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Quadratic Assignment Problem

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Abstract: Quadratic Assignment problem has gain very strong attention among the various combinatorial optimization problem, due to its easy to model nature to real life instances. In this paper, we have describe the concept of quadratic assignment problem, and its solving algorithm along with a survey of few developments and researches

.

Keywords: Optimization, Quadratic Assignment Problem, Exact Algorithm, Bound, Heuristic, NP-complete, Meta heuristic.

I. INTRODAUCTION

Father of the Quadratic assignment problem, Koopmans and Beckmann (QAP) who introduces QAP in 1957 as a mathematical model related to economic activities[1]. From there QAP has been utilized to model various applications in several areas like operations research, parallel and distributed computing, and combinatorial data analysis, for example: departments or offices within a plant , locating machines, forest management, assignment of runners in a relay team arranging indicators and controls in a control room to minimize eye fatigue, locating hospital departments , ……. etc.

II. DESCRIPTION OF THE PROBLEM

Linear assignment problem (LAP**)**, then, we will define the quadratic assignment problem (QAP) in this section of this paper.

2.1 The Linear Assignment Problem (LAP)

In order to assign n task to n agents,the linear assignment problem (LAP) was implemented by Hanan and Kurtzberg [2],. For every work assignment, there is a corresponding cost C_{ij} of assigning agent *i* to task *j*. The assignment of one agent to one and only one work with the objective of minimizes the total cost. Mathematical representation of the problem as follows:

$$\min \sum_{i=1}^{n} C_{i\pi(i)} \text{ ,over all permutations } \pi \in S_n$$

where S_n is the set of permutations of {1, 2,…, n}, and *j*= π(i) is the work assignment of worker or agent *i*. It is important to note that that there are *n* permutations from where to select the optimal assignment, i.e., there are n! different ways in which n task can be assigned to *n* agent. Also it is important that for higher values of n, checking of all possible permutations is not possible, For example, assigning *n= 12* workers to 12 work as above, then there are 12! to be examined , or approximately 3.63 million distinct permutations. Obviously, we require more optimal algorithms for solving bigger problems of the LAP.

2.2 The Quadratic Assignment Problem

Simply, QAP can be defined as: Given n facilities to assigned to *n* distinct locations, considering that a flow of value $f = [f_{ij}]$ has to go from facility *i* to facility *j* and the distance among the locations *k* and *l* is D= $[d_{kl}]$, and the assignment of facility to location must be such that we have a minimum total cost which is the addition of the products flow × distance [2]. For example: let the matrices P and Q are the distance and flow matrices of a facility location problem.

$$P = \begin{pmatrix} 0 & 3 & 6 & 4 & 2 \\ 3 & 0 & 2 & 3 & 3 \\ 6 & 2 & 0 & 3 & 4 \\ 4 & 3 & 3 & 0 & 1 \\ 2 & 3 & 4 & 1 & 0 \end{pmatrix}$$

$$Q = \begin{pmatrix} 0 & 10 & 15 & 0 & 7 \\ 1 & 00 & 5 & 6 & 0 \\ 15 & 5 & 0 & 4 & 2 \\ 0 & 6 & 4 & 0 & 5 \\ 7 & 0 & 2 & 5 & 0 \end{pmatrix}$$

As we have seen in LAP, there were n! permutations to select optimal solution from. But QAP is having distinct feature than LAP which make QAP a much more complex.

The difference is that the LAP assignment of task *j* to agent *i* was made independently of the assignments of the other agents. With QAP the assignments are not independent, so when assign agent *i* to task *j* we must also have a view on the assignments of all present agents who have non zero affinity toward agent *i* which signify the amount of face-to face communication between agents.

III. EXACT ALGORITHMS

Finding QAP optimal solution is a very hard. To gain optimality for QAP there are three different methods: the Branch and bound procedures, the cutting plane techniques [3] and the Dynamic programming [4]**.**

3.1 The Branch and Bound Method

Very initial method to obtain solution for the quadratic assignment problem is branch and bound method[19]. The possible solution for QAP is the Branch and bound as it is more efficient methodology. Gilmore [5] in 1962 firstly evolve branch and bound methodology who found out the solution for a QAP problem having instances of size n = 8. The name of the branch and bound depict its functionality. To apply The Branch and bound method; first, cho0se a heuristic procedure to get an initial feasible solution "suboptimal" and it is used as upper bound. Second, the problem is divided into small problems with a lower bound, then construct search tree by repeating the division and lower bounding of each sub problem .During iterations, best possible permutation is being constructed.

