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Thermoelastic Response Of A Thin Rectangular Plate Due To Heat Generation And Its Thermal Stresses

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Abstract: This paper is concerned with steady-state as well as transient thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a thin rectangular plate when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.

Keywords: Thin rectangular plate, transient problem, thermoelastic problem, thermal stresses, integral transform

INTRODUCTION

Adams et al. [1] studied thermoelastic vibrations of a laminated rectangular plate subjected to a thermal shock. Boley et al. [2] developed theory of thermal stresses. Carslaw et al. [3] studied conduction of heat in solids. Dhaliwal et al. [4] discussed generalized thermo elasticity for an isotropic media Ghume et al. [5] discussed deflection of a thick rectangular plate. Green et al. [6] studied Thermo elasticity of different objects. Grysa and Kozlowski [7, 8] studied one dimensional problems of temperature and heat flux determination at the surfaces of a thermoelastic slab. Hetnarski et al. [9] discussed generalized thermo elasticity: closed-form solutions. Ishihara et al. [10] discussed Theoretical analysis of thermoelastic plastic deformation of a circular plate due to partially distributed heat supply. Jadhav et al. [11] studied an inverse thermoelastic problem of a thin finite rectangular plate due to internal heat source. Khobragade et al. [12] discussed thermal deflection of a thick clamped rectangular plate. Khobragade et al. [13] studied an inverse unsteady-state thermoelastic problem of a thin rectangular plate. Lamba et al. [14] discussed thermoelastic problem of a thin rectangular plate due to partially distributed heat supply. Lekhnitskii et al. [15] studied theory of elasticity of an anisotropic body. Noda et al. [16] studied an inverse transient thermoelastic problem for a transversely isotropic body. Noda et al. [17] have written a book on thermal stresses. Noda et al Ishihara [18] discussed Theoretical analysis of thermoelastic-plastic deformation of a circular plate due to a partially distributed heat supply. Nowacki [19] studied thermo elasticity on different objects. Ozisik [20] discussed boundary value problems of heat conduction.

Recently Patil et al. [21] studied direct thermoelastic problem of heat conduction with internal heat generation and partially distributed heat supply in rectangular plate. Roy et al. [22] discussed thermal stresses of a semi infinite rectangular beam. Roy et al. [23] studied transient thermoelastic problem of an infinite rectangular slab. Roychoudhary [24] discussed thermoelastic vibrations of a simply supported rectangular plate produced by temperature prescribed on the faces. Sabherwal [25] studied an inverse problem of transient heat conduction. Sharma; Sharma and Sharma [26] discussed behavior of thermoelastic thick plate under lateral loads. Sneddon [27] has written a book on the use of integral transform. Sokolnikoff [28] developed Mathematical theory of elasticity. Sugano et al. [29] studied three-dimensional analysis of transient thermal stresses in a non homogenous plate. Sutar et al. [30] discussed an inverse thermoelastic problem of heat conduction with internal heat generation for the rectangular plate. Tanigawa et al. [31] studied thermal stress analysis of a rectangular plate and its thermal stress intensity factor for compressive stress field. Tanigawa et al. [32] discussed optimization of material composition to minimize thermal stresses in non homogeneous plate subjected to unsteady heat supply. Wankhede [33] studied the quasi- static thermal stresses in a circular plate.

In this paper, an attempt is made to determine the temperature distribution, displacement function and thermal stresses at any point of the plate occupying the space D: $\{-a \le x \le a, -b \le y \le b, -h \le z \le h\}$ with the known boundary conditions. Finite Marchi-Fasulo transform technique is used to find the solution of the problem.

STATEMENT OF THE PROBLEM-I

Consider a thin rectangular plate occupying the space D: $-a \le x \le a, -b \le y \le b, -h \le z \le h$ .The displacement components u_x, u_y and u_z in the x, y and z directions respectively as Tanigawa et al. [31] are

$$u_x = \int_{-a}^{a} \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} - \nu \frac{\partial^2 U}{\partial x^2} \right) + \lambda T \right] dx \tag{2.1}$$

$$u_y = \int_{-b}^{b} \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} - \nu \frac{\partial^2 U}{\partial y^2} \right) + \lambda T \right] dy \tag{2.2}$$

$$u_z = \int_{-h}^{h} \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial z^2} \right) + \lambda T \right] dz \tag{2.3}$$

where E, ν, and λ are the young's modulus, Poisson's ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and U (x,y,z) is the Airy's stress functions which satisfy the differential equation as Tanigawa et al. [31] is

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2 U(x,y,z) = -\lambda E\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\times T(x,y,z) \tag{2.4}$$

where T(x,y,z) denotes the temperature of a rectangular beam satisfy the following differential equation as Tanigawa et al**.** [31] is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{g(x,y,z)}{k}=0 \tag{2.5}$$

where k is the thermal conductivity of the material, subject to the boundary conditions:

$$T(x,y,z)+k_1\left[\frac{\partial T(x,y,z)}{\partial x}\right]_{x=-a} = f_1(y,z) \tag{2.6}$$

$$T(x,y,z)+k_2\left[\frac{\partial T(x,y,z)}{\partial x}\right]_{x=a} = f_2(y,z) \tag{2.7}$$

$$\left[T(x,y,z)+k_3\frac{\partial T(x,y,z)}{\partial y}\right]_{y=-b} = f_3(x,z) \tag{2.8}$$

$$\left[T(x,y,z)+k_4\frac{\partial T(x,y,z)}{\partial y}\right]_{y=b} = f_4(x,z) \tag{2.9}$$

$$\left[T(x,y,z)\right]_{z=-h} = f_5(x,y,t) \tag{2.10}$$

$$\left[T(x,y,z)\right]_{z=h} = f_6(x,y,t) \tag{2.11}$$

The stress components in terms of U(x, y, z, t) as Tanigawa et al. [31] are given by

$$\sigma_{xx} = \left[\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right] \tag{2.12}$$

$$\sigma_{yy} = \left[\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}\right] \tag{2.13}$$

$$\sigma_{zz} = \left[\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right] \tag{2.14}$$

Equations (2.1) to (2.14) constitute the mathematical formulation of the problem under consideration.

