ISSN No. 0976-5697
Volume 8, No. 5, May-June 2017
International Journal of Advanced Research in Computer Science
RESEARCH PAPER
Available Online at www.ijarcs.info

Thermoelastic Response Of A Thin Annular Disc Due To Partially Distributed Heat Supply And Its Thermal Stresses

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Abstract: This paper is concerned with transient thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a thin annular disc due to partially distributed heat supply, when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.   .

Keywords: Thermoelastic problem, thin annular disc partially distributed heat supply, thermal stresses, integral transforms

INTRODUCTION

In 1957, Nowacki [1] studied The state of stress in a thick circular plate due to a temperature field. Carslaw et al. [2] has written a book on Conduction of heat in solids. Boley et al.[3] developed Theory of thermal stresses. Nowacki [4] studied Thermo elasticity on different solids. Marchi et al. [5] discussed Heat conduction in hollow cylinder with radiation. Sabherwal [6] studied An inverse problem of transient heat conduction. Marchi et al.[7] discussed Heat conduction in sector of hollow cylinder with radiation. Ozisik [8] studied Boundary value problems of heat conduction. Patel [9] discussed Inverse problems of transient heat conduction with radiation. Roychaudhari [10] has succeeded in determining the quasi-static thermal stresses in a thin circular plate subjected to transient temperature along the circumference of circular upper face with lower face is at zero temperature and the fixed circular edge thermally insulated..

Wankhede [11] has determined the quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. Ishihara et al. [12] studied Theoretical analysis of thermoelastic plastic deformation of a circular plate due to partially distributed heat supply. Noda et al.[13] studied Thermal Stresses on different shapes of solid bodies. Ghadle et al. [14] discussed An inverse unsteady- state thermoelastic problem of a thin annular disc in Marchi-Fasulo transform domain. Durge et al. [15] studied An inverse steady- state thermoelastic problem of a thin annular disc in Marchi-Zgrablich transform domain. Singru et al. [16] discussed Steady-state thermoelastic problem of a thin annular disc in Marchi-Fasulo transform domain. Durge et al. [17] studied Analysis of stress functions in a thin annular disc due to partially distributed heat supply. Khobragade et al. [18] discussed An inverse transient thermoelastic problem of a thin annular disc. Durge et al. [19] studied An inverse unsteady- state thermoelastic problem of a thin annular disc in Marchi-Zgrablich transform domain, Gaikwad et al.[20] studied An inverse heat conduction problem in a thick annular disc.

In the present paper, an attempt is made to study the transient thermoelastic problem in which we need to determine the temperature (in heating and cooling process), displacement and stress functions of the disc occupying the space $a \le r \le b,\ 0 \le z \le h$ with the stated boundary condition. Here finite Fourier sine transform and Marchi-Zgrablich transform have been used to find the solution of problem. The numerical estimate for the temperature has been obtained at any point of the disc and depicted graphically.

STATEMENT OF THE PROBLEM (HEATING PROCESS)

Consider a thin annular disc occupying the space $D: a \le r \le b,\ 0 \le z \le h$. The initial temperature of the disc is the same as the temp of the surrounding medium which is kept constant for the time $t=0$ to $t=t_0$ the disc is subjected to a partially distributed axisymmetric heat supply $\left(-\frac{Q_0}{\lambda} f(r,t)\right)$ at point $z=0$. After that the heat supply is removed and disc is cooled by surrounding medium.

The derived equation governing the displacement function $U(r,z,t)$ as **(Roy Choudhary [10]**)is

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+\nu)a_r T \tag{2.1}$$

with $U_r = 0$ at $r = a$ and $r = b$.

where ν and a_t are Poisson's ratio and linear coefficient of thermal expansion of the material of the disc respectively.

