ISSN No. 0976-5697

Volume 8, No. 5, May-June 2017

International Journal of Advanced Research in Computer Science

RESEARCH PAPER

Available Online at www.ijarcs.info

Thermoelastic Problems of a Hollow Cylinder and its Thermal Stresses

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Abstract: In this paper, an attempt has been made to study thermoelastic response of a direct thermoelastic problem of a hollow cylinder occupying the space $D=\{(x,y,z)\in R^3 : a\le (x^2+y^2)^{1/2}\le b,\ 0\le z\le h\}$ with radiation type boundary conditions. We apply integral transform technique to find the thermoelastic solution.

***Keywords**:* Thermo elastic Response, hollow cylinder, Thermal Stresses, inverse problem.

INTRODUCTION

Dange et al. [1] studied Thermal Stresses of a finite length hollow cylinder due to heat generation. Gahane et al. [2] discussed Transient Thermoelastic Problem of A Semi-infinite Cylinder With Heat Sources. Gahane et al. [3] studied Transient Thermoelastic Problem of a cylinder with heat sources. Hiranwar et al. [4] discussed Thermoelastic Problem of A Cylinder With Internal Heat Sources. Jabbari et al. [5] studied Axisymmetric mechanical and thermal stresses in thick short length FGM cylinders. Jadhav et al. [6] discussed an Inverse Thermoelastic Problem of finite length thick hollow cylinder with internal heat sources. Kamdi et al. [7] studied Transient Thermoelastic Problem for a Circular Solid Cylinder with Radiation. Khobragade et al. [8] discussed Thermal Deflection of a Finite Length Hollow Cylinder due to Heat Generation. Khobragade [9] studied Thermal stresses of a hollow cylinder with radiation type conditions. Further Khobragade [10] discussed Thermoelastic analysis of a solid circular cylinder and Khobragade [11] studied Thermoelastic analysis of a thick hollow cylinder with radiation conditions.

Kulkarni et al. [12] discussed Thermal stresses of a finite length hollow cylinder. Lamba et al. [13] studied Stress functions in a hollow cylinder under heating and cooling processes and Lamba et al. [14] discussed Analysis of Coupled thermal Stresses in an Axisymmetric Hollow Cylinder. Lord et al. [15] developed a generalized dynamical theory of thermo elasticity. Love [16] has written a book entitled treatise on the mathematical theory of elasticity. Marchi et al. [17] studied Heat conduction in sector of hollow cylinder with radiation and Marchi et al. [18] discussed Heat conduction in hollow cylinder with radiation. Mehta [19] studied Interior value problem of heat conduction for a finite circular cylinder.

Noda et al. [21] discussed a three dimensional treatment of transient thermal stresses in a transversely isotropic semi infinite circular cylinder subjected to an asymmetric temperature on the cylindrical surface. Ozisik [22] studied boundary value problems of heat conduction. Ali et al. [23] studied Elastic–plastic stress analysis in a long functionally graded solid cylinder with fixed ends subjected to uniform heat generation. Pathak et al. [24] discussed Thermoelastic Problem of a Semi Infinite Cylinder with Internal Heat Sources. Rama Murthy [25] studied Thermal stresses in an anisotropic cylinder.

Raut et al. [26] discussed the plane strain and plane stress solutions of uniformly heated functionally graded solid cylinder or disc problems. Shao et al. [27] studied

Thermo-mechanical stresses in functionally graded circular hollow cylinder with linearly increasing boundary temperature, Composite Structures. Sherief et al. [28] discussed A Problem in generalized thermo elasticity for an infinitely long annular cylinder composed of two different materials. Sierakowski et al. [29] studied an exact solution to the elastic deformation of a finite length hollow cylinder. Sun et al. [30] discussed the axially symmetric deformation of a cylinder of finite length.

Takeuti et al. **[31]** discussed A three- dimensional treatment of transient thermal stresses in a circular cylinder due to an arbitrary heat supply and Takeuti et al. [32] studied Transient thermal stresses in a composite circular cylinder due to a band heat source. Tanigawa et al. [33] discussed One-dimensional transient thermal stress problem for non homogeneous hollow circular cylinder and its optimization of material composition for thermal stress relaxation. Yuriy et al. [34] discussed Analysis of residual stresses in a long hollow cylinder. Walde et al. [35] studied Thermal Stresses of a Solid Cylinder with Internal Heat Source. Warbhe et al. [36] discussed Numerical Study of Transient Thermoelastic Problem of A Finite Length Hollow Cylinder.