3.2 The Cutting Plane Method

This method has two division: traditional cutting plane methods and polyhedral cutting-plane or branch-and-cut methods. Traditional cutting plane algorithms for QAP was initially utilized by Kaufman and Broekx in 1978, these algorithms uses mixed integer linear programming (MILP) formulations to solve the problem of QAP.Generally, the time required for these methods is very much, and hence these methods may solve to optimality only very small QAPs. However, heuristics derived from cutting plane approaches produce good suboptimal solutions in early stages of the search. The polyhedral cutting planes or branch and cut algorithms utilizes the MILP formulations of QAP. The polyhedral cutting plane is not much used in QAP because of the lack of knowledge about QAP poly types.

3.3 The Dynamic Programming

The idea behind the dynamic programming method is easy to understand in comparison to other methodology. . In general, to solve a given problem, different parts of the problem "sub problems" need to be solved, and then the solutions of the sub problems should be combined to reach an overall solution. Christofides and Benavent [4] used a dynamic programming approach to solve a special case of QAP. Dynamic programming solves a problem of size n by starting from sub problems of size 1,2,…,n-1. After solving sub problems of size k it upgrades the solutions to size k+1. Problems may arise in dynamic programming if the solution to a sub problem or the upgrade procedure cannot be performed in polynomial time.

There have been fewer applications. For cutting plane and dynamic programming methods than the Branch and Bound method. As shown before, QAP is NP-hard due to the overwhelming complexity of QAP, most problems with large sizes remain nearly intractable by exact algorithms. Therefore, we use heuristics in order to get solutions with good quality for QAP in a reasonable computational time.

IV. HEURISTICS

Because of obvious difficulties experienced in the development of exact solution procedures, a wide variety of heuristic approaches has been developed for QAP. Heuristic algorithms generally not provide the guarantee of finding for the best solution. Thus, we take heuristic techniques as a procedure only to search of good quality of solutions. These approaches can be divided into the following division: Construction methods [6], Limited enumeration methods [7], Improvement methods, Improvement methods are frequently used in metaheuristics.)

4.1 Construction Methods (CM)

The construction method algorithm is taken one of the initial heuristics in use (Buffa, Armour and Vollmann (1964), Muller-Merbach (1970)). These methods obtain suboptimal permutations by initializing with a partial permutation that is initially blank. The permutation is explored by iterative assignments based on set selection criterion until the permutation is completely obtained.

4.2 Limited Enumeration Methods (LEM)

(West (1983), Burkard and Bonniger (1983)) Exact methods like branch and bound and cutting planes are strongly related to limited enumeration methods. The idea behind these algorithms is that a good suboptimal solution may be produced early in an enumerative search. In addition, an optimal solution may be found earlier in the search while the rest of the time is spent on proving the optimality of this solution. There are many ways to limit enumeration of the search space**;** one approach is to impose a time limit. Enumeration stops when the algorithm reaches a time limit or no improvement has been made in a predetermined time interval. These pre-specified parameters can be problem specific. A second option is to decrease the requirement for optimality. For example, if no improvement has been made after a certain pre-specified time interval, then the upper bound is decreased by a certain percentage resulting in deeper cuts in the enumeration tree. Although the optional solution may be cut off, it differs from the obtained solution by the above percentage.

4.3 Improvement Methods (IM)

Improvement methods are the most searched category of heuristic. Few important improvement methods are the local search and the tabu search. These two procedure work by starting with an initial basic feasible solution and then work to optimize it. The local search seeks a better solution in the neighborhood of the current solution, stop only when no better solution exists within that neighborhood.

V. META HEURISTICS

Meta heuristic is a very commonly used methodology which have appeared to adapt with the problem structure. Many of this methodology depends on some form of simulation of a natural behavior. With the merit of meta heuristics, QAP research has gained a new and enhanced interest which is reflected on QAP publications. Fig.1**.** shows the distribution of publications, classified by solution

techniques that were classified in this work as Heuristic Methods, Exact Methods and Meta heuristics. Fig.2 shows the distribution of meta heuristic solution techniques used to QAP. In this figure, we have greedy randomized adaptive search procedure (GRASP), genetic algorithm (GA), neural networks and others (NNO),tabu search (TS), ant colony(AC), scatter search (SS), variable neighborhood search (VNS), simulated annealing (SA) and hybrid algorithms (HA). Hybrid procedures that result from different metaheuristic compositions are the most practiced solution procedure. However, when we study for a comparison among pure meta heuristics, the procedures based on more traditional simulated annealing and more recently defined ant colony are the most popular.