$T(r,z,t)$ is the heating temperature of the disc at time t satisfying the equation (**Roy Choudhary** [**10**]):

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \tag{2.3}$$

Subjected to initial condition

$$T(r,z,t)|_{t=0} = F(r,z) \tag{2.4}$$

The boundary condition

$$\left[[T(r,z,t) + k_1 \frac{\partial T(r,z,t)}{\partial r} \right]_{r=a} = f_1(z,t) \tag{2.5}$$

$$\left[[T(r,z,t) + k_2 \frac{\partial T(r,z,t)}{\partial r} \right]_{r=b} = f_2(z,t) \tag{2.6}$$

$$[T(r,z,t)]_{z=0} = -\frac{Q_0}{\lambda} f(r,t) \tag{2.7}$$

$$[T(r,z,t)]_{z=h} = g(r,t) \tag{2.8}$$

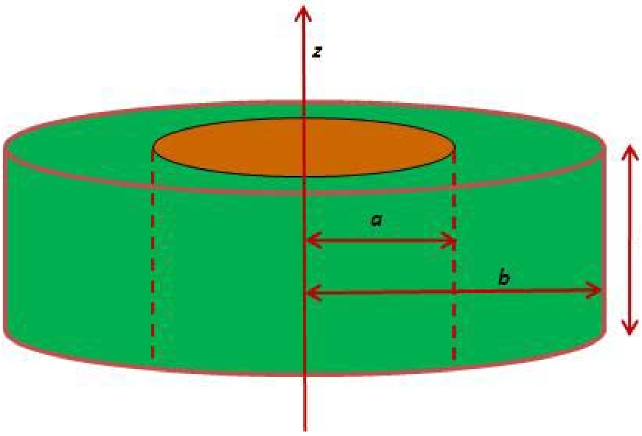
Where k & λ are the thermal diffusivity and conductivity of the material of the disc respectively, k_1 & k_2 are radiation constant on the curved surface of the disc respectively.

The stress function σ_{rr} and $\sigma_{\theta\theta}$ are given by (**Roy Choudhary** [**10**]) :

$$\sigma_{rr} = -2\mu \frac{1}{r}\frac{\partial U}{\partial r} \tag{2.9}$$

$$\sigma_{\theta\theta} = -2\mu \frac{\partial^2 U}{\partial r^2} \tag{2.10}$$

Equations (2.1) to (2.10) constitute the mathematical formulation of the problem under consideration.



SOLUTION OF THE PROBLEM

Applying finite **Marchi-Zgrablich transform [5]** to the equation (2.3), one obtains

$$\frac{d^2\overline{T}}{dz^2} - \mu_n^2 \overline{T}(n,z,t) + \psi = \frac{1}{k}\frac{d\overline{T}}{dt} \tag{3.1}$$

Where

$$\psi = \frac{b}{k_2} S_0(k_1,k_2,\mu_n b) f_2 - \frac{a}{k_1} S_0(k_1,k_2,\mu_n a) f_1 \tag{3.2}$$

Now applying **Fourier sine transform** to the equation (3.1) we get

$$\frac{d\overline{T}^*}{dt} + kp^2 \overline{T}^* = Q_1 \overline{g}(n,t) - Q_2 \overline{f}(n,t) + \psi_1 \tag{3.3}$$

where,

$p^2 = \mu_n^2 + q_m^2$, $q_m = \frac{m\pi}{h}$, $Q_1 = \frac{m\pi}{h}(-1)^{m+1}k$, $Q_2 = \frac{kq_m\phi_0}{\lambda}$, $\psi_1 = k\psi^*$

The solution of differential equation (3.3) is

$$\overline{T}^*(n,m,t) = e^{-kp^2 t} \times \left[\int_0^t (Q_1\overline{g}(n,t') - Q_2 \overline{f}(n,t') + \psi_1) e^{kp^2 t'} dt' + C \right]$$

At t = 0, $T = F(r,z) \Rightarrow C = \overline{F}^*(n,m)$

Therefore

$$\overline{T}^*(n,m,t) = e^{-kp^2 t} \times \left[\int_0^t (Q_1\overline{g}(n,t') - Q_2 \overline{f}(n,t') + \psi_1) e^{kp^2 t'} dt' + \overline{F}^*(n,m) \right] \tag{3.4}$$

Applying inverse Fourier sine transform and inverse Marchi-Zgrablich transform to equation (3.4) we get

$$T(r,z,t) = \frac{2}{h}\sum_m \sum_n \left(\frac{\sin q_m z}{c_n} \right) e^{-kp^2 t} \times \left[\int_0^t (Q_1\overline{g}(n,t') - Q_2 \overline{f}(n,t') + \psi_1) e^{kp^2 t'} dt' + \overline{F}^*(n,m) \right] \times S_0(k_1,k_2,\mu_n r) \tag{3.5}$$

