Volume 8, No. 5, May-June 2017
International Journal of Advanced Research in Computer Science
RESEARCH PAPER
Available Online at www.ijarcs.info
ISSN No. 0976-5697

Transient Thermoelastic Problem of Semi Infinite Rectangular Beam

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Abstract: This paper is concerned with transient thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a semi- infinite rectangular beam when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem.

Keywords: Thermoelastic problem**,** semi- infinite rectangular beam, transient problem, integral transform, heat source

INTRODUCTION

In 2014, Bhongade and Durge [1] studied quasi static thermal stresses in a thin rectangular plate with internal heat generation. Dange; Khobragade and Durge [2] discussed three dimensional inverse transient thermoelastic problem of a thin rectangular plate. Ghume and Khobragade [3] studied deflection of a thick rectangular plate. Jadhav and Khobragade [4] discussed an inverse thermoelastic problem of a thin finite rectangular plate due to internal heat source. Khobragade; Payal Hiranwar; Roy and Lalsingh Khalsa [5] studied thermal deflection of a thick clamped rectangular plate**.** Nasser M; EI-Maghraby [6] discussed two dimensional problems in generalized thermoelasticity with heat sources. Roy; Bagade and Khobragade [7] studied thermal stresses of a semi infinite rectangular beam. Further Roy and Khobragade [8] discussed transient thermoelastic problem of an infinite rectangular slab and Sutar and Khobragade [9] studied an inverse thermoelastic problem of heat conduction with internal heat generation for the rectangular plate

In this paper , an attempt has been made to determine the temperature distribution , displacement function and thermal stresses of a semi infinite rectangular beam occupying the region D : $-a \le x \le a$;$-b \le y \le b$, $0 \le z < \infty$ with known boundary conditions. Here Marchi-Fasulo transforms and Fourier cosine transform techniques have been used to find the solution of the problem.

STATEMENT OF THE PROBLEM

Consider a thin rectangular plate occupying the space D:$-a \le x \le a$; $-b \le y \le b$, $0 \le z < \infty$. The displacement components u_x , u_y and u_z in the x and y and z directions respectively as Tanigawa et al. [10] are

$$u_x = \int_{-a}^{a} \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} - \nu \frac{\partial^2 U}{\partial x^2} \right) + \lambda T \right] dx \tag{2.1}$$

$$u_y = \int_{-b}^{b} \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} - \nu \frac{\partial^2 U}{\partial y^2} \right) + \lambda T \right] dy \tag{2.2}$$

$$u_z = \int_{0}^{\infty} \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial z^2} \right) + \lambda T \right] dz \tag{2.3}$$

where E, ν and λ are the young's modulus, Poisson's ratio and the linear coefficient of the thermal expansion of the material of the beam respectively and U (x,y,z,t) is the Airy's stress functions which satisfy the differential equation as Tanigawa et al. [10] is

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)^2 U(x,y,z,t) = -\lambda E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \times T(x,y,z,t) \tag{2.4}$$

where T(x,y,z,t) denotes the temperature of a rectangular beam satisfy the following differential equation as Tanigawa et al. [10] is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{g(x,y,z,t)}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \tag{2.5}$$

where k is the thermal conductivity and α is the thermal diffusivity of the material,
subject to initial condition

$$T(x,y,z,0) = f(x,z,t) \tag{2.6}$$

The boundary conditions are

$$\left[T(x,y,z,t) + k_1 \frac{\partial T(x,y,z,t)}{\partial x} \right]_{x=a} = f_1(y,z,t) \tag{2.7}$$

$$\left[T(x,y,z,t) + k_2 \frac{\partial T(x,y,z,t)}{\partial x} \right]_{x=-a} = f_2(y,z,t) \tag{2.8}$$

$$\left[T(x,y,z,t) + k_3 \frac{\partial T(x,y,z,t)}{\partial y} \right]_{y=b} = f_3(x,z,t) \tag{2.9}$$

$$\left[T(x,y,z,t) + k_4 \frac{\partial T(x,y,z,t)}{\partial y} \right]_{y=-b} = f_4(x,z,t) \tag{2.10}$$

$$\left. \frac{\partial T(x,y,z,t)}{\partial z} \right|_{z=0} = 0 \tag{2.11}$$

$$\left.\frac{\partial T(x,y,z,t)}{\partial z}\right|_{z=\infty} = h(x,y,t) \tag{2.12}$$