FORMULATION OF THE PROBLEM-I

Consider a hollow cylinder as shown in the figure 1. The material of the cylinder is isotropic, homogenous and all properties are assumed to be constant. We assume that the cylinder is of a small thickness and its boundary surfaces remain traction free. The initial temperature of the cylinder is the same as the temperature of the surrounding medium, which is kept constant.

The displacement function $\phi(r,z)$ satisfying the differential equation as Khobragade [9-11] is

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{\partial^2\phi}{\partial z^2}=\left(\frac{1+\nu}{1-\nu}\right)a_t T \tag{2.1}$$

with $\phi=0$ at $r=a$ and $r=b$ (2.2)

where ν and a_t are Poisson ratio and linear coefficient of thermal expansion of the material of the cylinder respectively and $T(r,z)$ is the heating temperature of the cylinder satisfying the differential equation as Khobragade [9-11] is

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]+\frac{g(r,z)}{k}=0 \qquad (2.3)$$

where $\kappa = K/\rho c$ is the thermal diffusivity of the material of the cylinder, K is the conductivity of the medium, c is its specific heat and ρ is its calorific capacity (which is assumed to be constant) respectively,
subject to the boundary conditions

$$M_r(T,1,\overline{\overline{k}}_1,a)=F_1(z)\text{, for all } -h \le z \le h\ , \qquad (2.4)$$

$$M_r(T,1,\overline{\overline{k}}_2,b)=F_2(z) \quad \text{for all } -h \le z \le h\ , \qquad (2.5)$$

$$M_z(T,1,k_3,-h)=F_3(r) \text{ for all } a \le r \le b\ , \qquad (2.6)$$

$$M_z(T,1,k_4,h)=G(r) \quad \text{for all } a \le r \le b\ , \qquad (2.7)$$

being: $M_{\vartheta}(f,\overline{k},\overline{\overline{k}},\$)=(\overline{k}\ f+\overline{\overline{k}}\ \hat{f})_{\vartheta=\$}$
where the prime (^) denotes differentiation with respect to ϑ, radiation constants are $\overline{k}$ and $\overline{\overline{k}}$ on the curved surfaces of the plate respectively.
The radial and axial displacement U and W satisfy the uncoupled thermoelastic equation as **Khobragade** [9-11] are

$$\nabla^2 U-\frac{U}{r^2}+(1-2\nu)^{-1}\frac{\partial e}{\partial r}=2\left(\frac{1+\nu}{1-2\nu}\right)a_t\frac{\partial T}{\partial r} \qquad (2.8)$$

$$\nabla^2 W+(1-2\nu)^{-1}\frac{\partial e}{\partial z}=2\left(\frac{1+\nu}{1-2\nu}\right)a_t\frac{\partial T}{\partial z} \qquad (2.9)$$

where

$$e=\frac{\partial U}{\partial r}+\frac{U}{r}+\frac{\partial W}{\partial r} \qquad (2.10)$$

$$U=\frac{\partial \phi}{\partial r}, \qquad (2.11)$$

$$W=\frac{\partial \phi}{\partial z} \qquad (2.12)$$

The stress functions are given by

$$\tau_{rz}(a,z)=0,\tau_{rz}(b,z)=0,\tau_{rz}(r,0)=0 \qquad (2.13)$$

$$\sigma_r(a,z)=p_i,\sigma_r(b,z)=-p_o,\sigma_z(r,0)=0 \qquad (2.14)$$

where p_i and p_o are the surface pressure assumed to be uniform over the boundaries of the cylinder. The stress functions are expressed in terms of the displacement components by the following relations as Khobragade [9-11] are

$$\sigma_r=(\lambda+2G)\frac{\partial U}{\partial r}+\lambda\left(\frac{U}{r}+\frac{\partial W}{\partial z}\right) \qquad (2.15)$$

$$\sigma_z=(\lambda+2G)\frac{\partial W}{\partial z}+\lambda\left(\frac{\partial U}{\partial r}+\frac{U}{r}\right) \qquad (2.16)$$

$$\sigma_\theta=(\lambda+2G)\frac{U}{r}+\lambda\left(\frac{\partial U}{\partial r}+\frac{\partial W}{\partial z}\right) \qquad (2.17)$$

$$\tau_{rz}=G\left(\frac{\partial W}{\partial r}+\frac{\partial U}{\partial z}\right) \qquad (2.18)$$

where $\lambda=2G\nu/(1-2\nu)$ is the lame's constant, G is the shear modulus and U, W are the displacement components, ease of use