STATEMENT OF THE PROBLEM (COOLING PROCESS)

The temperature change $T'(r,z,t)$ for the cooling process satisfies the equation (**Noda et.al**.[**13**]):

$$\frac{\partial^2 T'}{\partial r^2} + \frac{1}{r}\frac{\partial T'}{\partial r} + \frac{\partial^2 T'}{\partial z^2} = \frac{1}{k}\frac{\partial T'}{\partial t} \tag{4.1}$$

$$[T'(r,z,t)]_{t=t_0} = T'(r,z,t_0) \tag{4.2}$$

The boundary condition

$$\left[T'(r,z,t) + k_1 \frac{\partial T'}{\partial r}(r,z,t) \right]_{r=a} = f_1(z,t) \tag{4.3}$$

$$\left[T'(r,z,t) + k_2 \frac{\partial T'}{\partial r}(r,z,t) \right]_{r=b} = f_2(z,t) \tag{4.4}$$

$$[T(r,z,t)]_{z=0} = f(r,t) \tag{4.5}$$

$$[T'(r,z,t)]_{z=h} = g(r,t) \tag{4.6}$$

Where $T'(r,z,t)$ is the cooling temperature of the disc at time *t*.

DETERMINATION OF TEMPERATURE

Applying finite **Marchi-Zgrablich transform** [**5**] to equation (4.1) one obtains

$$\frac{\partial^2 \overline{T}'}{\partial z^2} - \mu_n^2 \overline{T}' + \psi = \frac{1}{k}\frac{d\overline{T}}{dt} \tag{5.1}$$

where,

$$\psi = \frac{b}{k_2} S_0(k_1,k_2,\mu_n b) f_2 - \frac{a}{k_1} S_0(k_1,k_2,\mu_n a) f_1$$

Now applying finite **Fourier sine transform** to the equation (5.1), one obtains

$$\frac{d\overline{\overline{T}}'}{dt} + kp^2 \overline{\overline{T}}' = Q_1' \overline{g}(n,t) + Q_2' \overline{f}(n,t) + \psi_1 \tag{5.2}$$

where,

$$p^2 = \mu_n^2 + q_m^2, \quad q_m = \frac{m\pi}{h},$$

$$Q_1' = (-1)^{m+1} q_m k, \quad Q_2' = kq_m, \quad \psi_1 = k\overline{\psi}$$

The solution of differential equation (5.2) is given by

$$\overline{\overline{T}}'(n,m,t) = e^{-kp^2 t}$$

$$\times\left[\int_0^t (Q_1{}'\bar{g}(n,t')+Q_2{}'\bar{f}(n,t')+\psi_1)e^{kp^2t'}dt'+\bar{F}^*(n,m)\right] \quad (5.3)$$

Applying inverse Fourier sine transform and inverse Marchi-Zgrablich transform, we get

$$T'(r,z,t)=\frac{2}{h}\sum_m\sum_n\frac{\sin q_m z}{c_n}e^{-kp^2t}$$
$$\times\left[\int_0^{t_0}(Q_1{}'\bar{g}(n,t')+Q_2{}'\bar{f}(n,t')+\psi_1)e^{kp^2t'}dt'+\bar{F}^*(n,m)\right]$$
$$\times S_0(k_1,k_2,\mu_n r) \quad (5.4)$$

which is the required solution.

DISPLACEMENT FUNCTION

Substituting value of $T(r,z,t)$ from equation (3.5) in equation (2.1) one obtains thermoelastic displacement function $U(r,z,t)$ as

$$U(r,z,t)=\frac{-2(1+v)a_t}{h}\times\sum_m\sum_n\frac{\sin q_m z}{c_n}e^{-kp^2t}$$
$$\times\left[\int_0^{t_0}(Q_1\bar{g}(n,t')-Q_2\bar{f}(n,t')+\psi_1)e^{kp^2t'}dt'+\bar{F}^*(n,m)\right]$$
$$\times S_0(k_1,k_2,\mu_n r) \quad (6.1)$$