The stress components in terms of U(x, y, z , t) Durge et al. [1] are given by

$$\sigma_{xx} = \left[\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right] \tag{2.13}$$

$$\sigma_{yy} = \left[\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2}\right] \tag{2.14}$$

$$\sigma_{zz} = \left[\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right] \tag{2.15}$$

Equations (2.1) to (2.15) constitute the mathematical formulation of the problem under consideration.

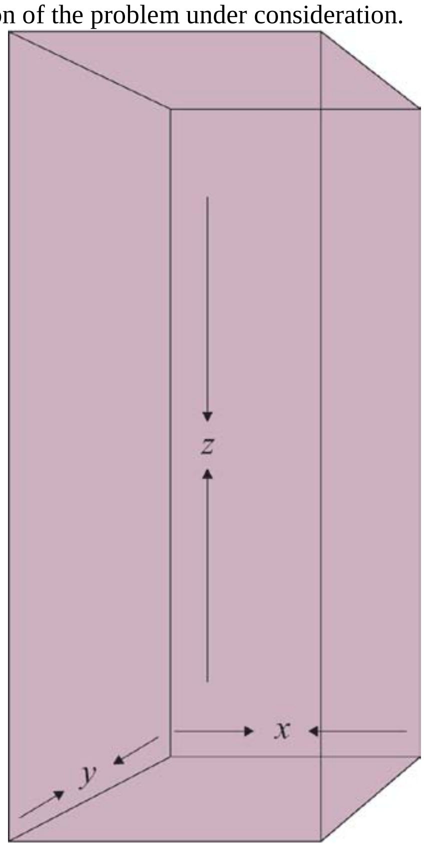


Figure 1: Geometry of the problem

SOLUTION OF THE PROBLEM

Applying finite Marchi-Fasulo transform defined in [9] and Fourier cosine transform to the equations, we get

$$\frac{d\overline{\overline{T}}^*}{dt} + \alpha q^2 \overline{\overline{T}}^* = \frac{\alpha \overline{\overline{g}}^*}{k} + \Psi$$

This is a linear equation whose solution is given by

$$\overline{\overline{T}}^*(m,n,\eta,t) = e^{-\alpha q^2 t}\left(\overline{\overline{f}}^* + \int_0^t \left[\frac{\alpha \overline{\overline{g}}^*}{k} + \Psi\right] e^{\alpha q^2 t'} dt'\right) \tag{3.1}$$

where,

$$\Psi = \left[\frac{P_n(a)}{k_1} f_1 - \frac{P_n(-a)}{k_2} f\right] + \left[\frac{R_m(b)}{k_3} f_3 - \frac{R_m(-b)}{k_4} f_4\right]$$

Now , applying inversion of Fourier Cosine transform and finite Marchi-Fasulo transform to the equation (3.1),one obtains the expression for temperature distribution as

$$T(x,y,z,t) = \left(\frac{2\eta}{\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\Lambda(z) \tag{3.2}$$

Where $\Lambda(z) = \int_0^\infty B(t)\cos(\eta z)dz$,

$$B(t) = e^{-\alpha q^2 t}\left(\overline{\overline{f}}^* + \int_0^t \left[\frac{\alpha \overline{\overline{g}}^*}{k} + \Psi\right] e^{\alpha q^2 t'} dt'\right)$$

$$q^2 = \left(\lambda_n^{\ 2} + \mu_m^{\ 2}\right)$$

Equation (3.2) is the required solution.