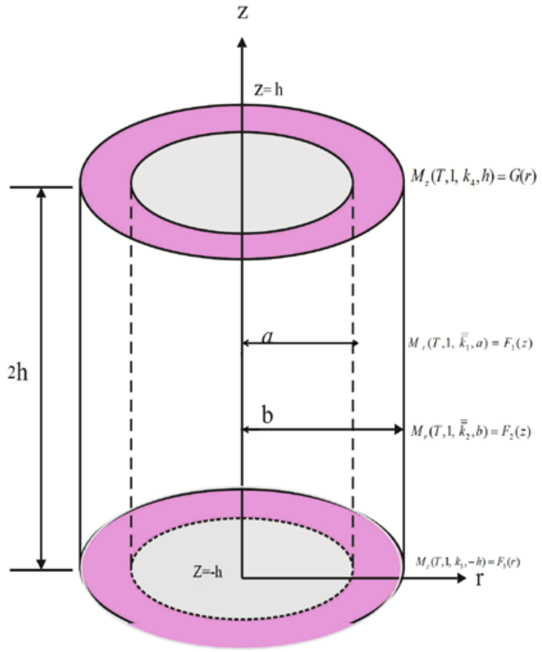


Figure 1: Geometry of the problem

SOLUTION OF THE OF THE PROBLEM

Applying Marchi-Fasulo transform on equation (2.1), we get

$$\frac{\partial^2\overline{T}}{\partial r^2}+\frac{1}{r}\frac{\partial\overline{T}}{\partial r}-\lambda_n^2\ \overline{T}=\Psi$$

where $\Psi=\frac{-P_n(h)}{k_3}F_3(r)\frac{-P_n(h)}{k_4}G(r)-\frac{\overline{g}}{k}$

Equation (2.6) is a Bessel's equation whose solution yields
$\overline{T}=AI_0(\lambda_n r)+BK_0(\lambda_n r)+F(r)$
where $F(r)$ is the P.I.
As $r\to 0, K_0\to\infty$, But $\overline{T}$ is finite
$\therefore B=0$

$$A=\frac{\overline{F_1}(z)-k_1F'(a)-F(a)}{I_0(\lambda_n a)+k_1I_0'(\lambda_n a)}$$

$$\therefore \overline{T}=\frac{\overline{F_1}(z)-k_1F'(a)-F(a)}{I_0(\lambda_n a)+k_1I_0'(\lambda_n a)}I_0(\lambda_n r)+F(r)$$

Applying inverse Marchi-Fasulo transform an equation (3.3) we get,

$$T=\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\left[\frac{\overline{F_1}(z)-k_1F'(a)-F(a)}{I_0(\lambda_n a)+k_1I_0'(\lambda_n a)}I_0(\lambda_n r)+F(r)\right]$$

DETERMINATION OF DISPLACEMENT AND STRESS COMPONENTS

Substituting the value of temperature distribution from (3.4) in equation (2.1) one obtains the thermo elastic displacement function $\phi(r,z)$ as

$$\phi=\frac{r^2}{4}\left(\frac{1+\nu}{1-\nu}\right)a_t$$

$$\times \sum_{n=1}^{\infty} \frac{P_n(z)}{\lambda_n} \left[\frac{\overline{F_1}(z) - k_1 F'(a) - F(a)}{I_0(\lambda_n a) + k_1 I_0'(\lambda_n a)} I_0(\lambda_n r) + F(r) \right] \quad (4.1)$$

The stress components are

$$U = \frac{a_t}{4}\left(\frac{1+\nu}{1-\nu}\right) \sum_{n=1}^{\infty} \frac{P_n(z)}{\lambda_n} \left\{ \frac{\overline{F_1}(z) - k_1 F'(a) - F(a)}{I_0(\lambda_n a) + k_1 I_0'(\lambda_n a)} \times \left[r^2 I_0'(\lambda_n r) + 2r I_0(\lambda_n r) \right] + r^2 F'(r) + 2rF(r) \right\} \quad (4.2)$$

$$W = \frac{r^2}{4}\left(\frac{1+\nu}{1-\nu}\right) a_t \sum_{n=1}^{\infty} \frac{P_n'(z)}{\lambda_n} \left[\frac{\overline{F_1}(z) - k_1 F'(a) - F(a)}{I_0(\lambda_n a) + k_1 I_0'(\lambda_n a)} I_0(\lambda_n r) + F(r) \right] \quad (4.3)$$