SOLUTION OF THE PROBLEM

By applying Marchi-Fasulo transform defined in [12] w.r.to x and y successively , we get

$$\left(\frac{\partial^2 \bar{\bar{T}}}{\partial z^2}\right)-\mu\bar{\bar{T}}=\psi \tag{3.1}$$

where, $\mu = \left(\lambda_n^2+\mu_m^2\right)$

$$\Psi = \frac{P_n(a)}{k_1}f_2-\frac{P_n(-a)}{k_2}f_1+\frac{Q_m(b)}{k_4}f_4-\frac{Q_m(-b)}{k_3}f_3+\frac{g}{k}$$

Equation (3.1) is a second order differential equation whose solution is given by

$$\bar{\bar{T}} = \mathrm{A}e^{\mu z}+\mathrm{B}e^{-\mu z}+\Pi \tag{3.2}$$

where, $P.I = \frac{1}{D^2-\mu}\psi = \Pi$

Using equations (2.10) and (2.11) in equation (3.2) we get

$$\bar{\bar{T}} = \frac{\left[f_5+\Pi_{-h}\right]}{\sinh(2\mu h)}\sinh\mu(z-h)+\frac{\left[f_6+\Pi_h\right]}{\sinh(2\mu h)}\sinh\mu(z+h) \tag{3.3}$$

Further applying inversion of Marchi-Fasulo transform to the equation (3.3) one obtains the expression for temperature distribution as

$$T(x,y,z) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{P_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\Omega \tag{3.4}$$

where m, n are the positive integers.

$$\Omega = \frac{\left[f_5+\Pi_{-h}\right]}{\sinh(2\mu h)}\sinh\mu(z-h)+\frac{\left[f_6+\Pi_h\right]}{\sinh(2\mu h)}\sinh\mu(z+h)$$

Substituting equation (3.2) in equation (2.4), we get

$$U = -\lambda E\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{P_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\Omega \tag{3.5}$$

Substituting equation (3.4) in equations (2.1) - (2.3), the displacement components are obtained as

$$u_x = \lambda\left\{\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{Q_m(y)}{\mu_m}\Omega-\frac{Q_m''(y)}{\mu_m}\Omega-\frac{Q_m(y)}{\mu_m}\Omega''\right]\right\} \times\frac{1}{\lambda_n}\int_{-a}^{a}P_n(x)dx+v\frac{P'_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_n}\Lambda \tag{3.6}$$

$$u_y = \lambda\left\{\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{P_n(x)}{\lambda_n}\Omega-\frac{P''_n(x)}{\lambda_n}\Omega-\frac{P_n(x)}{\lambda_n}\Omega''\right]\right\} \times\frac{1}{\mu_m}\int_{-b}^{b}Q_m(y)dy+v\frac{P'_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_n}\Omega \tag{3.7}$$

$$u_z = \lambda\left\{\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{P'_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_n}-\frac{P''_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_n}-\frac{P'_n(x)}{\lambda_n}\frac{Q''_m(y)}{\mu_n}\right]'\right\} \times\int_{-h}^{h}\Omega dz+v\frac{P'_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_n}\Omega \tag{3.8}$$

DETERMINATION OF STRESS FUNCTION

Substituting the value of Airy's stress function U(x,y,z) from equation (3.2) in the equations (2.13) to (2.14) one obtain the stress functions as,

$$\sigma_{xx} = -\lambda E\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{P_n(x)}{\lambda_n}\frac{Q''_m(y)}{\mu_m}\Omega+\frac{P_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\Omega''\right] \tag{4.1}$$

$$\sigma_{yy} = -\lambda E\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{P''_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\Omega+\frac{P_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\Omega''\right] \tag{4.2}$$

$$\sigma_{zz} = -\lambda E\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{P''_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\Omega+\frac{P_n(x)}{\lambda_n}\frac{Q''_m(y)}{\mu_m}\Omega\right] \tag{4.3}$$

SPECIAL CASE

Set $f_5(x,y) = (x^2+ax)(y^2+ay)(-h)$,

$f_6(x,y) = (x^2+ax)(y^2+ay)(h)$

$$\bar{\bar{f}}_5(x,y) = \left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{a_n^{\ 2}}\right] \times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{b_m^{\ 2}}\right]\times(-h) \tag{5.1}$$

$$\bar{\bar{f}}_6(x,y) = \left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{a_n^{\ 2}}\right] \times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{b_m^{\ 2}}\right]\times(h) \tag{5.2}$$

Substitute these values in equations (3.4) to (3.5) one obtains

$$T(x,y,z)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{P_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\left[\frac{\left[\left[\frac{a_n\cos^2(a_na)-\cos(a_na)\sin(a_na)}{a_n^{2}}\right]\times\left[\frac{b_m\cos^2(b_mb)-\cos(b_mb)\sin(b_mb)}{b_m^{2}}\right]\times(-h)+Z_{-h}\right]}{\sinh 2\mu_p h}\sinh\mu_p(z-h)+\frac{\left[\left[\frac{a_n\cos^2(a_na)-\cos(a_na)\sin(a_na)}{a_n^{2}}\right]\times\left[\frac{b_m\cos^2(b_mb)-\cos(b_mb)\sin(b_mb)}{b_m^{2}}\right]\times(h)+Z_{h}\right]}{\sinh 2\mu_p h}\sinh\mu_p(z+h)\right] \tag{5.3}$$

$$U=-\lambda E\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{P_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\left[\frac{\left[\left[\frac{a_n\cos^2(a_na)-\cos(a_na)\sin(a_na)}{a_n^{2}}\right]\times\left[\frac{b_m\cos^2(b_mb)-\cos(b_mb)\sin(b_mb)}{b_m^{2}}\right]\times(-h)+Z_{-h}\right]}{\sinh 2\mu_p h}\sinh\mu_p(z-h)+\frac{\left[\left[\frac{a_n\cos^2(a_na)-\cos(a_na)\sin(a_na)}{a_n^{2}}\right]\times\left[\frac{b_m\cos^2(b_mb)-\cos(b_mb)\sin(b_mb)}{b_m^{2}}\right]\times(h)+Z_{h}\right]}{\sinh 2\mu_p h}\sinh\mu_p(z+h)\right] \tag{5.4}$$