AIRY'S STRESS FUNCTIONS

Substituting the value of temperature distribution T(x,y,z,t) from (3.2) in equation (2.4) one obtains

$$U(x,y,z,t) = -\left(\frac{2\eta\pi E}{\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\Lambda(z) \tag{4.1}$$

Where $\Lambda(z) = \int_0^\infty B(t)\cos(\eta z)dz$

DISPLACEMENT COMPONENTS

Substituting the values of Airy's stress function from equation (4.1) in equations (2.1) to (2.3), one obtains

$$u_x = -\left(\frac{2\eta\lambda}{\pi}\right)\Lambda(z)\int_{-a}^{a}\left\{\begin{array}{l}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m''(y)}{\mu_m}\right) - \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} -\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right) \\ -\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\left(\frac{R_m(y)}{\mu_m}\right) + \lambda\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\end{array}\right\}dx \tag{5.1}$$

$$u_y = -\left(\frac{2\eta\lambda}{\pi}\right)\Lambda(z)\int_{-b}^{b}\left\{\begin{array}{l}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right) - \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\left(\frac{R_m(y)}{\mu_m}\right) \\ +\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m''(y)}{\mu_m}\right)) + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\end{array}\right\}dy \tag{5.2}$$

$$u_z = -\left(\frac{2\eta\lambda}{\pi}\right)\Lambda(z)\int_{0}^{\infty}\left\{\begin{array}{l}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\left(\frac{R_m(y)}{\mu_m}\right) + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m''(y)}{\mu_m}\right) \\ +\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right) - \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\end{array}\right\}dz \tag{5.3}$$

DETERMINATION OF STRESS FUNCTION

Substituting the value of Airy's stress function U(x,y,z,t) from equation (4.1) in the equation (2.13) to (2.15) one obtain the stress functions as,

$$\sigma_{xx} = -\left(\frac{2\eta\lambda E}{\pi}\right)\Lambda(z)\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left[\sum_{m=1}^{\infty}\left(\frac{R_m''(y)}{\mu_m}\right) - \eta^2\sum_{m=1}^{\infty}\left(\frac{R_m(y)}{\mu_m}\right)\right] \tag{6.1}$$

$$\sigma_{yy} = -\left(\frac{2\eta\lambda E}{\pi}\right)\Lambda(z)\sum_{m=1}^{\infty}\left(\frac{R_m(y)}{\mu_m}\right)\left[\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right) - \eta^2\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\right] \tag{6.2}$$

$$\sigma_{zz} = -\left(\frac{2\eta\lambda E}{\pi}\right)\Lambda(z)\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\left(\frac{R_m(y)}{\mu_m}\right)+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m''(y)}{\mu_m}\right)\right] \tag{6.3}$$

SPECIAL CASE

Set

$$f(x,y,z,t) = (x-a)^2(x+a)^2(y-b)^2(y+b)^2(z+e^{-z})(1-e^{-t}) \tag{7.1}$$

$$\overline{\overline{f}}(n,m,z,t) = (z+e^{-z})(1-e^{-t}) \times\left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{{a_n}^2}\right] \times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{{b_m}^2}\right] \tag{7.2}$$

Substituting the above value in equations (3.2) to (6.3) one obtains

$$T(x,y,z,t) = \left(\frac{2\eta}{\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\ \Lambda(z) \times\left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{{a_n}^2}\right] \times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{{b_m}^2}\right] \times(z+e^{-z})(1-e^{-t}) \tag{7.3}$$

$$U(x,y,z,t) = -\frac{2\eta\pi E}{\pi}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\ \Lambda(z) \times\left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{{a_n}^2}\right] \times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{{b_m}^2}\right] \times(z+e^{-z})(1-e^{-t}) \tag{7.4}$$

$$u_x = \left(-\frac{2\eta\lambda}{\pi}\right)\Lambda(z)\int_{-a}^{a}\left\{\begin{array}{l}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m''(y)}{\mu_m}\right)-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}-\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\\ -\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\left(\frac{R_m(y)}{\mu_m}\right)+\lambda\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\end{array}\right\}dx \times\left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{{a_n}^2}\right] \times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{{b_m}^2}\right] \times(z+e^{-z})(1-e^{-t}) \tag{7.5}$$