DETERMINATION OF STRESS FUNCTION

Substituting the value of (4.2) and (4.3) in equations (2.15)-(2.18) one obtains the thermal stresses as

$$\sigma_r = \frac{a_t}{4}\left(\frac{1+\nu}{1-\nu}\right) \sum_{n=1}^{\infty} \frac{1}{\lambda_n} \left\{ \frac{\overline{F_1}(z) - k_1 F'(a) - F(a)}{I_0(\lambda_n a) + k_1 I_0'(\lambda_n a)} \left\{ P_n(z) \left[(\lambda + 2G)\left(r^2 I_0''(\lambda_n r) + 4r I_0'(\lambda_n r) + 2 I_0(\lambda_n r) \right) + \lambda \left(r I_0'(\lambda_n r) + 2r I_0(\lambda_n r) \right) \right] + P_n''(z) \lambda r^2 I_0(\lambda_n r) \right\} + P_n(z) \left[(\lambda + 2G)\left(r^2 F''(r) + 4rF'(r) + 2F(r) + \lambda \left(rF'(r) + 2F(r) \right) \right) \right] + P_n''(z) \lambda r^2 F(r) \right\} \quad (5.1)$$

$$\sigma_z = \frac{a_t}{4}\left(\frac{1+\nu}{1-\nu}\right) \sum_{n=1}^{\infty} \frac{1}{\lambda_n} \left\{ \frac{\overline{F_1}(z) - k_1 F'(a) - F(a)}{I_0(\lambda_n a) + k_1 I_0'(\lambda_n a)} \left\{ P_n''(z)(\lambda + 2G) r^2 I_0(\lambda_n r) + P_n(z) \lambda \left[r^2 I_0''(\lambda_n r) + 5r I_0'(\lambda_n r) + 4 I_0(\lambda_n r) \right] + P_n''(z)(\lambda + 2G) r^2 P_n(z) \lambda \left[r^2 F''(r) + 5rF'(r) + 4F(r) \right] \right\} \right\} \quad (5.2)$$

$$\sigma_\theta = \frac{a_t}{4}\left(\frac{1+\nu}{1-\nu}\right) \sum_{n=1}^{\infty} \frac{1}{\lambda_n} \left\{ \frac{\overline{F_1}(z) - k_1 F'(a) - F(a)}{I_0(\lambda_n a) + k_1 I_0'(\lambda_n a)} \left\{ P_n(z) \left[(\lambda + 2G)(r^2 I_0'(\lambda_n r) + 2r I_0(\lambda_n r) + \lambda \left(r^2 I_0''(\lambda_n r) + 4r I_0'(\lambda_n r) + 2 I_0(\lambda_n r) \right) \right] + P_n''(z) \lambda r^2 I_0(\lambda_n r) \right\} + P_n(z) \left[(\lambda + 2G)\left(rF'(r) + 2rF(r) + 2\lambda \left(r^2 F''(r) + 4F'(r) + 2F(r) \right) \right) \right] + P_n''(z) \lambda r^2 F(r) \right\} \quad (5.3)$$

$$\tau_{rz} = \frac{a_t}{2}\left(\frac{1+\nu}{1-\nu}\right) \sum_{n=1}^{\infty} \frac{P_n'(z)}{\lambda_n} \left\{ \frac{\overline{F_1}(z) - k_1 F'(a) - F(a)}{I_0(\lambda_n a) + k_1 I_0'(\lambda_n a)} \left[r^2 I_0'(\lambda_n r) + 2r I_0(\lambda_n r) \right] + r^2 F'(r) + 2rF(r) \right\} \quad (5.4)$$

SPECIAL CASE

Set

$$f(r,t) = (1 - e^{-t})\, \delta(r - r_0) \quad (6.1)$$

Applying finite transform defined in Marchi Zgrablich [2] to the equation (32) one obtains

$$\bar{f}(n,t) = (1 - e^{-t})\, r_0\, S_0(k_1, k_2, \mu_n r_0) \quad (6.2)$$

Substituting the value of (32) in the equations (21) to (31) one obtains

$$T = \sum_{n=1}^{\infty} \frac{P_n(z)}{\lambda_n} \left[\frac{\overline{F_1}(z) - k_1 F'(a) - F(a)}{I_0(\lambda_n a) + k_1 I_0'(\lambda_n a)} I_0(\lambda_n r) + F(r) \right] \quad (6.3)$$

NUMERICAL RESULTS, DISCUSSION AND REMARKS

To interpret the numerical computation we consider material properties of low carbon steel (AISI 1119), which can be used for medium duty shafts, studs, pins, distributor cams, cam shafts, and universal joints having mechanical and thermal properties

$\kappa = 13.97[\mu m / s^2]$ $\upsilon = 0.29$, $\lambda = 51.9\ [W/(m-K)]$ and $a_t = 14.7\ \mu m / m -^0 C$.