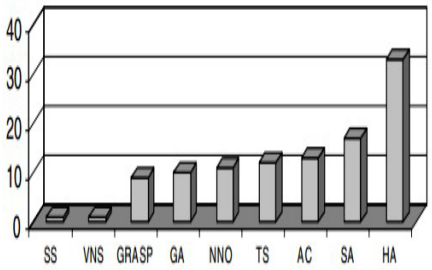

Fig. 1. Publications: solution techniques Metaheuristic

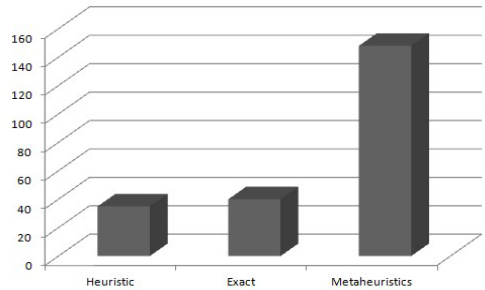

Fig. 2. Publications metaheuristics used to the QAP.

5.1 Simulated Annealing (SA)

Simulated annealing is an important type of meta heuristic approach employed to solve different optimization problem. It is developed by Burkard and Rendl (1984),Wilhelm and Ward (1987). Name of this approach depict the principle of its working, which is utilized to get over the problem of local optima. This methodology, is known to be annealing which leads excite high energy particles to lower energy states by minimizing of the temperature, and last we work for cooling a material to a steady state. At the beginning state of the heuristic, the algorithm is lenient and capable of moving to a worse solution present within the domain. However, with each iteration the algorithm becomes stricter requiring a better solution at each step.

5.2Genetic Algorithms (GA)

Among the wide range of functionality and usability of genetic algorithm, GA has gain its popularity even to solve QAP.Name of the genetic algorithm only depict the functionality of its algorithm which work on the basis of Darwin's theory of natural selection[10]. Genetic algorithms makes a group of solutions and then iterate to replace the obtained solutions with better ones check by the basis of predefined fitness criterion, usually the objective function value. Genetic algorithms mostly work in parallel framework and helpful when applied in such an environment.

5.3 Particle Swarm Optimization

Particle swarm optimization algorithm based on food searching pattern of swarm. This algorithm is comparatively much easier to implement than other metaheuristic algorithm. Russell Eberhart and James Kennedy in 1995 has invented Particle Swarm Optimization (PSO). PSO algorithm also known to be Particle swarm Optimization has got its base from livelihood behavior patterns and habits of organisms that live and interact within other similar swarm in a large groups. In particular, PSO incorporates swarming behaviors observed in flocks of birds, schools of fish, or swarms of bees, and even human social behavior, from which the Swarm Intelligence (SI) paradigm has emerged [14]. It could be implemented and applied easily to solve various function optimization problems, or the problems that can be transformed to function optimization problems. As an algorithm, the very unique and important feature of Particle Swarm Optimization is its nature of quick convergence, which compares favorably with many different global optimization algorithms.

5.4Ant Colonies (AC)

Maniezzo, Colorni, and Dorigo developed an ant based methodology to solve optimization problem named as ant colony optimization[13]**.** The algorithm works on the principle of utilizing the interconnection methodology, an ant group is capable of searching the minimum weighted path among any two points. Whenever ant go out for food search**,** a pheromone named chemical trail is leaved by them on the path they follow to reach the food. Aim of leading this trace on path is to guide other incoming ant to follow the path to reach over the food. Quantity of pheromone is directly proportional to optimality of path. Moreover, this chemical substance has a decreasing action over with time, and the quantity left by one ant depends on the amount of food found and the number of ants using this trail. Applied on QAP, The AC is depends on a hybridization of the ant system with a local search method, every ant being associated with an integer permutation. Modifications based on the pheromone trail are then used in every permutation. The solutions (ants) found so far are then make efficient by utilizing a local search method, update of the pheromone trail simulates the evaporation and takes into account the solutions obtained in the search strategy.