NUMERICAL RESULTS

Set a = 1m, b = 2m, h = 2m,t = 1sec and k= 0.86 in equation (5.3) to (5.4), we obtain

$$T(x,y,z)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{P_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\left[\frac{\left[\left[\frac{a_n\cos^2(a_n)-\cos(a_n)\sin(a_n)}{a_n^{2}}\right]\times\left[\frac{b_m\cos^2(2b_m)-\cos(2b_m)\sin(2b_m)}{b_m^{2}}\right]\times(-2)+Z_{-2}\right]}{\sinh 4\mu_p}\sinh\mu_p(z-2)+\frac{\left[\left[\frac{a_n\cos^2(a_n)-\cos(a_n)\sin(a_n)}{a_n^{2}}\right]\times\left[\frac{b_m\cos^2(2b_m)-\cos(2b_m)\sin(2b_m)}{b_m^{2}}\right]\times(2)+Z_{2}\right]}{\sinh 4\mu_p}\sinh\mu_p(z+2)\right] \tag{6.1}$$

$$U=-\lambda E\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{P_n(x)}{\lambda_n}\frac{Q_m(y)}{\mu_m}\left[\frac{\left[\left[\frac{a_n\cos^2(a_n)-\cos(a_n)\sin(a_n)}{a_n^{2}}\right]\times\left[\frac{b_m\cos^2(2b_m)-\cos(2b_m)\sin(2b_m)}{b_m^{2}}\right]\times(-2)+Z_{-2}\right]}{\sinh 4\mu_p}\sinh\mu_p(z-2)+\frac{\left[\left[\frac{a_n\cos^2(a_n)-\cos(a_n)\sin(a_n)}{a_n^{2}}\right]\times\left[\frac{b_m\cos^2(2b_m)-\cos(2b_m)\sin(2b_m)}{b_m^{2}}\right]\times(2)+Z_{2}\right]}{\sinh 4\mu_p}\sinh\mu_p(z+2)\right] \tag{6.2}$$

STATEMENT OF THE PROBLEM-II

Consider a thin rectangular plate occupying the space D: $-a\le x\le a,-b\le y\le b,-h\le z\le h$ .The displacement components u_x and u_y u_z in the x and y and z directions respectively as Tanigawa et al. [31] are

$$u_x=\int_{-a}^{a}\left[\frac{1}{E}\left(\frac{\partial^2U}{\partial y^2}+\frac{\partial^2U}{\partial z^2}-v\frac{\partial^2U}{\partial x^2}\right)+\lambda T\right]dx \tag{7.1}$$

$$u_y=\int_{-b}^{b}\left[\frac{1}{E}\left(\frac{\partial^2U}{\partial z^2}+\frac{\partial^2U}{\partial x^2}-v\frac{\partial^2U}{\partial y^2}\right)+\lambda T\right]dy \tag{7.2}$$

$$u_z=\int_{-h}^{h}\left[\frac{1}{E}\left(\frac{\partial^2U}{\partial x^2}+\frac{\partial^2U}{\partial y^2}-v\frac{\partial^2U}{\partial z^2}\right)+\lambda T\right]dz \tag{7.3}$$

where E, ν, and λ are the young's modulus, Poisson's ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and U (x,y,z,t) is the Airy's stress functions which satisfy the differential equation as Tanigawa et al. [31] is

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2U(x,y,z,t)=-\lambda E\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\times T(x,y,z,t) \tag{7.4}$$

where T(x,y,z,t) denotes the temperature of a rectangular beam satisfy the following differential equation as Tanigawa et al**.** [31] is

$$\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y^2}+\frac{\partial^2T}{\partial z^2}+\frac{g(x,y,z,t)}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{7.5}$$

where k is the thermal conductivity and α is the thermal diffusivity of the material,
subject to the initial and boundary conditions:

$$T(x,y,z,0)=F(x,y,z) \tag{7.6}$$

$$T(x,y,z,t)+k_1\left[\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=-a}=f_1(y,z,t) \tag{7.7}$$

$$T(x,y,z,t)+k_2\left[\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=a}=f_2(y,z,t) \tag{7.8}$$

$$\left[T(x,y,z,t)+k_3\frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=-b}=f_3(x,z,t) \tag{7.9}$$

$$\left[T(x,y,z,t)+k_4\frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=b}=f_4(x,z,t) \tag{7.10}$$

$$\left[T(x,y,z,t)+k_5\frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=-h}=f_5(x,y,t) \tag{7.11}$$

$$\left[T(x,y,z,t)+k_6\frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=h}=f_6(x,y,t) \tag{7.12}$$

The stress components in terms of U(x, y, z, t) as Tanigawa et al. [31] are given by

$$\sigma_{xx}=\left[\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right] \tag{7.13}$$

$$\sigma_{yy}=\left[\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}\right] \tag{7.14}$$

$$\sigma_{zz}=\left[\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right] \tag{7.15}$$

Equations (7.1) to (7.15) constitute the mathematical formulation of the problem under consideration.

SOLUTION OF THE PROBLEM

By applying Marchi-Fasulo transform defined in [12] w.r.to x , y and z successively, we get

$$\left(\frac{\partial \overline{\overline{T}}^*}{\partial t}\right)+\alpha p^2\overline{\overline{T^*}}=\psi \tag{8.1}$$

Where $p^2=\left(\lambda_l{}^2+\mu_m{}^2+\eta_n{}^2\right)$

$$\Psi=\frac{P_l(a)}{k_1}f_2-\frac{P_l(-a)}{k_2}f_1+\frac{Q_m(b)}{k_4}f_4-\frac{Q_m(-b)}{k_3}f_3+\frac{R_n(h)}{k_6}-\frac{R_n(-h)}{k_5}+\frac{g}{k}$$

This is linear equation.