Setting the physical parameter with $a = 0.5$, $b = 1$ and $h = 3$.

FORMULATION OF THE PROBLEM-II

Consider a hollow cylinder as shown in the figure 1. The material of the cylinder is isotropic, homogenous and all properties are assumed to be constant. We assume that the cylinder is of a small thickness and its boundary surfaces remain traction free. The initial temperature of the cylinder is the same as the temperature of the surrounding medium, which is kept constant.

The displacement function $\phi(r, z, t)$ satisfying the differential equation as Khobragade [9-11] is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left(\frac{1+\nu}{1-\nu}\right) a_t T \quad (8.1)$$

with $\phi = 0$ at $r = a$ and $r = b$ (8.2)

where ν and a_t are Poisson ratio and linear coefficient of thermal expansion of the material of the cylinder respectively and $T(r, z, t)$ is the heating temperature of the cylinder at time t satisfying the differential equation as Khobragade [9-11] is

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] + \frac{g(r,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (8.3)$$

where $\kappa = K/\rho c$ is the thermal diffusivity of the material of the cylinder, K is the conductivity of the medium, c is its specific heat and ρ is its calorific capacity (which is assumed to be constant) respectively, subject to the initial and boundary conditions

$$M_t(T, 1, 0, 0) = F \quad \text{for all} \quad a \le r \le b,\ -h \le z \le h \quad (8.4)$$

$$M_r(T, 1, \bar{\bar{k}}_1, a) = F_1(z,t), \ \text{for all} \quad -h \le z \le h,\ t > 0 \quad (8.5)$$

$$M_r(T, 1, \bar{\bar{k}}_2, b) = F_2(z,t), \ \text{for all} \quad -h \le z \le h,\ t > 0 \quad (8.6)$$

$$M_z(T, 1, k_3, -h) = F_3(r,t), \ \text{for all} \quad a \le r \le b,\ t > 0 \quad (8.7)$$

$$M_z(T, 1, k_4, h) = G(r,t), \ \text{for all} \quad a \le r \le b,\ t > 0 \quad (8.8)$$

being: $M_\vartheta(f, \bar{k}, \bar{\bar{k}}, \$) = (\bar{k}\ f + \bar{\bar{k}}\ \hat{f})_{\vartheta = \$}$

where the prime (^) denotes differentiation with respect to ϑ, radiation constants are $\bar{k}$ and $\bar{\bar{k}}$ on the curved surfaces of the plate respectively.

The radial and axial displacement U and W satisfy the uncoupled thermoelastic equation as Khobragade [9-11] are

$$\nabla^2 U - \frac{U}{r^2} + (1-2\nu)^{-1}\frac{\partial e}{\partial r} = 2\left(\frac{1+\nu}{1-2\nu}\right) a_t \frac{\partial T}{\partial r} \tag{8.9}$$

$$\nabla^2 W + (1-2\nu)^{-1}\frac{\partial e}{\partial z} = 2\left(\frac{1+\nu}{1-2\nu}\right) a_t \frac{\partial T}{\partial z} \tag{8.10}$$

where

$$e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial r} \tag{8.11}$$

$$U = \frac{\partial \phi}{\partial r}, \tag{8.12}$$

$$W = \frac{\partial \phi}{\partial z} \tag{8.13}$$

The stress functions are given by

$$\tau_{rz}(a,z,t) = 0\,,\ \tau_{rz}(b,z,t) = 0\,,\ \tau_{rz}(r,0,t) = 0 \tag{8.14}$$

$$\sigma_r(a,z,t) = p_i\,,\ \sigma_r(b,z,t) = -p_o\,,\ \sigma_z(r,0,t) = 0 \tag{8.15}$$

where p_i and p_o are the surface pressure assumed to be uniform over the boundaries of the cylinder. The stress functions are expressed in terms of the displacement components by the following relations as Khobragade [9-11] are

