\omega(y(\omega)_{2n+1}, y(\omega)_{2n+2}) \le \Psi[p_\omega(y(\omega)_{2n}, y(\omega)_{2n+1})].$$

Similarly $p_\omega(y(\omega)_n, y(\omega)_{n+1}) \le \Psi[p_\omega(y(\omega)_{n-1}, y(\omega)_{n+1})]$.

$\Rightarrow p_\omega\left(y(\omega)_n, y(\omega)_{n+p}\right) \le \Psi^n[\delta_{p\omega}(X)]$

Where $\delta_{p\omega}(X) = \sup\{p(x(\omega), y(\omega))/x(\omega), y(\omega) \in X \times \Omega\}$.

Then by Ψ_2 and lemma 1.1(c) we have $\{y(\omega)_n\}$ is Cauchy sequence in X.

Remaining proof is same as in theorem 2.1.

Theorem 2.3: Let $(X \times \Omega, \vartheta(\omega))$ be a random Hausdorff uniform space and p be an $E - distance$ on $p - cauchy$ complete X. Let A^r, B^r, S^s and T^s are sequences defined on X such that

(I) $A^r(X \times \Omega) \subseteq T^s(X \times \Omega)$ and $B^r(X \times \Omega) \subseteq S^s(X \times \Omega)$;

(II) The pair (A^r, S^s) is semi compatible and (B^r, T^s) is weak commutative;

(III) one of A^r, B^r, S^s and T^s is continuous;

(IV) $p_\omega(A^r(x(\omega)), B^r(y(\omega))) \le \Psi[\max\{p_\omega\left(S^s(x(\omega)), T^s(y(\omega))\right),$

$p_\omega\left(S^s(x(\omega)), A^r(x(\omega))\right) + p_\omega\left(T^s(y(\omega)), A^r(x(\omega))\right),$

$$p_\omega\left(T^s(y(\omega)), B^r(y(\omega))\right) + p_\omega\left(T^s(y(\omega)), A^r(x(\omega))\right),$$

$1/2\{p_\omega\left(S^s(x), B^r(y)\right) + p_\omega\left(T^s(y(\omega)), A^r(x(\omega))\right)\}\}].$

Where r and s are positive integers. Then A^r, B^r, S^s and T^s have unique common fixed point.

Proof: Same as theorem 2.2.

Examples

Examples 1: Let $X \times \Omega = [\omega, 10(\omega)]$ random uniform space and d is usual metric on X. Define $B, T: X \times \Omega \to X \times \Omega$ by

$$Bx(\omega) = \begin{cases} \omega & if\ \ \omega \le x(\omega) < 3(\omega) \\ \dfrac{\omega + x(\omega)}{4} & if\ \ x(\omega) \ge 3(\omega) \end{cases}$$

and

$$Tx(\omega) = \begin{cases} \dfrac{x(\omega) = \omega}{2} & if\ \ \omega \le x(\omega) < 2(\omega) \\ \dfrac{2x(\omega) + \omega}{5} & if\ \ x(\omega) \ge 2(\omega) \end{cases}$$

Also consider the sequence $x_n(\omega) = 2(\omega) + (1/n)$.

Clearly, T and S weak commutative

$\Rightarrow BT2(\omega) = TB2(\omega)$ and $T2(\omega) = S2(\omega) = (\omega)$. Similarly, we define $A, S: X \times \Omega \to X \times \Omega$ by

$Ax(\omega) = \begin{cases} (\omega) & if\ \ \omega \le\ x(\omega) < 4(\omega) \\ \frac{2(\omega) + x(\omega)}{6} & if\ \ x(\omega) \ge 4(\omega) \end{cases}$ and $Sx(\omega) = \begin{cases} x(\omega) & if\ \ 1 \le\ x(\omega) < 2 \\ \frac{x(\omega) + (\omega)}{6} & if\ \ x(\omega) \ge 2 \end{cases}$

Clearly, the pair (A, S) is semi compatible. We observe that A, B, S and T satisfying the conditions of theorem 2.1 and hence (ω) is the fixed point.

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