STRESS FUNCTIONS

Substituting the value of equation (6.1) in equation (2.9) and (2.10) one obtains

$$\sigma_{rr}=\frac{4\mu(1+v)a_t}{h}\times\sum_n\frac{\mu_n\sin q_m z}{c_n}e^{-kp^2t}$$
$$\times\left[\int_0^{t_0}(Q_1\bar{g}(n,t')-Q_2\bar{f}(n,t')+\psi_1)e^{kp^2t'}dt'+\bar{F}^*(n,m)\right]$$
$$\times\frac{S_0'(k_1,k_2,\mu_n r)}{r} \quad (7.1)$$

$$\sigma_{\theta\theta}=\frac{4\mu(1+v)a_t}{h}\times\sum_n\frac{\mu_n\sin q_m z}{c_n}e^{-kp^2t}$$
$$\times\left[\int_0^{t_0}(Q_1\bar{g}(n,t')-Q_2\bar{f}(n,t')+\psi_1)e^{kp^2t'}dt'+\bar{F}^*(n,m)\right]$$
$$\times S_0''(k_1,k_2,\mu_n r) \quad (7.2)$$

SPECIAL CASE

Set $F(r,z)=-\dfrac{Q_0}{\lambda}e^z e^{-r}$ (8.1)

Applying **Marchi-Zgrablich transform** [**5**] to the above equation we get

$$\bar{F}(n,z)=-\frac{Q_0}{\lambda}e^z\int_a^b re^{-r}S_0(k_1,k_2,\mu_n r)dr$$

Where

$$S_p(k_1,k_2,\mu_n r)=J_p(\mu_n r)[y_p(k_1,\mu_n a)+y_p(k_2,\mu_n b)]$$
$$-y_p(\mu_n r)[J_p(k_1,\mu_n a)+J_p(k_2,\mu_n b)]$$

And $J_p(\mu r)$ and $y_p(\mu r)$ are Bessel's function of 1st and 2nd kind respectively

The eigen values μ_n are the positive roots of the characteristic equation

$$J_0(k_1,\mu a)+y_0(k_2,\mu b)]-J_0(k_2,\mu b)y_0(k_1,\mu a)]=0$$

Solving above we get

$$\bar{F}(n,z)=-\frac{Q_0}{\lambda}e^z e^{-a}\mu_n[J_0(\mu_n a)y_0(\mu_n a)+y_0(\mu_n b)J_0(\mu_n b)]$$

$$\bar{F}(n,z)=-\frac{Q_0}{\lambda}e^z e^{-a}\mu_n D_n \quad (8.2)$$

Where

$$D_n=J_0(\mu_n a)y_0(\mu_n a)+y_0(\mu_n b)J_0(\mu_n b)$$

Applying Fourier sine transform to above equation we get

$$\bar{F}^*(n,m)=-\frac{Q_0}{\lambda}e^{-a}\mu_n D_n\int_0^h e^z\left(\frac{\sin m\pi z}{h}\right)dz$$

$$\bar{F}^*(n,m)=-\frac{Q_0\pi h}{\lambda}e^{-a}\sum_m\sum_n\frac{m\mu_n D_n}{h^2+m^2\pi^2}(1-(-1)^m e^h) \quad (8.3)$$

Now set

$$g(r,t)=-\frac{Q_0}{\lambda}e^{-t}e^{-r}e^h \quad (8.4)$$

Applying Marchi-Zgrablich transform to above equation, we get

$$\bar{g}(n,t')=-\frac{Q_0}{\lambda}e^{-t'}e^h e^{-a}\mu_n D_n \quad (8.5)$$

Also set

$$f(r,t)=-\frac{Q_0}{\lambda}e^{-t}e^{-r} \quad (8.6)$$

Applying Marchi-Zgrablich transform to above equation we get

$$f'(n,t')=-\frac{Q_0}{\lambda}e^{-t'}e^{-a}\mu_n D_n \quad (8.7)$$

Thus the final expression for temperature distribution is

$$T(r,z,t)=\frac{2}{h}\sum_m\sum_n\frac{\sin q_m z}{c_n}e^{-kp^2t}$$
$$\times\left[\begin{array}{l}\displaystyle\int_0^t\left(Q_1\left(\frac{Q_0}{\lambda}e^{-t'}e^h e^{-a}\mu_n D_n\right)-Q_2\left(\frac{Q_0}{\lambda}e^{-t'}e^h e^{-a}\mu_n D_n\right)+\psi_1\right)e^{kp^2t'}dt'\\ \displaystyle+\left\{-\frac{Q_0\pi he^{-a}}{\lambda}\sum_m\sum_n\frac{m\mu_n D_n}{h^2+m^2\pi^2}(1-(-1)^m e^h)\right\}\end{array}\right]$$
$$\times S_0(k_1,k_2,\mu_n r)$$
$$(8.8)$$