Further using their inverses in equation (8.1), one obtains the expression for temperature distribution as

$$T(x,y,z,t)=e^{\alpha p^2 t}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_l(x)}{\lambda_l}\,\frac{Q_m(y)}{\mu_m}\,\frac{R_n(z)}{\eta_n}\times\left[\overline{\overline{\overline{F}}}+\int_0^t\psi e^{\alpha p^2 t'}dt'\right] \tag{8.2}$$

where l, m, n are the positive integers.

Substituting equation (8.2) in equation (2.4) we get

$$U=-\lambda E e^{\alpha p^2 t}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_l(x)}{\lambda_l}\,\frac{Q_m(y)}{\mu_m}\,\frac{R_n(z)}{\eta_n}\times\left[\overline{\overline{\overline{F}}}+\int_0^t\psi e^{\alpha p^2 t'}dt'\right] \tag{8.3}$$

Substituting equation (3.3) in equations (8.1) - (8.3), the displacement components are obtained as

$$u_x=\lambda\int_{-a}^{a}\frac{e^{\alpha p^2 t}}{\lambda_l\mu_m\eta_n}\left[\overline{\overline{F}}+\int_0^t\psi e^{\alpha p^2 t'}dt'\right]\times\Big(P_l(x)Q_m(y)R_n(z)-P_l(x)Q_m{}''(y)R_n(z)-P_l(x)Q_m(y)R_n''(z)+\nu P_l''(x)Q_m(y)R_n(z)\Big)\,\mathrm{dx} \tag{8.4}$$

$$u_y=\lambda\int_{-b}^{b}\frac{e^{\alpha p^2 t}}{\lambda_l\mu_m\eta_n}\left[\overline{\overline{F}}+\int_0^t\psi e^{\alpha p^2 t'}dt'\right]\times\Big(P_l(x)Q_m(y)R_n(z)-P_l''(x)Q_m(y)R_n''(z)-P_l(x)Q_m(y)R_n''(z)+\nu P_l(x)Q_m''(y)R_n(z)\Big)dy \tag{8.5}$$

$$u_z=\lambda\int_{-h}^{h}\frac{e^{\alpha p^2 t}}{\lambda_l\mu_m\eta_n}\left[\overline{\overline{F}}+\int_0^t\psi e^{\alpha p^2 t'}dt'\right]\times\Big(P_l(x)Q_m(y)R_n(z)-P_l''(x)Q_m(y)R_n(z)-P_l(x)Q_m(y)R_n(z)+\nu P_l(x)Q_m(y)R_n''(z)\Big)dz \tag{8.6}$$

DETERMINATION OF STRESS FUNCTION

Substituting the value of Airy's stress function U(x,y,z,t) from equation (8.2) in the equations (7.13) to (7.15) one obtain the stress functions as,

$$\sigma_{xx}=-\lambda E e^{\alpha p^2 t}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_l(x)}{\lambda_l\mu_m\eta_n}\times\left[\overline{\overline{\overline{F}}}+\int_0^t\psi e^{\alpha p^2 t'}dt'\right]\times\left[Q_m{}''(y)R_n(z)+Q_m(y)R_n{}''(z)\right] \tag{9.1}$$

$$\sigma_{yy}=-\lambda E e^{\alpha p^2 t}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{Q_m(y)}{\lambda_l\mu_m\eta_n}\times\left[\overline{\overline{\overline{F}}}+\int_0^t\psi e^{\alpha p^2 t'}dt'\right]\times\left[P_l''(x)R_n(z)\,(7.15)+P_l{}''(x)R_n(z)\right] \tag{9.2}$$

$$\sigma_{zz}=-\lambda E e^{\alpha p^2 t}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{R_n(z)}{\lambda_l\mu_m\eta_n}\times\left[\overline{\overline{\overline{F}}}+\int_0^t\psi e^{\alpha p^2 t'}dt'\right]\times\left[P_l{}''(x)Q_m(y)+P_l(x)Q_m{}''(y)\right] \tag{9.3}$$

SPECIAL CASE

Set $f(x,y,t)=(e^{-t})(x+a)^2(x-a)^2(y+b)^2(y-b)^2$,

$F(x,y,z)=(e^{-t})(x+a)^2(x-a)^2(y+b)^2(y-b)^2(z+h)^2(z-h)^2$

$$\overline{\overline{F}}^*(x,y,z)=(e^{-t})\times\left[\frac{a_l\cos^2(a_la)-\cos(a_na)\sin(a_la)}{a_l{}^2}\right]\times\left[\frac{b_m\cos^2(b_mb)-\cos(b_mb)\sin(b_mb)}{b_m{}^2}\right]\times\left[\frac{h_n\cos^2(h_nh)-\cos(h_nh)\sin(h_nh)}{h_n{}^2}\right] \tag{10.1}$$

Substituting this value in equations (8.2)- (9.3) we get

$$T(x,y,z,t)=e^{\alpha p^2}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_l(x)}{\lambda_l}\,\frac{Q_m(y)}{\mu_m}\,\frac{R_n(z)}{\eta_n}\left\{(e^{-t})\times\left[\frac{a_l\cos^2(a_la)-\cos(a_na)\sin(a_la)}{a_l{}^2}\right]\times\left[\frac{b_m\cos^2(b_mb)-\cos(b_mb)\sin(b_mb)}{b_m{}^2}\right]\times\left[\frac{h_n\cos^2(h_nh)-\cos(h_nh)\sin(h_nh)}{h_n{}^2}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\right\} \tag{10.2}$$

$$U=-\lambda E e^{\alpha p^2 t}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_l(x)}{\lambda_l}\,\frac{Q_m(y)}{\mu_m}\,\frac{R_n(z)}{\eta_n}\left\{(e^{-t})\times\left[\frac{a_l\cos^2(a_la)-\cos(a_na)\sin(a_la)}{a_l{}^2}\right]\times\left[\frac{b_m\cos^2(b_mb)-\cos(b_mb)\sin(b_mb)}{b_m{}^2}\right]\times\left[\frac{h_n\cos^2(h_nh)-\cos(h_nh)\sin(h_nh)}{h_n{}^2}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\right\} \tag{10.3}$$