VI. 6. APPLICATION

QAP has been utilized to solve many real life problems Koopmans and Beckmann (1957**)** initially defined QAP as a mathematical model regarding the economic activities. From then , it has used in several practical applications[17]. Along with all this uses QAP is also utilized to lower down the number of connection within backboard wring by Steinberg in 1961.QAP has enhances its branches from computer and mathematical problem to solve economic problems by Heffley (1972, 1980**)**, Francis and White (1974**)** obtain framework for arranging police posts, supermarkets, schools within a particular cities for serving a given set of clients, Geoffrion and Graves (1976) concentrated on scheduling problems, QAP is also used to obtain the best design for

typewriter keyboards and control panels by Pollatschek et al**.** (1976. Benjaafar (2002) introduced a formulation of the facility layout design problem for minimizing work-in-process (WIP). Ben-David and Malah (2005) looked into a special case of QAP is known to be index assignment in order to minimize channel errors in vector-quantization. Vector-quantization is used when mapping speech or pictures to digital signals. A similar mapping problem is also found when configuring the layout of microarrays, which is a problem in bioinformatics presented as a QAP by de Carvalho Jr. and Rahmann. A more modern application of the same problem is the design of keyboards on touch screen devices (Dell'Amico et al., 2009). The main difference in this approach is that on a touch screen only one finger is used, and the letters can be positioned at any place on the screen instead of in a rectangle as with normal keyboards[16].

VII. SURVEY

Many different research has been done on the quadratic assignment problem**.** QAP publications have the different classification: applications, theory (i.e., formulations, complexity studies and lower bounding techniques) and algorithms.We observe that an explosion of interest in algorithm development**.**

Table I. We will give some PhD thesis, dissertations and articles about development on QAP:

AUTHOR	TYPE	TITLE	UNIVERSITY	OBJECTIVE
_KOOPMANS & BECKMANN(1957)	Article	Assignment Problems and the Location of Economic Activities	Yale University,America	In 1957 by Koopmans and Beckmann model a plant location problem which have result to the introduction of QAP[1]. From then the QAP has become object of numerous researches
JENS CLAUSEN MICHAEL PERREGAARD(1997)	PhD thesis	Solving Large Quadratic Assignment Problems in Parallel	DIKU, Dept. of Computer Science, University of Copenhagen	Proposed the linkage of one of the efficient bound functions used within the Branch-and-Bound algorithm (the Gilmore-Lawler bound) and their various testing, define new bounds among the branching when implemented within parallel Branch and Bound algorithm[9}
Thomas St¨utzle , Marco Dorigo (2000) ,	Article	Local Search and Metaheuristics for the Quadratic Assignment Problem	IRIDIA, Universit´ e Libre de Bruxelles	Gave an overview of existing algorithmic approaches to tackle the QAP. The best performing algorithms for the QAP are currently those based on tabu search and hybrid algorithms[12].
Thomas Stützle(2006)	Article	Iterated local search for the quadratic assignment problem	Darmstadt, Germany	Present and analyze the application of ILS to the quadratic assignment problem (QAP)[18].
Yi-Rong Zhu(2007)	dissertation	Recent advances and challenges in QAP	the University of Pennsylvania	Contribute to the theoretical, algorithmic and applicable understanding of quadratic assignment and its related problems[11].

Hongbo Liu1,3, Ajith Abraham 1,2, and Jianying Zhang3(2007)	Master thesis	A Particle Swarm Approach to Quadratic Assignment Problems	Dalian Maritime University, Dalian, 116024, China	Introduce a novel approach based on PSO for QAPs[14]. The representations of the position and velocity of the particles in the conventional PSO is extended from the real vectors to fuzzy matrices
Duman et al. (2012)	Article	Migrating Birds Optimization: A new metaheuristic approach and its performance on quadratic assignment problem[8].	Ozyegin University, Department of Industrial Engineering,Istanbul, Turkey Dogus University, Department of Computer Engineering, Istanbul, Turkey [8]	Formulate a new nature based metaheuristic methodology on the V flight construction of the migrating birds which result to an optimized formation in energy saving. Its power is tested on quadratic assignment problem instances evolve from a real life problem and highly efficient results are obtained[8].
M. Fatih Tasgetiren .tr Quan-Ke Pan (2013)	Master thesis	Metaheuristic Algorithms for the Quadratic Assignment Problem	Industrial Engineering Department, Selcuk Yasar Campus,Yasar University, Izmir, Turkey	Uses the two meta-heuristic algorithms to find out the optimal solution the quadratic assignment problem. Greedy algorithm work on repetitive manner has two main parts, which are named as destruction and construction procedures
Gamal Abd El-Nasser A. Said. (2014)	Article	A Comparative Study of Meta-heuristic Algorithms for Solving Quadratic Assignment Problem	Computer Science Department Faculty of Computer & Information Sciences, Ain Shams University Cairo, Egypt	Presents a comparison among the various Meta-heuristic algorithms: Genetic Algorithm, Tabu Search, and Simulated annealing for obtaining fruitful result for real life QAP[15].
Pramod. H. Yelmewad1, ED. B. Kulkarni2,(2016)	PHD thesis	Local Search Metaheuristic Algorithm using GPGPU	Department of Information Technology, Walchand College of Engineering, Sangli, MS, India1,	This paper gives detailed information about local searching strategy to solve optimization problem on GPU based computing.