$$U(r,z,t)=\frac{-2(1+v)a_t}{h}\times\sum_m\sum_n\frac{\sin q_m z}{c_n}e^{-kp^2t}$$
$$\times\left[\int_0^{t_0}(Q_1\bar{g}(n,t')-Q_2\bar{f}(n,t')+\psi_1)e^{kp^2t'}dt'+\bar{F}^*(n,m)\right]$$
$$\times S_0(k_1,k_2,\mu_n r) \quad (8.9)$$

$$\sigma_{rr} = \frac{4\mu(1+v)a_t}{h} \times \sum_n \frac{\mu_n \sin q_m z}{c_n} e^{-kp^2 t} \times \left[\int_0^{t_0} (Q_1 \bar{g}(n,t') - Q_2 \bar{f}(n,t') + \psi_1) e^{kp^2 t'} dt' + \bar{F}^*(n,m) \right] \times \frac{S_0'(k_1,k_2,\mu_n r)}{r} \tag{8.10}$$

$$\sigma_{\theta\theta} = \frac{4\mu(1+v)a_t}{h} \times \sum_n \frac{\mu_n \sin q_m z}{c_n} e^{-kp^2 t} \times \left[\int_0^{t_0} (Q_1 \bar{g}(n,t') - Q_2 \bar{f}(n,t') + \psi_1) e^{kp^2 t'} dt' + \bar{F}^*(n,m) \right] \times S_0''(k_1,k_2,\mu_n r) \tag{8.11}$$

NUMERICAL RESULTS

Set

$k_1 = 0.2,\ k_2 = 0.2,\ h = 0.25\,ft,\ \pi = 3.14,\ a = 1\,ft, b = 2\,ft,\ t = 2$ in seconds, $k = 0.5$

$$T(r,z,t) = 2\sum_m \sum_n \frac{\sin q_m z}{c_n} e^{-0.5p^2 t} \times \left[\begin{array}{l} \int_0^t \left(Q_1 \left(\frac{Q_0}{\lambda} e^{-t'} e^h e^{-2} \mu_n D_n \right) - Q_2 \left(\frac{Q_0}{\lambda} e^{-t'} e^h e^{-2} \mu_n D_n \right) + \psi_1 \right) e^{kp^2 t'} dt' \\ + \left\{ -\frac{Q_0 (3.14) h e^{-a}}{\lambda} \sum_m \sum_n \frac{m \mu_n D_n}{1 + m^2 (3.14)^2} (1 - (-1)^m e^h) \right\} \end{array} \right] \times S_0(0.2, 0.2, \mu_n r) \tag{9.1}$$

$$U(r,z,t) = \frac{-2(1+v)a_t}{h} \times \sum_m \sum_n \frac{\sin q_m z}{c_n} e^{-kp^2 t} \times \left[\int_0^{t_0} (Q_1 \bar{g}(n,t') - Q_2 \bar{f}(n,t') + \psi_1) e^{kp^2 t'} dt' + \bar{F}^*(n,m) \right] \times S_0(k_1,k_2,\mu_n r) \tag{9.2}$$