$$u_x=\lambda\int_{-a}^{a}\frac{e^{\alpha p^2 t}}{\lambda_l\mu_m\eta_n}\left\{(e^{-t})\times\left[\frac{a_l\cos^2(a_la)-\cos(a_na)\sin(a_la)}{a_l{}^2}\right]\right.$$

$$\times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{b_m{}^2}\right]$$

$$\times\left[\frac{h_n\cos^2(h_n h)-\cos(h_n h)\sin(h_n h)}{h_n{}^2}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\Bigg\}$$

$$\times\left(P_l(x)Q_m(y)R_n(z)-P_l(x)Q''_m(y)R_n(z)-P_l(x)Q_m(y)R''_n(z)+\nu P''_l((x)Q_m(y)R_n(z))\right)dx \tag{10.4}$$

$$u_y=\lambda\int_{-b}^{b}\frac{e^{\alpha p^2 t}}{\lambda_l\mu_m\eta_n}\Bigg\{(e^{-t})\times\left[\frac{a_l\cos^2(a_l a)-\cos(a_n a)\sin(a_l a)}{a_l{}^2}\right]$$

$$\times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{b_m{}^2}\right]$$

$$\times\left[\frac{h_n\cos^2(h_n h)-\cos(h_n h)\sin(h_n h)}{h_n{}^2}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\Bigg\}$$

$$\times\left(P_l(x)Q_m(y)R_n(z)-P''_l(x)Q_m(y)R_n(z)-P_l(x)Q_m(y)R''_n(z)+\nu P_l(x)Q''_m(y)R_n(z)\right)dy \tag{10.5}$$

$$u_z=\lambda\int_{-h}^{h}\frac{e^{\alpha p^2 t}}{\lambda_l\mu_m\eta_n}\Bigg\{(e^{-t})\times\left[\frac{a_l\cos^2(a_l a)-\cos(a_n a)\sin(a_l a)}{a_l{}^2}\right]$$

$$\times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{b_m{}^2}\right]$$

$$\times\left[\frac{h_n\cos^2(h_n h)-\cos(h_n h)\sin(h_n h)}{h_n{}^2}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\Bigg\}$$

$$\times\left(P_l(x)Q_m(y)R_n(z)-P''_l(x)Q_m(y)R_n(z)-P_l(x)Q''_m(y)R_n(z)+\nu P_l(x)Q_m(y)R''_n(z)\right)dz \tag{10.6}$$

$$\sigma_{xx}=-\lambda Ee^{\alpha p^2 t}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_l(x)}{\lambda_l\mu_m\eta_n}$$

$$\times\Bigg\{(e^{-t})\times\left[\frac{a_l\cos^2(a_l a)-\cos(a_n a)\sin(a_l a)}{a_l{}^2}\right]$$

$$\times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{b_m{}^2}\right]$$

$$\times\left[\frac{h_n\cos^2(h_n h)-\cos(h_n h)\sin(h_n h)}{h_n{}^2}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\Bigg\}$$

$$\times\left[Q''_m(y)R_n(z)+Q_m(y)R''_n(z)\right] \tag{10.7}$$

$$\sigma_{yy}=-\lambda Ee^{\alpha p^2 t}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{Q_m(y)}{\lambda_l\mu_m\eta_n}$$

$$\times\Bigg\{(e^{-t})\times\left[\frac{a_l\cos^2(a_l a)-\cos(a_n a)\sin(a_l a)}{a_l{}^2}\right]$$

$$\times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{b_m{}^2}\right]$$

$$\times\left[\frac{h_n\cos^2(h_n h)-\cos(h_n h)\sin(h_n h)}{h_n{}^2}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\Bigg\}$$

$$\times\left[P_l(x)R''_n(z)+P''_l(x)R_n(z)\right] \tag{10.8}$$

$$\sigma_{zz}=-\lambda Ee^{\alpha p^2 t}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{R_n(z)}{\lambda_l\mu_m\eta_n}$$

$$\times\Bigg\{(e^{-t})\times\left[\frac{a_l\cos^2(a_l a)-\cos(a_n a)\sin(a_l a)}{a_l{}^2}\right]$$

$$\times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(2b_m)}{b_m{}^2}\right]$$

$$\times\left[\frac{h_n\cos^2(h_n h)-\cos(h_n h)\sin(h_n h)}{h_n{}^2}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\Bigg\}$$

$$\times\left[P''_l(x)Q_m(y)+P_l(x)Q''_m(y)\right] \tag{10.9}$$

NUMERICAL RESULTS

Set a = 1, b = 2, h = 2,t = 1sec and k= 0.86 in equation (10.1)-(10.9), we obtain

$$T(x,y,z,t)=e^{\alpha p^2}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_l(x)}{\lambda_l}\frac{Q_m(y)}{\mu_m}\frac{R_n(z)}{\eta_n}$$

$$\times\Bigg\{(e^{-t})\times\left[\frac{a_l\cos^2(a_l)-\cos(a_n)\sin(a_l)}{a_l{}^2}\right]$$

$$\times\left[\frac{b_m\cos^2(2b_m)-\cos(2b_m)\sin(2b_m)}{b_m{}^2}\right]$$

$$\times\left[\frac{h_n\cos^2(2h_n)-\cos(2h_n)\sin(2h_n)}{h_n{}^2}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\Bigg\} \tag{11.1}$$

$$U=-\lambda Ee^{\alpha p^2 t}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_l(x)}{\lambda_l}\frac{Q_m(y)}{\mu_m}\frac{R_n(z)}{\eta_n}$$

$$\times\Bigg\{(e^{-t})\times\left[\frac{a_l\cos^2(a_l)-\cos(a_n)\sin(a_l)}{a_l{}^2}\right]$$

