Volume 8, No. 4, May 2017 (Special Issue)
International Journal of Advanced Research in Computer Science
REVIEW ARTICLE
Available Online at www.ijarcs.info

ISSN No. 0976-5697

Natural frequency analysis of simply supported thin centrally attached hole rectangular plate with attached concentrated masses

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***Abstract*- In the present work, free vibration response of a simply supported rectangular plate with attached concentrated masses has been analysed. The plate material is linearly elastic, homogenous and isotropic. Galerkin method is used for finding fundamental frequencies of plate with attached concentrated masses. The candidate mode shape of plate is approximated by the sine series, which satisfy the boundary conditions of simply supported at all the edges of the plate. The modal values obtained by Galerkin method are compared with that of obtained by ANSYS. The result obtained by Galerkin method is in good agreement with those of ANSYS for concentrated mass upto 6.7% of the plate mass. It has been shown that by using one term of approximation for candidate mode in Galerkin method, the error in natural frequency of the plate is less than 0.06% when attached concentrated mass is less than 6.7% of plate mass. This saves a lot of computation.**

Keywords: Isotropic concentrated mass rectangular plate; modal analysis; natural frequency; Galerkin method.

I. Introduction

Vibration of plates with and without concentrated mass is very common engineering application. Rectangular plate has wide applications in mechanical and civil engineering. Plates form an essential part of many aerospace, marine and automobile structures [1]. It has been observed that rectangular plate with attached discrete masses are often encountered in engineering practices such as engineering slabs and cladding panel in building structure, bridge and ship decks [2]. Warburton investigated natural frequency of rectangular plate by Rayleigh – Ritz method [3]. Cohen and Handelman [4] have reviewed fundamental mode shapes and fundamental frequency of a rectangular plate having rigidly attached mass by Rayleigh – Ritz method. Leissa [5] and Laura et al. [6] have reviewed analytically free vibrations of plates with concentrated masses attached at discrete locations on plate. The effect of mass loading on natural frequency and mode modification of plate is done by Wong [7], Kopmaz and Telli [8]. Jian et al. [9] used Galerkin method for free vibration analysis of arbitrary laminated plate with clamped boundary conditions at all the edges of the plate. Dynamic analysis of plate and shell is done by L.W.Andreev [10]. The analytical solution of free and force vibration of cantilever, square and rectangular plate carrying point masses are investigated by Yu [11]. Vinayak and Ghosh [12, 13] have analysed free and force vibration of plate with attached masses and patches using Rayleigh – Ritz and finite element method

The problem of free transverse vibration of rectangular simply supported plate with attached concentrated masses is considered here. The solution of problem is obtained by Galerkin method. The deflected surface of the plate is approximated by sine series, which satisfy the simply supported boundary conditions. The modal analysis of the plate with attached concentrated mass is also calculated by ANSYS and results are compared

II. ANALYTICAL MODELLING

A) A thin rectangular isotropic plate without concentrated mass and perforation with dimensions a and b is shown in Fig. 1.

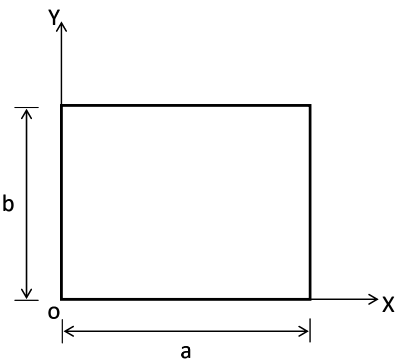

Fig. 1. Rectangular plate without concentrated mass

The fourth order partial differential equation governing the undamped, natural or free, linear vibration of a uniform isotropic rectangular plate is given by [14]

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\rho h\frac{\partial^2 w}{\partial t^2}=0 \qquad (1)$$

where w is the deflection of middle surface of the plate and is function of space and time, ρ is the density, h is the plate thickness and D is the flexural rigidity given as [14, 15]

$$D=\frac{Eh^3}{12\left(1-\upsilon^2\right)}$$ where E is the modulus of elasticity

The assumed solution of equation (1) is considered to have form [14],

CONFERENCE PAPERS
National Conference on Emerging Trends on Engineering & Technology (ETET-2017)
On 21st April 2017
University Inst. of Engg. & Tech. & University Inst. of Computer, SBBS University, Punjab (India)

$$w(x,y,t) = (A\cos\omega t + B\sin\omega t)W(x,y) \qquad (2)$$

This is the separable solution of the shape function W(x, y) describing the modes of the vibration and some harmonic function of time; ω is the natural frequency of the plate vibration which is related to vibration time period T by the relationship ω = 2Π/T. Introducing equation (2) into equation (1) we get,

$$D\nabla^2\nabla^2 W(x,y) - \omega^2 \rho h W = 0 \qquad (3)$$

Assuming that the shape function of plate is in the form of series (x, y) which satisfy the simply supported boundary conditions [14], we have

$$W(x,y) = \sum_{i=1}^{n} C_i W_i(x,y) \qquad (4)$$

where C_i are unknown coefficients.

In general procedure of the Galerkin method [14], the unknown coefficients C_i can be obtained by orthogonality conditions as given by equation (5)

$$\iint_A \left(D\sum_{i=1}^{n} C_i \nabla^2\nabla^2 W_i - \omega^2 \rho h \sum_{i=1}^{n} C_i W_i \right) W_k \, dxdy = 0;$$

where k = 1, 2, 3…, n (5)

The numerical implementation of equation (5) leads to the Galerkin system of linear algebraic homogeneous equation in the following form

$a_{11}C_1 + a_{12}C_2 + \ldots\ldots\ldots\ldots\ldots\ldots = 0$

…………………………………..