Where,

$$\psi = \frac{b}{k_2} S_0(k_1,k_2,3\mu_n) f_2 - \frac{a}{k_1} S_0(k_1,k_2,\mu_n) f_1$$

$$p^2 = \mu_n^2 + q_m^2,\ q_m = \frac{m\pi}{0.25},\ Q_1' = (-1)^{m+1} q_m k,\ Q_2' = k q_m,\ \psi_1 = 0.5\bar{\psi}$$

$$\sigma_{rr} = \frac{4\mu(1+v)a_t}{h} \times \sum_n \frac{\mu_n \sin q_m z}{c_n} e^{-0.25p^2 t} \times \left[\int_0^{t_0} (Q_1 \bar{g}(n,t') - Q_2 \bar{f}(n,t') + \psi_1) e^{kp^2 t'} dt' + \bar{F}^*(n,m) \right] \times \frac{S_0'(k_1,k_2,\mu_n r)}{r} \tag{9.3}$$

$$\sigma_{\theta\theta} = \frac{4\mu(1+v)a_t}{h} \times \sum_n \frac{\mu_n \sin q_m z}{c_n} e^{-0.25p^2 t} \times \left[\int_0^{t_0} (Q_1 \bar{g}(n,t') - Q_2 \bar{f}(n,t') + \psi_1) e^{kp^2 t'} dt' + \bar{F}^*(n,m) \right] \times S_0''(k_1,k_2,\mu_n r) \tag{9.4}$$

MATERIAL PROPERTIES

The numerical calculation has been carried out for an Aluminum (Pure) annular disc with the material properties as,

Density ρ = 169 lb/ft3

Specific heat = 0.208 Btu/ lb0F

Thermal conductivity k = 117 Btu/(hr.ft0F)

Thermal diffusivity α = 3.33 ft2/hr

Poisson ratio ν = 0.35

Coefficient of linear thermal expansion $\alpha_t = 12.84 \times 10^{-6}$ 1/F

Lame constant μ = 26.67

DIMENSIONS

The constants associated with the numerical calculation are taken as

Radius of disc *a* = 1.5ft

Radius of disc b = 2ft

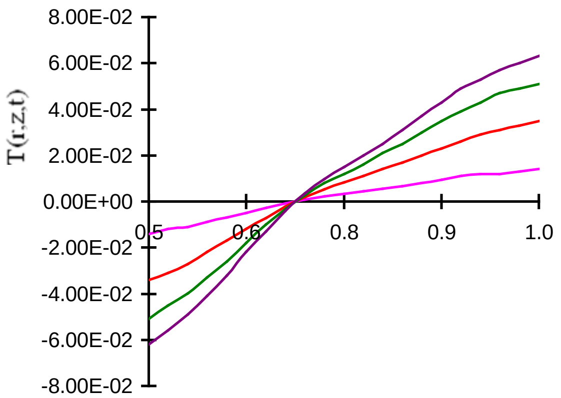
Height of disc h = 0.25ft

CONCLUSION

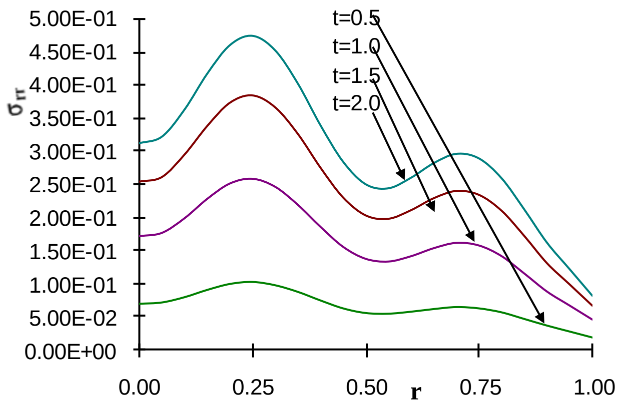
The temperature distribution, displacement function and thermal stresses at any point of the disc have been determined when the other boundary condition are known with the aids of finite Fourier sine transform and Marchi Zgrablich transform techniques.

The expression are obtained in the form of infinite series and are represented graphically. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the equation.

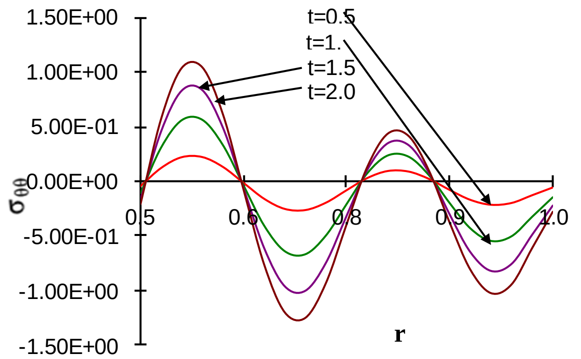
The results presented here will be more useful in engineering problems particularly in the determination of the state of strain in the disc constituting the foundation of container for hot gases or liquid in foundation for furnaces etc.