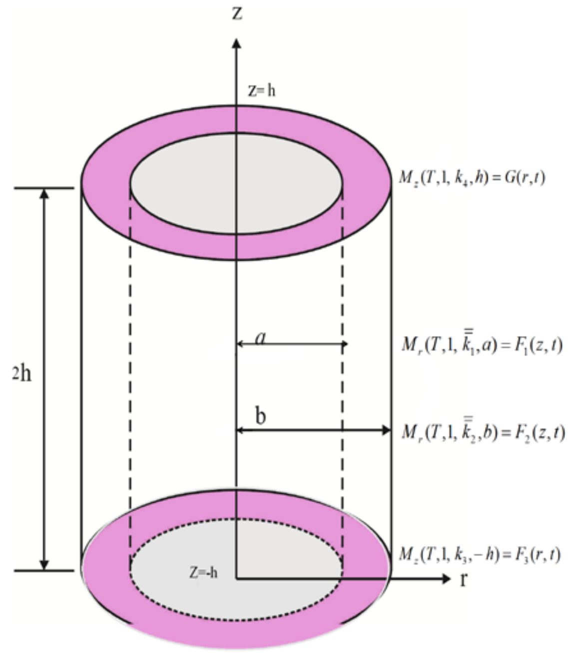


Figure 2: Geometry of the problem

$$\sigma_r = (\lambda+2G)\frac{\partial U}{\partial r} + \lambda\left(\frac{U}{r} + \frac{\partial W}{\partial z}\right) \tag{8.16}$$

$$\sigma_z = (\lambda+2G)\frac{\partial W}{\partial z} + \lambda\left(\frac{\partial U}{\partial r} + \frac{U}{r}\right) \tag{8.17}$$

$$\sigma_\theta = (\lambda+2G)\frac{U}{r} + \lambda\left(\frac{\partial U}{\partial r} + \frac{\partial W}{\partial z}\right) \tag{8.18}$$

$$\tau_{rz} = G\left(\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right) \tag{8.19}$$

where $\lambda = 2G\nu/(1-2\nu)$ is the Lame's constant, G is the shear modulus and U, W are the displacement components.

Equations (8.1)-(8.19) constitute the mathematical formulation of the problem under consideration.

SOLUTION OF THE OF THE PROBLEM-II

Applying transform defined in [18] to the equations (8.3), (8.4) and (8.6) over the variable r having $p=0$ with responds to the boundary conditions of type (8.5) and then Marchi-Fasulo transform , one obtains

$$\bar{T}^*(n,z,s) = e^{-\alpha p^2 t}\left[\bar{F}^* + \int_0^t \psi\, e^{\alpha P^2 t'} dt'\right] \tag{9.1}$$

where constants involved $\bar{T}^*(n,z,s)$ are obtained by using boundary conditions (8.6). Finally applying the inversion theorems of transform defined in [18] and inverse Marchi-Fasulo transform, one obtains the expressions of the temperature distribution $T(r,z,t)$ for heating processes as

$$T(r,z,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{P_m(z)\, S_0(\bar{k}_1,\bar{k}_2,\mu_n r)}{\mu_n^2 \lambda_m} \times e^{-\alpha p^2 t}\left[\bar{F}^* + \int_0^t \psi\, e^{\alpha p^2 t'} dt'\right] \tag{9.2}$$

where n is the transformation parameter as defined in appendix, m is the Marchi-Fasulo transform parameter.

DETERMINATION OF DISPLACEMENT AND STRESS FUNCTION

Substituting the value of temperature distribution from (9.2) in equation (8.1) one obtains the thermo elastic displacement function $\phi(r,z,t)$ as

$$\phi(r,z,t) = \frac{r^2 a_t}{4}\left(\frac{1+\nu}{1-\nu}\right)\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{P_m(z)\, S_0(\bar{k}_1,\bar{k}_2,\mu_n r)}{\mu_n^2 \lambda_m} \times e^{-\alpha p^2 t}\left[\bar{F}^* + \int_0^t \psi\, e^{\alpha p^2 t'} dt'\right] \tag{10.1}$$

Using (10.1) in the equations (8.11) and (8.12) one obtains

$$U = \frac{a_t(1+\nu)}{4(1-\nu)}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{P_m(z)}{\mu_n^2\lambda_m}\left[2rS_0(k_1,k_2,\mu_n r) + r\mu_n S'_0(k_1,k_2,\mu_n r)\right] \times e^{-\alpha p^2 t}\left[\bar{F}^* + \int_0^t \psi\, e^{\alpha p^2 t'} dt'\right] \tag{10.2}$$

$$W = \frac{r^2\, a_t(1+\nu)}{4(1-\nu)}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{P'_m(z)}{\mu_n^2\lambda_m} S_0(k_1,k_2,\mu_n r) \times e^{-\alpha p^2 t}\left[\bar{F}^* + \int_0^t \psi\, e^{\alpha p^2 t'} dt'\right] \tag{10.3}$$