$a_{n1}C_n + a_{n2}C_2 + \ldots\ldots\ldots\ldots\ldots\ldots = 0$

(6)

where $a_{ik} = a_{ki} = \iint (D\nabla^2\nabla^2 W_i - \omega^2 \rho h W_i) W_k \, dxdy$

(7)

This system of homogeneous equations has nontrivial solution if its determinant Δ (ω) is equal to zero. Therefore, we obtain n^{th} order characteristic equation for finding natural frequencies [14] i.e.

Δ (ω) = 0 (8)

This equation has infinite number of solutions. The lowest frequency is called as fundamental or natural frequency and all other frequencies are called frequencies of higher harmonic.

2.2 Determination of the fundamental natural frequency of plates with attached concentrated masses.

The plate is simply supported along all of its edges. Rotary inertia of the plate has not been considered. The deflection of middle surface of the plate is approximated by using shape function W(x,y) which satisfy the boundary conditions on the edges x = 0, x = a and y = 0, y = b. By applying the Galerkin method, we find the solution of simply supported rectangular plate with attached concentrated masses as shown in Fig. 2.

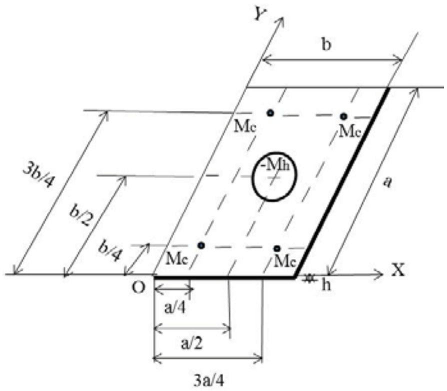

Fig. 2. Coordinates of the plate simply supported on all edges carrying concentrated masses with attached hole

The modified coefficients a_{ik} of Galerkin's system of equations is given by [14]:

$$a_{ik} = a_{ki} = \iint_A \left[D\nabla^2\nabla^2 W_i - m\omega^2 W_i \right] W_k \, dxdy - \sum (-M_h)\omega^2 W_i W_k - \sum M_c \omega^2 W_i W_k , \qquad (9)$$

where m is the mass of plate, M_c is mass of each concentrated mass and (- M_h) is negative mass of hole in plate.

The deflected surface of the plate is approximated by series as given by equation (10)

$$W(x,y) = \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} C_{ik} \sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{i\pi y}{b}\right) \qquad (10)$$

This satisfies the boundary conditions of simply supported at all the edges of the plate.

The first approximation (i.e. for i = 1 and j = 1) the coefficient a_{11} is given by equation (11)

$$a_{11} = \int_0^a \int_0^b \left[D\left(\frac{\partial^4 W_1}{\partial x^4} + 2\frac{\partial^2 W_1}{\partial x^2}\frac{\partial^2 W_1}{\partial y^2} + \frac{\partial^4 W_1}{\partial y^4} \right) - m\omega^2 W_1 \right] W_1 dx \qquad (11)$$

For i = 1, j =1, we get from (10), one term of approximation of W (x, y)

$$W_1 = \sin\frac{\pi x}{a}\sin\frac{\pi y}{b} \qquad (12)$$

Substituting W_1 in (11) and equating it to zero, we get the expression for natural frequency.

III. FINITE ELEMENT MODELLING

The result obtained by Galerkin method is compared with that obtained by ANSYS, a finite element tool. For the plate material, the element type used is SOLID 185 with 8 node brick element and for rigidly attached concentrated mass, element chosen is structural mass in ANSYS. A convergence study has been carried out for mesh generated. The dimension of plate and attached concentrated masses are shown in Table 1. The plate thickness doesnot vary and plate is simply supported along of its edges. The plate material i.e. mild steel is homogenous, linear elastic and isotropic. The material properties for all the specimens are $E = 2.051*10^{11}$ N/m^2, $\mu = 0.3$, $\rho = 7850$ Kg/m^3.

Table 1: Rectangular plate specimen with different concentrated masses attached

Specimen No	a(m)	b(m)	h(m)	Concentrated mass M (Kg)
1	1	1.5	0.005	4
2	1	1.5	0.005	4.5
3	1	1.5	0.005	5
4	1	1.5	0.005	10
5	1	1.5	0.005	15
6	1	1.5	0.005	20
7	1	1.5	0.005	25

IV. RESULTS AND DISCUSSIONS

Now, the comparison of natural frequencies of the plate with attached concentrated masses for the first mode is given in Table 2. Last column of the Table 2 shows percentage error between natural frequency values obtained by Galerkin and ANSYS.

Table 2: Fundamental frequency results of numerical (Galerkin) and FEM simulations

Specimen No	M(kg) Attached concentrated mass in % of plate mass	ω (Hz) (Galerkin)	ω (Hz) (ANSYS)	% Error
1	6.7	52.769	52.802	0.06
2	7.6	52.070	50.435	3.2
3	8.4	51.429	48.336	6.39
4	16.9	48.994	41.969	16.73
5	25.4	46.887	38.593	21.49
6	33.9	45.923	36.861	24.58
7	42.4	44.163	34.013	32.72

In this, we have used one term of approximation for candidate mode in Galerkin method. Result shows that when concentrated mass is upto 6.7% of the plate mass, the error in natural frequency is less than 0.06% which can be an acceptable value. It is obvious that more number of terms in candidate mode shape may be required for higher value of attached concentrated mass. With increase in attached concentrated masses, deviation in natural frequencies increase.

CONCLUSIONS

In this paper, we have used one term of approximation for candidate mode shape in Galerkin method. Results shows that when concentrated mass is upto 6.7% of the plate mass the error in natural frequency is less than 0.06%, which can be an acceptable value. With increase in concentrated mass, error in natural frequency increases with one term approximation in candidate mode shape.

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Available Online at www.ijarcs.info