$$u_y = \left(-\frac{2\eta\lambda}{\pi}\right)\Lambda(z)\int_{0}^{b}\left\{\begin{array}{l}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\left(\frac{R_m(y)}{\mu_m}\right)\\ +\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m''(y)}{\mu_m}\right)+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\end{array}\right\}dy \times\left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{{a_n}^2}\right] \times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{{b_m}^2}\right] \times(z+e^{-z})(1-e^{-t}) \tag{7.6}$$

$$u_z = \left(-\frac{2\eta\lambda}{\pi}\right)\Lambda(z)\int_{0}^{\infty}\left\{\begin{array}{l}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\left(\frac{R_m(y)}{\mu_m}\right)+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m''(y)}{\mu_m}\right)\\ +\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\end{array}\right\}dz \times\left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{{a_n}^2}\right] \times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{{b_m}^2}\right] \times(z+e^{-z})(1-e^{-t}) \tag{7.7}$$

$$\sigma_{xx} = \left(-\frac{2\eta\lambda E}{\pi}\right)\Lambda(z)\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left[\sum_{m=1}^{\infty}\left(\frac{R_m''(y)}{\mu_m}\right)-\eta^2\sum_{m=1}^{\infty}\left(\frac{R_m(y)}{\mu_m}\right)\right] \times\left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{{a_n}^2}\right] \times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{{b_m}^2}\right] \times(z+e^{-z})(1-e^{-t}) \tag{7.8}$$

$$\sigma_{yy} = \left(-\frac{2\eta\lambda E}{\pi}\right)\Lambda(z)\sum_{m=1}^{\infty}\left(\frac{R_m(y)}{\mu_m}\right)\left[\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)-\eta^2\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\right] \times\left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{{a_n}^2}\right] \times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{{b_m}^2}\right] \times(z+e^{-z})(1-e^{-t}) \tag{7.9}$$

$$\sigma_{zz} = \left(-\frac{2\eta\lambda E}{\pi}\right)\Lambda(z)\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\left(\frac{R_m(y)}{\mu_m}\right)+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m''(y)}{\mu_m}\right)\right] \times\left[\frac{a_n\cos^2(a_n a)-\cos(a_n a)\sin(a_n a)}{{a_n}^2}\right] \times\left[\frac{b_m\cos^2(b_m b)-\cos(b_m b)\sin(b_m b)}{{b_m}^2}\right] \times(z+e^{-z})(1-e^{-t}) \tag{7.10}$$

NUMERICAL RESULTS

Set $a = 2$, $k = 0.86$, , $b = 3$, $t = 1$ sec in the equations (7.3)- (7.10) to obtain

$$T(x,y,z,t) = \left(\frac{\eta}{\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\ \Lambda(z) \times\left[\frac{a_n\cos^2(2a_n)-\cos(2a_n)\sin(2a_n)}{{a_n}^2}\right]$$

$$\times\left[\frac{b_m\cos^2(3b_m)-\cos(3b_m)\sin(3b_m)}{{b_m}^2}\right]$$
$$\times(z+e^{-z})(1-e^{-t}) \tag{8.1}$$

$$U(x,y,z,t)=-\left(\frac{\eta\pi E}{\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\Lambda(z)$$
$$\times\left[\frac{a_n\cos^2(2a_n)-\cos(2a_n)\sin(2a_n)}{{a_n}^2}\right]$$
$$\times\left[\frac{b_m\cos^2(3b_m)-\cos(3b_m)\sin(3b_m)}{{b_m}^2}\right]$$
$$\times(z+e^{-z})(1-e^{-t}) \tag{8.2}$$

$$u_x=-\left(\frac{\eta\lambda}{\pi}\right)\Lambda(z)\int_{-a}^{a}\left\{\begin{array}{l}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m''(y)}{\mu_m}\right)-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}-\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\\ -\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\left(\frac{R_m(y)}{\mu_m}\right)+\lambda\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\end{array}\right\}dx$$
$$\times\left[\frac{a_n\cos^2(2a_n)-\cos(2a_n)\sin(2a_n)}{{a_n}^2}\right]$$
$$\times\left[\frac{b_m\cos^2(3b_m)-\cos(3b_m)\sin(3b_m)}{{b_m}^2}\right]$$
$$\times(z+e^{-z})(1-e^{-t}) \tag{8.3}$$