$$\times\left[\frac{b_m\cos^2(2b_m)-\cos(2b_m)\sin(2b_m)}{b_m{}^2}\right]$$

$$\times\left[\frac{h_n\cos^2(2h_n)-\cos(2h_n)\sin(2h_n)}{h_n{}^2}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\Bigg\} \tag{11.2}$$

$$u_x=\lambda\int_{-1}^{1}\frac{e^{\alpha p^2}}{\lambda_l\mu_m\eta_n}\Bigg\{(e^{-t})\times\left[\frac{a_l\cos^2(a_l)-\cos(a_n)\sin(a_l)}{a_l{}^2}\right]$$

$$\times\left[\frac{b_m\cos^2(2b_m)-\cos(2b_m)\sin(2b_m)}{b_m{}^2}\right]$$

$$\times\left[\frac{h_n\cos^2(2h_n)-\cos(2h_n)\sin(2h_n)}{h_n{}^2}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\Bigg\}$$

$$\times\left(P_l(x)Q_m(y)R_n(z)-P_l(x)Q''_m(y)R_n(z)-P_l(x)Q_m(y)R''_n(z)+\nu P''_l(x)Q_m(y)R_n(z)\right)\mathrm{dx} \tag{11.3}$$

$$u_y=\lambda\int_{-2}^{2}\frac{e^{\alpha p^2}}{\lambda_l\mu_m\eta_n}\Bigg\{\left[\frac{a_l\cos^2(a_l)-\cos(a_n)\sin(a_l)}{a_l{}^2}\right](e^{-1})$$

$$\times\left[\frac{b_m\cos^2(2b_m)-\cos(2b_m)\sin(2b_m)}{b_m{}^2}\right]$$

$$\times\left[\frac{h_n\cos^2(2h_n)-\cos(2h_n)\sin(2h_n)}{h_n{}^2}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\Bigg\}$$

$$\times\left(P_l(x)Q_m(y)R_n(z) - P''_l(x)Q_m(y)R_n(z) - P_l(x)Q_m(y)R''_n(z) + \nu P_l(x)Q_m(y)R_n(z) \right)dy \tag{11.4}$$

$$u_z = \lambda\int_{-2}^{2}\frac{e^{\alpha p^2}}{\lambda_l\mu_m\eta_n}\left\{(e^{-1})\times\left[\frac{a_l\cos^2(a_l)-\cos(a_n)\sin(a_l)}{a_l^{\,2}}\right]\times\left[\frac{b_m\cos^2(2b_m)-\cos(2b_m)\sin(2b_m)}{b_m^{\,2}}\right]\times\left[\frac{h_n\cos^2(2h_n)-\cos(2h_n)\sin(2h_n)}{h_n^{\,2}}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\right\}$$
$$\times\left(P_l(x)Q_m(y)R_n(z) - P''_l(x)Q_m(y)R_n(z) - P_l(x)Q_m(y)R_n(z) + \nu P_l(x)Q_m(y)R''_n(z) \right)dz \tag{11.5}$$

$$\sigma_{xx} = -\lambda E e^{\alpha p^2}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_l(x)}{\lambda_l\mu_m\eta_n}\times\left\{(e^{-1})\times\left[\frac{a_l\cos^2(a_l)-\cos(a_n)\sin(a_l)}{a_l^{\,2}}\right]\times\left[\frac{b_m\cos^2(2b_m)-\cos(2b_m)\sin(2b_m)}{b_m^{\,2}}\right]\times\left[\frac{h_n\cos^2(2h_n)-\cos(2h_n)\sin(2h_n)}{h_n^{\,2}}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\right\}$$
$$\times\left[Q_m''(y)R_n(z)+Q_m(y)R_n''(z)\right] \tag{11.6}$$

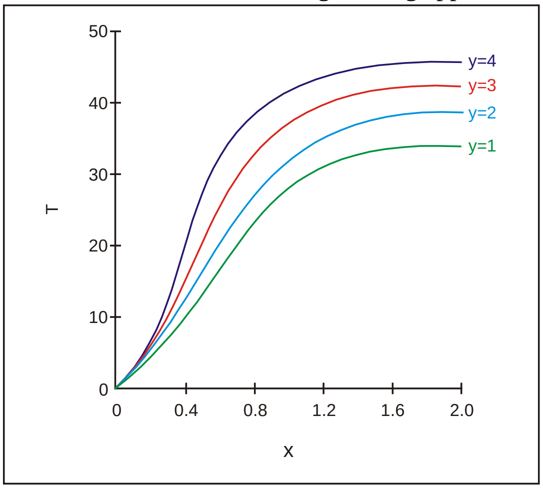
$$\sigma_{yy} = -\lambda E e^{\alpha p^2}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{Q_m(y)}{\lambda_l\mu_m\eta_n}\times\left\{(e^{-1})\times\left[\frac{a_l\cos^2(a_l)-\cos(a_n)\sin(a_l)}{a_l^{\,2}}\right]\times\left[\frac{b_m\cos^2(2b_m)-\cos(2b_m)\sin(2b_m)}{b_m^{\,2}}\right]\times\left[\frac{h_n\cos^2(2h_n)-\cos(2h_n)\sin(2h_n)}{h_n^{\,2}}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\right\}$$
$$\times\left[P_l(x)R_n''(z)+P_l''(x)R_n(z)\right] \tag{11.7}$$

$$\sigma_{zz} = -\lambda E e^{\alpha p^2}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{R_n(z)}{\lambda_l\mu_m\eta_n}\times\left\{(e^{-1})\times\left[\frac{a_l\cos^2(a_l)-\cos(a_n)\sin(a_l)}{a_l^{\,2}}\right]\times\left[\frac{b_m\cos^2(2b_m)-\cos(2b_m)\sin(2b_m)}{b_m^{\,2}}\right]\times\left[\frac{h_n\cos^2(2h_n)-\cos(2h_n)\sin(2h_n)}{h_n^{\,2}}\right]+\int_0^1\psi e^{\alpha p^2 t'}dt'\right\}$$
$$\times\left[P_l''(x)Q_m(y)+P_l(x)Q_m''(y)\right] \tag{11.8}$$