VIII. CONCLUSION

The Quadratic Assignment Problem (QAP) which is a type of combinatorial optimization problem is still very tough for researchers when input size of the problem enhances. This study paper focuses on basic concepts of QAP with real world example and mathematical expression. Some of the prominent applications of QAP are discussed and later on complexity issues are discussed. The significance of QAP in a real world scenario is candid and straightforward from the applications described in this paper. Fine description about the applicability and significance of the QAP and this work can be enhanced towards solution of other real world applications which have been successfully modeled as QAP.

IX. ACKNOWLEDGMENT

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X. REFERENCES

[1] Koopmans, Tjalling C., and Martin Beckmann. "Assignment problems and the location of economic activities." Econometrica: journal of the Econometric Society. ,pp.53-76, 1957.

[2] Hanan, Maurice, and Jerome M. Kurtzberg. "A review of the placement and quadratic assignment problems." Siam Review 14, no. 2 (1972): 324-342.

[3] M. S. Bazaraa and H. D. Sherali. “On the Use of Exact and Heuristic Cutting Plane Methods for the Quadratic Assignment Problem”, The Journal of the Operational Research Society, Volume 33, Issue 11, Novmber 1982.

[4] Christofides, Nicos, and Enrique Benavent. "An exact algorithm for the quadratic assignment problem on a tree." Operations Research, vol. 37.5, pp. 760-768,1989.

[5] Gilmore, Paul C. "Optimal and suboptimal algorithms for the quadratic assignment problem." Journal of the society for industrial and applied mathematics, vol.10.2, pp.305-313, 1962 .

[6] Armour, Gordon C., and Elwood S. Buffa. "A heuristic algorithm and simulation approach to relative location of facilities.",Management Science, vol. 9.2, pp. 294-309, 1963.

[7] Kaveh, Arman. "Algorithms and theoretical topics on selected combinatorial optimization problems.", PhD diss., Science: Department of Mathematics, 2010.

[8] Duman, Ekrem, Mitat Uysal, and Ali Fuat Alkaya., "Migrating Birds Optimization: A new metaheuristic approach and its performance on quadratic assignment problem.", Information Sciences, vol. 217, pp. 65-77, 2012.

[9] Clausen, Jens, and Michael Perregaard. "Solving large quadratic assignment problems in parallel.", *Computational Optimization and Applications, vol.* 8.2 pp. 111-127, 1997.

[10] Tate, David M., and Alice E. Smith., "A genetic approach to the quadratic assignment problem.", *Computers & Operations Research*, vol. 22.1, pp. 73-83, 1995.

[11] Zhu, Yi-Rong., "Recent advances and challenges in quadratic assignment and related problems.", PhD diss., University of Pennsylvania, 2007.

[12] Stützle, Thomas, and Marco Dorigo., "Local search and metaheuristics for the quadratic assignment problem.", Technical Report, 2001.

[13] E.-G. Talbia,∗, O. Rouxb, C. Fonluptb, D. Robillardb(2001), “Parallel Ant Colonies for the quadratic assignment problem”, E.-G. Talbi et al./Future Generation Computer Systems, vol. 17 ,pp. 441–449, (2001).

[14] Liu, Hongbo, Ajith Abraham, and Jianying Zhang. "A particle swarm approach to quadratic assignment problems." Soft computing in industrial applications. Springer Berlin Heidelberg, pp. 213-222, 2007.

[15] Said, Gamal Abd El-Nasser A., Abeer M. Mahmoud, and El-Sayed M. El-Horbaty. "A comparative study of meta-heuristic algorithms for solving quadratic assignment problem.", *arXiv preprint arXiv, pp. 1407.4863*, 2014.

[16] Dell’Amico, Mauro, et al. "The single-finger keyboard layout problem." Computers & Operations Research , vol.36.11, pp.3002-3012, 2009.

[17] COMMANDER, CLAYTON WARREN. "A survey of the quadratic assignment problem, with applications." PhD diss., UNIVERSITY OF FLORIDA, 2003.

[18] Stützle, Thomas. "Iterated local search for the quadratic assignment problem.", *European Journal of Operational Research, vol.* 174.3, pp. 1519-1539, 2006.

[19] Brüngger,Adrian, et al., "Solving large-scale QAP problems in parallel with the search library ZRAM.", *Journal of Parallel and Distributed Computing*, vol. 50.1-2 , pp. 157-169, 1998.