Graph 1. Temperature distribution vs. radius



Graph 2. Radial stresses vs. radius



Graph 3. Tangential stresses vs. radius

REFERENCES

[1] W. Nowacki, "The state of stress in a thick circular plate due to a temperature field", Bull. Acad. Polon. Sci. Ser. Sci. Tech.5, P.27. 1957

[2] H. S. Carslaw and J. C. Jaeger**,** "Conduction Of Heat In Solids, 2nd Ed. Oxford Clarendon Press. 1959

[3] B. A. Boley and J. H. Weiner, Theory Of Thermal Stresses, Johan Wiley and Sons, New York. 1960

[4] W. Nowacki, Thermo elasticity, Addition- Wisely Publishing Comp. Inc. London, 1962

[5] E. Marchi and J. Zgrablich, "Heat conduction in hollow cylinder with radiation", proceeding Edinburgh Math. Soc. Vol. 14 (Series 11) Part. 2, pp. 159-164, 1964

[6] K. C. Sabherwal**,** "An inverse problem of transient heat conduction", Indian J. Pure and Appl. Physics, Vol.3, No.10, Pp.397-398. 1965

[7] E. Marchi and A. Fasulo, "Heat conduction in sector of hollow cylinder with radiation", Atti, Della Acc. Sci. di. Torino, 1, 373-382. 1967

[8] N. M. Ozisik, Boundary Value Problems Of Heat Conduction, International Text Book Co. Scranton, Pennsylvania. 1968

[9] S. R. Patel, "Inverse problems of transient heat conduction with radiation", The Math. Edn. Vol. V, No.4. 173-177, 1971

[10] S. K. Roychoudhary, "A note on the quasi-static thermal stresses in a thin circular plate due to transient temperature applied along the circumferences of a circle over the upper face", Bull. Acad. Polon. Sci. ser., Tech. No. 1, pp.20-21, 1972.

[11] P.C. Wankhede, "On the Quasi-static thermal stresses in a circular plate," Indian J.pur and Appl. Math. 13(11), pp. 1273-1277, 1982.

[12] Ishihara, Noda and Tanigawa, "Theoretical analysis of thermoelastic plastic deformation of a circular plate due to partially distributed heat supply", Journal of Thermal Stresses, Vol.20; p.203-233. 1997

[13] N. Noda; R. B. Hetnarski and Y. Tanigawa**,** **"**Thermal Stresses", 2nd Edition's, Taylor & Francis, New York. 2003

[14] K. P. Ghadle and N. W. Khobragade, "An inverse unsteady-state thermoelastic problem of a thin annular disc in Marchi-Fasulo transform domain", Varahmihir J. Math. Sci. vol.3 (No. 2), pp 263-279. 2003

[15] M. H. Durge and N. W. Khobragade**,** "An inverse steady-state thermoelastic problem of a thin annular disc in Marchi-Zgrablich transform domain", Varahmihir J. Math. Sci. Vol. 4, No. 2, pp. 535-544. 2004

[16] S. S. Singru and N. W. Khobragade**,** "Steady-state thermoelastic problem of a thin annular disc in Marchi-Fasulo transform domain", Bull. of Pure and Appl. Sci, vol. 23E(No.1), pp. 53-62. 2004

[17] M. H. Durge and N. W. Khobragade**,** "Analysis of stress functions in a thin annular disc due to partially distributed heat supply", Far East J. of Appl. Math, 21 (1), pp.105-115. 2005

[18] K. W. Khobragade, V. Varghese and N. W. Khobragade, "An inverse transient thermoelastic problem of a thin annular disc", Appl. Math. E-Notes, 6, pp.17-25. 2005

[19] M. H. Durge and N. W. Khobragade**,** "An inverse unsteady-state thermoelastic problem of a thin annular disc in Marchi-Zgrablich transform domain", Bull. of the Cal. Math. Soc. 98 (3), pp. 255-266. 2006

[20] K. R. Gaikwad and K. P. Ghadle**,** "An inverse heat conduction problem in a thick annular disc", International Journal of Applied Mathematics and Mechanics, Vol-7(16), pp. 27-41, 2011