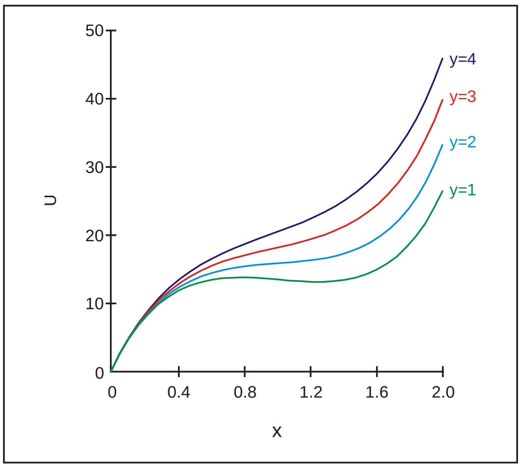
CONCLUSION

In both the problems, the temperature distribution, displacement function and thermal stresses of a thin rectangular plate have been derived, with the aid of finite Marchi-Fasulo transform technique when the stated boundary conditions are known. The results are obtained in terms of Bessel's function in the form of infinite series. The series solutions converge provided if we take sufficient number of terms in the series. The numerical results are calculated and depicted graphically.

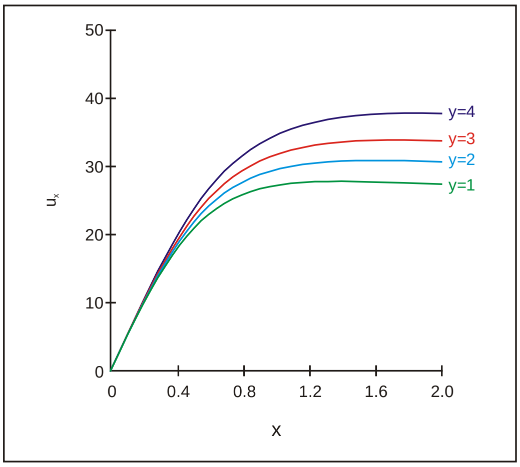
The results that are obtained can be applied to the design of useful structures or machines in engineering applications.



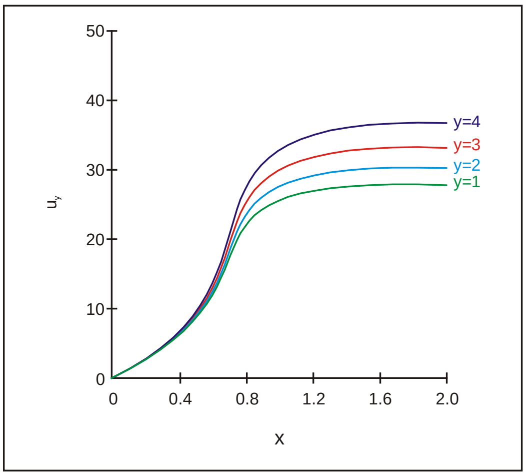
Graph. 1: Temperature distribution vs. x