Substitution the value of (10.2), (10.3) in (8.16) to (8.19) one obtains the stress functions as

$$\sigma_r = \frac{a_t}{4}\left(\frac{1+\nu}{1-\nu}\right)\sum_{m,n=1}^{\infty}\Phi_{nm}\left[P_m(z)\cdot\right.$$

$$\left[(\lambda+2G)\left(2rS_0'\mu_n + r^2S_0''\mu_n^2 + S_0'2r + 2S_0) + \frac{\partial}{r}(k_1, k_2, \mu_n r)\right)(rS_0'\mu_n + 2S_0)\right.$$
$$\left. + \lambda r^2 S_0(\bar{k}_1, \bar{k}_2, \mu_n r) \times P_m''\right] \tag{10.4}$$

$$\sigma_z = \frac{a_t}{4}\left(\frac{1+v}{1-v}\right)\sum_{m,n=1}^{\infty}\Phi_{nm}\left[\left(r^2 S_0(\bar{k}_1, \bar{k}_2, \mu_n r)P_{(m)}''\right] + P_m(z)\lambda\right.$$
$$\left.\left(2rS_0'\mu_n + r^2 S_0''\mu_n^2 + S_0'2r + 2S_0) + \frac{1}{r}rS_0'\mu_n + 2S_0\right)\right] \tag{10.5}$$

$$\sigma_\theta = \frac{a_t}{4}\left(\frac{1+v}{1-v}\right)\sum_{m,n=1}^{\infty}\Phi_{nm}\left[P_m(z)\left[(\lambda+2G)\frac{1}{r}(rS_0'\ \mu_n + 2S_0)\right.\right.$$
$$\left. + \lambda\left(2rS_0^{"}\mu_n + r^2 S_0''\mu_n^2 + S_0 2r + 2S_0\right)\right] + \lambda r^2 S_0(\bar{k}_1, \bar{k}_2, \mu_n r)P_m'' \tag{10.6}$$

$$\tau_{rz} = P_m' \frac{a_t}{4}\left(\frac{1+v}{1-v}\right)\sum_{m,n=1}^{\infty}\Phi_{nm}\ (2Gr^2 S_0'\mu_n + 4GS_0 r) \tag{10.7}$$

Where $\Phi_{nm} = \dfrac{e^{-\alpha p^2 t}}{\mu_n^2\lambda_m}\left[\overline{F}^* + \displaystyle\int_0^t \psi\, e^{\alpha p^2 t'}dt'\right]$

Special Case

Set $f(r,t) = (1-e^{-t})\,\delta(r-r_0)$ (11.1)

Applying finite transform defined in Marchi Zgrablich [18] to the equation (11.1) one obtains

$$\bar{f}(n,t) = (1-e^{-t})\, r_0\, S_0(k_1, k_2, \mu_n r_0) \tag{11.2}$$

Substituting the value of (11.2) in the equations (9.2) one obtains

$$T(r,z,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{P_m(z)\, S_0(\bar{k}_1, \bar{k}_2, \mu_n r)}{\mu_n^2\lambda_m} \times e^{-\alpha p^2 t}\left[\overline{F}^* + \int_0^t \psi\, e^{\alpha p^2 t'}dt'\right] \tag{11.3}$$

Numerical Results

Set $a = 0.5$, $b = 1$ and $h = 3$ **t**= 1 sec k_1=0.25, k_2 = 0.25 in equation (11.3) we get

$$T(r,z,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{P_m(z)\, S_0(0.25, 0.25, \mu_n r)}{\mu_n^2\lambda_m} \times e^{-\alpha p^2}\left[\overline{F}^* + \int_0^1 \psi\, e^{\alpha p^2 t'}dt'\right] \tag{12.1}$$

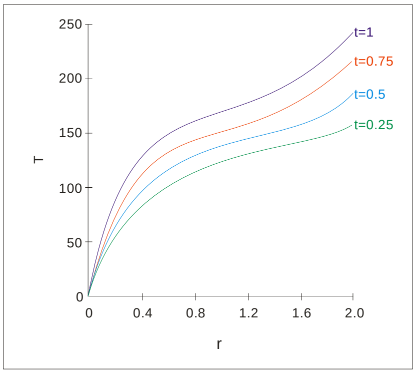
NUMERICAL RESULTS, DISCUSSION AND REMARKS

To interpret the numerical computation we consider material properties of low carbon steel (AISI 1119), which can be used for medium duty shafts, studs, pins, distributor cams, cam shafts, and universal joints having mechanical and thermal properties