$$u_y=-\left(\frac{\eta\lambda}{\pi}\right)\Lambda(z)\int_{-b}^{b}\left\{\begin{array}{l}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\left(\frac{R_m(y)}{\mu_m}\right)\\ +\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m''(y)}{\mu_m}\right)+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\end{array}\right\}dy$$
$$\times\left[\frac{a_n\cos^2(2a_n)-\cos(2a_n)\sin(2a_n)}{{a_n}^2}\right]$$
$$\times\left[\frac{b_m\cos^2(3b_m)-\cos(3b_m)\sin(3b_m)}{{b_m}^2}\right]$$
$$\times(z+e^{-z})(1-e^{-t}) \tag{8.4}$$

$$u_z=-\left(\frac{\eta\lambda}{\pi}\right)\Lambda(z)\int_{0}^{\infty}\left\{\begin{array}{l}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\left(\frac{R_m(y)}{\mu_m}\right)+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m''(y)}{\mu_m}\right)\\ +\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m(y)}{\mu_m}\right)\end{array}\right\}dz$$
$$\times\left[\frac{a_n\cos^2(2a_n)-\cos(2a_n)\sin(2a_n)}{{a_n}^2}\right]$$
$$\times\left[\frac{b_m\cos^2(3b_m)-\cos(3b_m)\sin(3b_m)}{{b_m}^2}\right]$$
$$\times(z+e^{-z})(1-e^{-t}) \tag{8.5}$$

$$\sigma_{xx}=-\left(\frac{\eta\lambda E}{\pi}\right)\Lambda(z)\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left[\sum_{m=1}^{\infty}\left(\frac{R_m''(y)}{\mu_m}\right)-\eta^2\sum_{m=1}^{\infty}\left(\frac{R_m(y)}{\mu_m}\right)\right]$$
$$\times\left[\frac{a_n\cos^2(2a_n)-\cos(2a_n)\sin(2a_n)}{{a_n}^2}\right]$$
$$\times\left[\frac{b_m\cos^2(3b_m)-\cos(3b_m)\sin(3b_m)}{{b_m}^2}\right]$$
$$\times(z+e^{-z})(1-e^{-t}) \tag{8.6}$$

$$\sigma_{yy}=-\left(\frac{\eta\lambda E}{\pi}\right)\Lambda(z)\sum_{m=1}^{\infty}\left(\frac{R_m(y)}{\mu_m}\right)\left[\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)-\eta^2\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\right]$$
$$\times\left[\frac{a_n\cos^2(2a_n)-\cos(2a_n)\sin(2a_n)}{{a_n}^2}\right]$$
$$\times\left[\frac{b_m\cos^2(3b_m)-\cos(3b_m)\sin(3b_m)}{{b_m}^2}\right]$$
$$\times(z+e^{-z})(1-e^{-t}) \tag{8.7}$$

$$\sigma_{zz}=-\left(\frac{\eta\lambda E}{\pi}\right)\Lambda(z)\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\left(\frac{R_m(y)}{\mu_m}\right)+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(\frac{R_m''(y)}{\mu_m}\right)\right]$$
$$\times\left[\frac{a_n\cos^2(2a_n)-\cos(2a_n)\sin(2a_n)}{{a_n}^2}\right]$$
$$\times\left[\frac{b_m\cos^2(3b_m)-\cos(3b_m)\sin(3b_m)}{{b_m}^2}\right]$$
$$\times(z+e^{-z})(1-e^{-t}) \tag{8.8}$$

MATERIAL PROPERTIES

The numerical calculations has been carried out for an Aluminum (pure) rectangular beam with the material properties as:

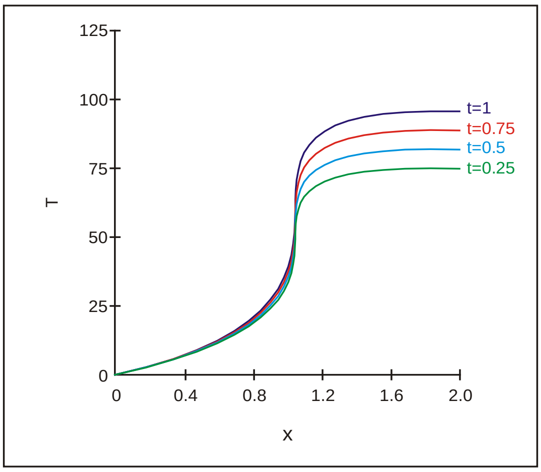
Density ρ =169 lb/ft ^{3}
Specific heat = 0.208 Btu/lbOF
Thermal conductivity K = 117Btu/(hr. ftOF)
Thermal diffusivity α = 3.33 ft^2/hr.
Poisson ratio ν = 0.35
Coefficient of linear thermal expansion α_t = 12.84 x 10^{-6}1/F
Lame constant μ = 26.67
Young's modulus of elasticity E = 70G Pa

DIMENSIONS

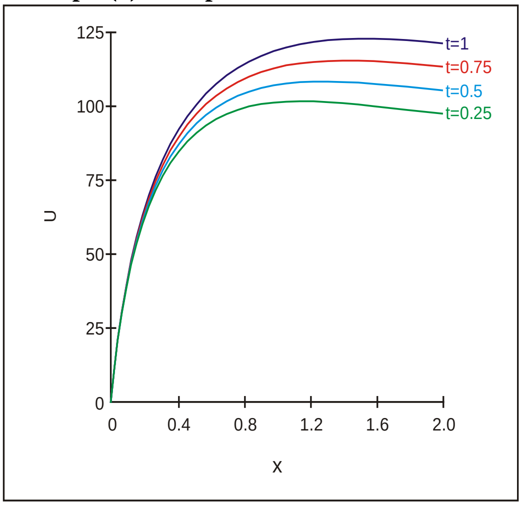
The constants associated with the numerical calculation are taken as
Length of rectangular beam x = 2ft
Breath of rectangular beam y = 3ft
Height of rectangular beam z = 10^{3}ft

CONCLUSION

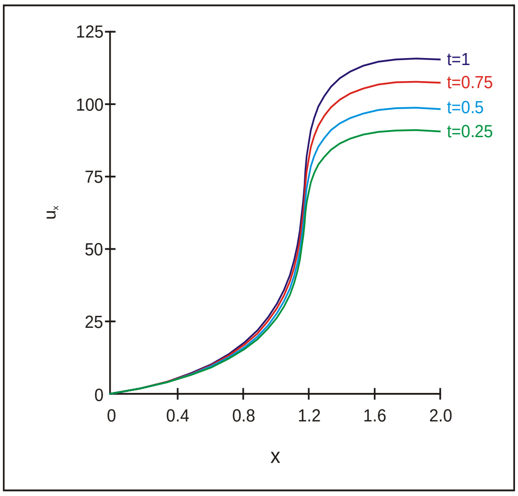
The temperature distribution, displacements and thermal stresses at any point of a semi-infinite rectangular beam have been obtained; when the boundary conditions are known with the aid of finite Marchi-Fasulo transform and Fourier cosine transform techniques. The results are obtained in terms of Bessel's function in the form of infinite series. The results obtained here are useful in engineering problems particularly in the determination of state of stress in a thin rectangular plate, base of furnace of boiler of a thermal power plant and gas power plant.



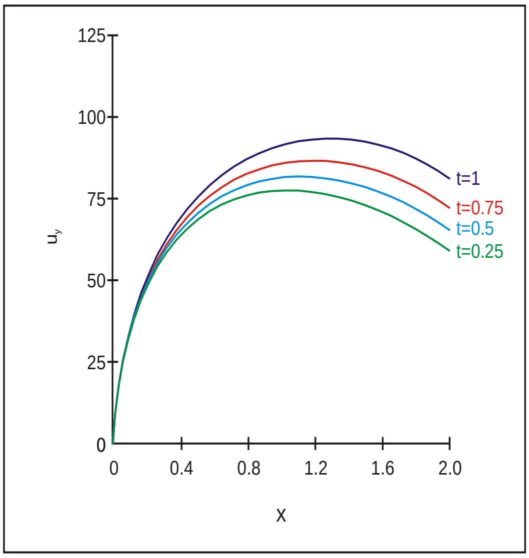
Graph (1): Temperature distribution vs. x