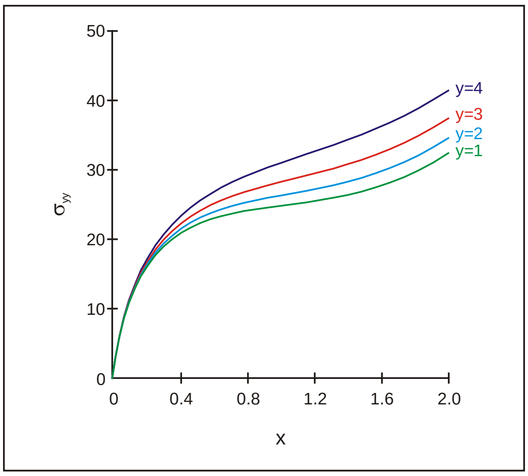
Graph. 2: Airy's stress function vs. x



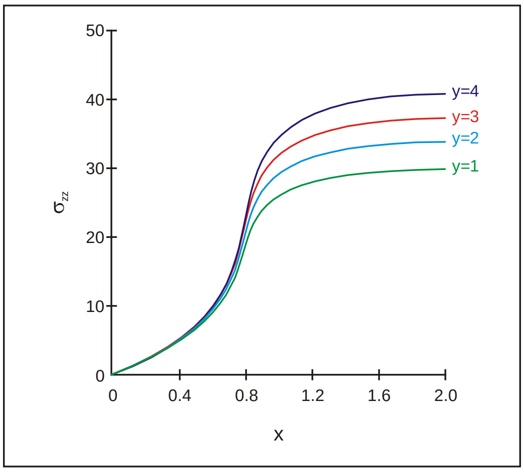
Graph. 3: Displacement component vs. x



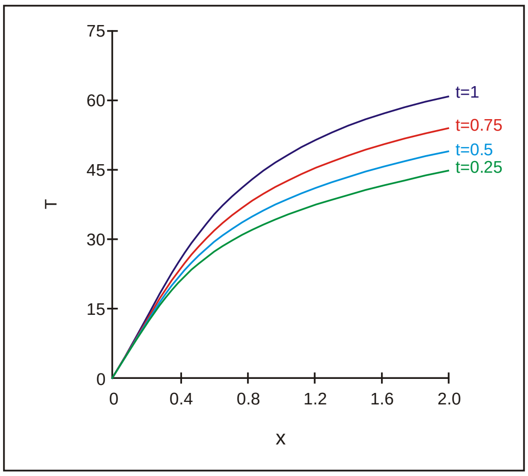
Graph. 4: Displacement component vs. x



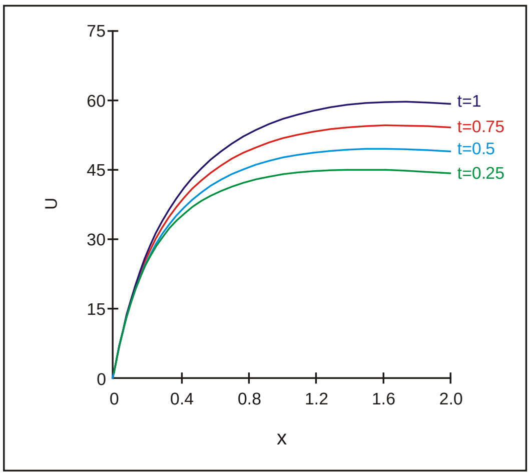
Graph. 5: Stress function vs. x



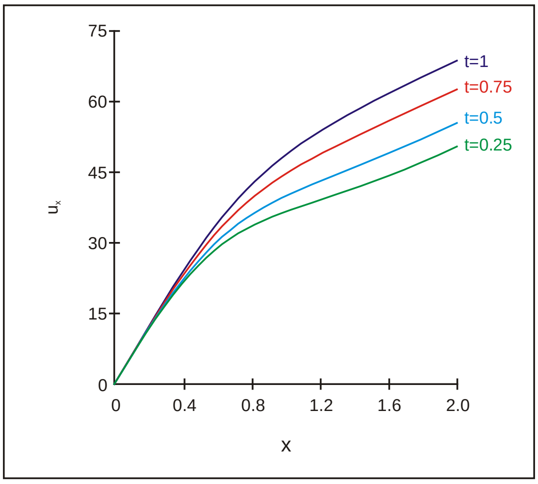
Graph. 6: Stress function vs. x



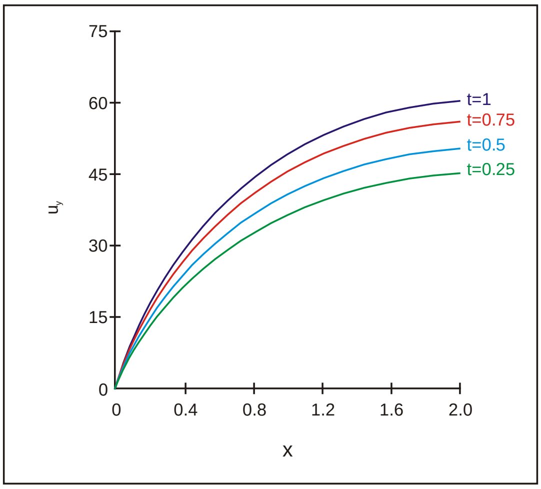
Graph 7: Temperature distribution vs x



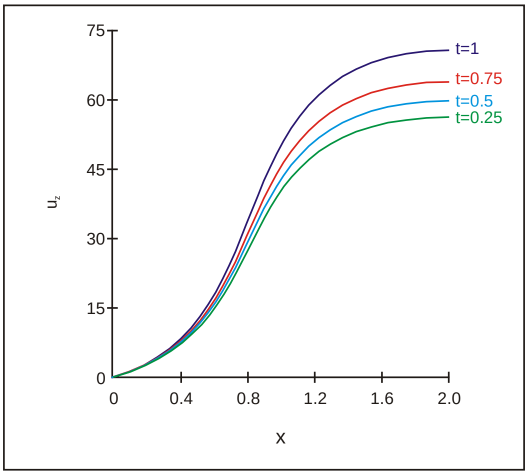
Graph 8: Airy's stress function vs x



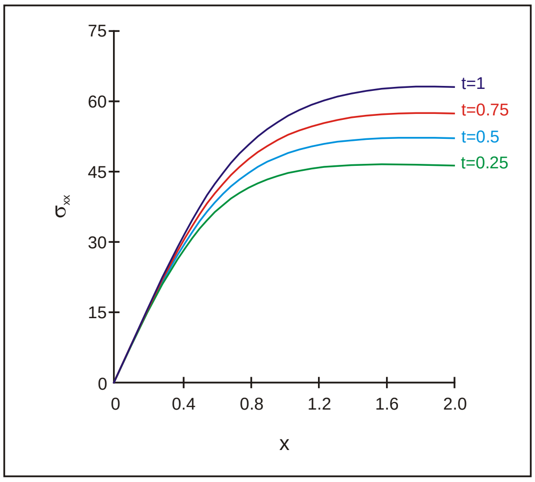
Graph 9: Displacement component vs x



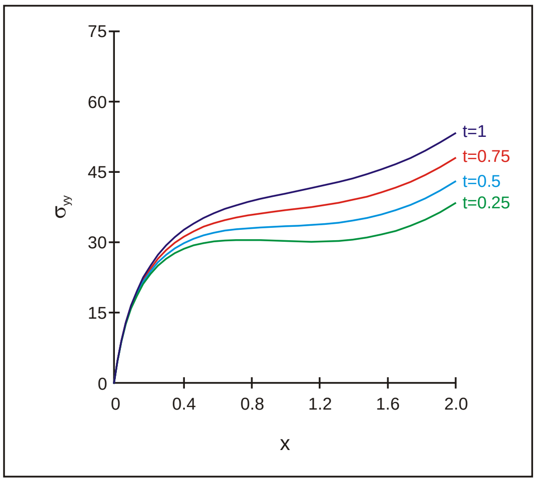
Graph 10: Displacement component vs x



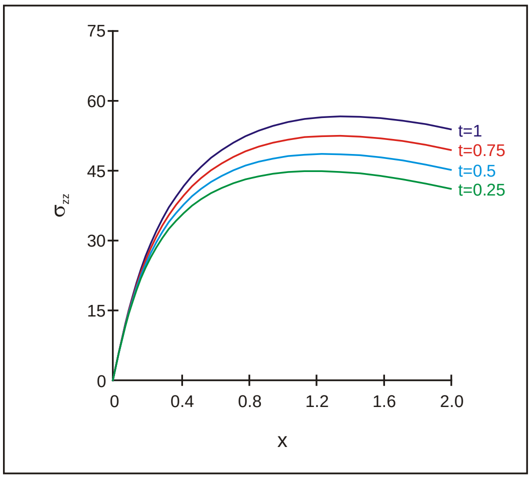
Graph 11: Displacement component vs x



Graph 12: Stress function vs x



Graph 13: Stress function vs x



Graph 14: Stress function vs x

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