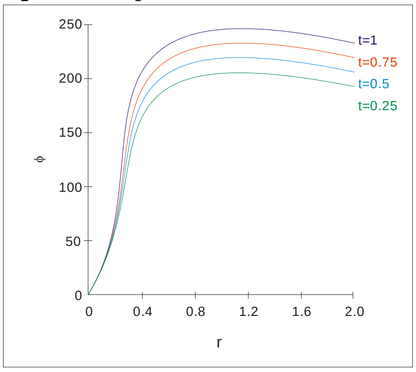
$\kappa = 13.97[\mu m/s^2]$ $\upsilon = 0.29$, $\lambda = 51.9\ [W/(m-K)]$ and $a_t = 14.7\ \mu m/m-^0 C$.

Conclusion

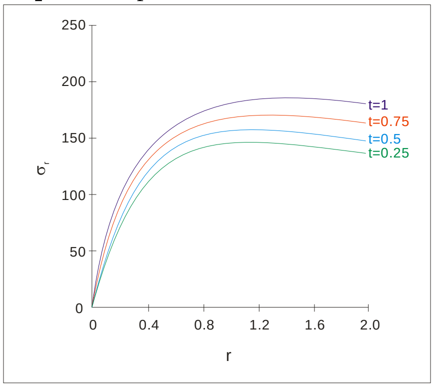
In this paper, we modify the conceptual idea proposed by **Khobragade et al**. [174] for hollow cylinder and the temperature distributions, displacement and stress functions at the edge $z = h$ occupying the region of the cylinder $a \le r \le b$, $-h \le z \le h$ have been obtained with the known boundary conditions. We develop the analysis for the temperature field by introducing the transformation defined by Zgrablich et al., finite Fourier sine transform and Laplace transform techniques with boundaries conditions of radiations type. The series solutions converge provided we take sufficient number of terms in the series. Since the thickness of cylinder is very small, the series solution given here will be definitely convergent. Assigning suitable values to the parameters and functions in the series expressions can derive any particular case. The temperature, displacement and thermal stresses that are obtained can be applied to the design of useful structures or machines in engineering applications.



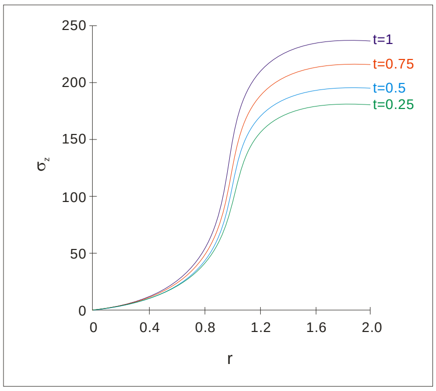
Graph 1: Temperature distribution versus r



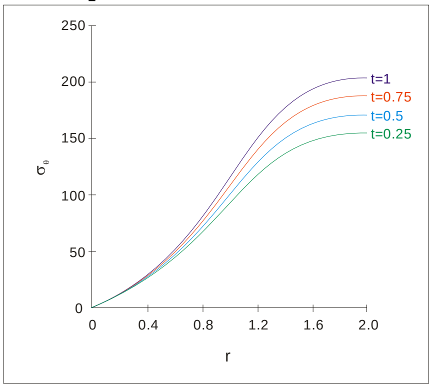
Graph 2: Displacement function versus r



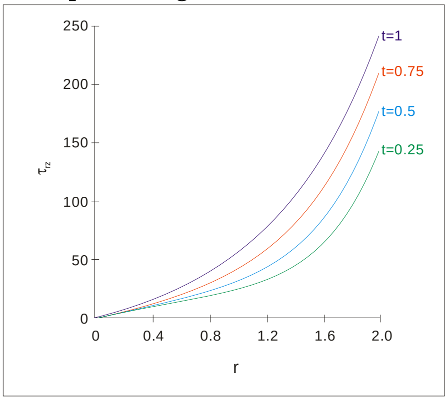
Graph 3: Radial stresses versus r



Graph 4: Axial stresses versus r



Graph 5: Tangential stresses versus r



Graph 6: Shear stresses versus r

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