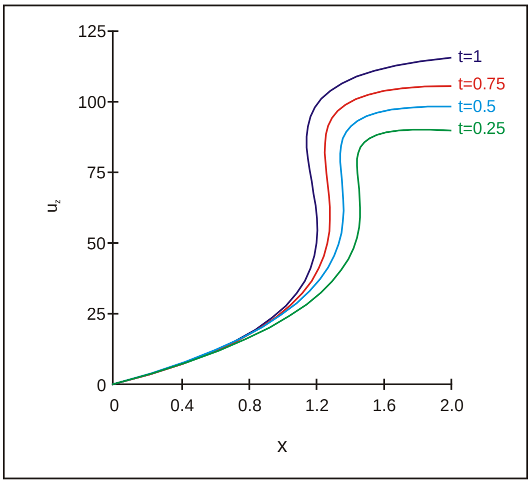
Graph (2): Airy's stress function vs. x



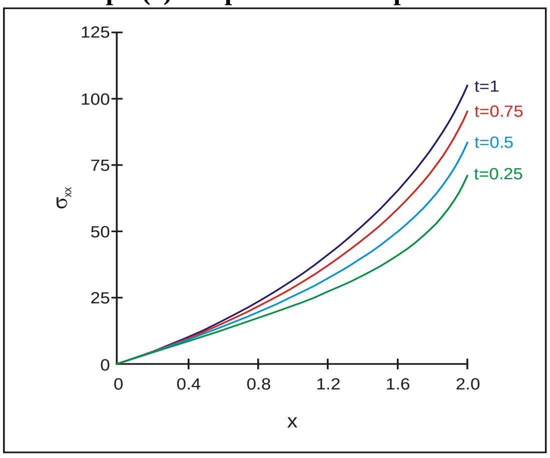
Graph (3): Displacement component vs. x



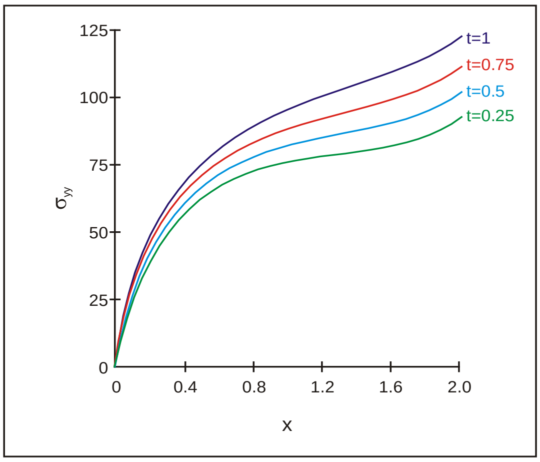
Graph (4): Displacement component vs. x



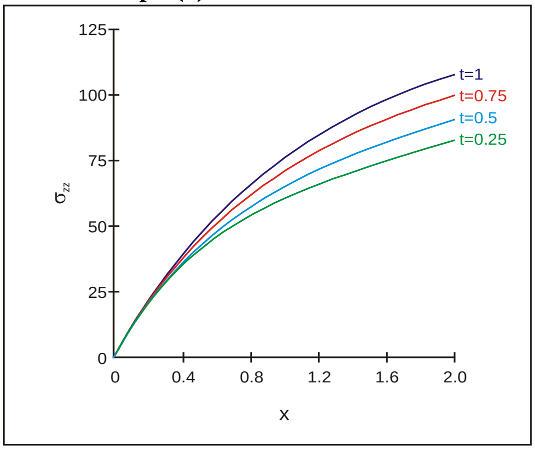
Graph (5): Displacement component vs. x



Graph (6): Thermal stresses vs. x



Graph (7): Thermal stresses vs. x



Graph (8): Thermal stresses vs. x

ACKNOWLEDGMENT

Authors are thankful to anonymous referee for his helpful suggestions.

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