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Model Order Reduction and Stability Analysis of Interval System

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Abstract: The model order reduction tool has been utilized for decrementing the order of the interval model. The main prospective of this paper is to demonstrate the procedure for determining stability analysis of continuous time interval system. This paper concentrate mainly on, how we get the Kharitonov polynomials and how to use the algorithms given by Kharitonov for plotting Kharitonov rectangle. With the plot of Kharitonov rectangle we may determine the stability of interval system graphically. The conventional method for determining stability is also compared with the graphical method for determining the stability. The presented paper demonstrates stability analysis of decreased order system in accordance to the original higher order interval system. The stability analysis has been performed by using Kharitonov algorithm and MATLAB tool. Simulation results have been shown for comparison of stability of higher order interval system and decreased system.

Keyword: Kharitonov; algorithm; interval systems; MOR; stability.

I. INTRODUCTION

In any physical system, there are various parameters which affect the performance and accuracy of system. These parameters are affected by many uncertainties present in the physical world. Due to these uncertainties, there will be variation in the parameters, which must be included while we are modeling any physical system, to achieve robustness of system. These uncertainties in the parameters are in certain limits. While including these uncertainties in parameters, whose variations are in a certain limit, in modeling any physical system, leads to development of interval system[12]. The developed mathematical model, in many cases are of higher order, whose analysis and controller design, is more complex, thus, its needed to shrivel the order of original mathematical model of that physical system, for easiness in study of performance and characteristics of physical system. For this purpose model order reduction (MOR) tool is used. Model Order Reduction [1-11] of interval system[13-14] are discussed in Section III. In section III, we are discussing Routh Padé approximation [13-21].In section IVthe detailed stability analysis of interval system is done. In section V simulation results are discussed in which the detailed graphical analysis and algorithms are discussed. Once we developed reduced system for our previous system, we design a controller for smaller system and check response of smaller order model as well as original system with designed controller and if compared results are identical, then this controller is directly used in original system for controlling purpose. Designing of reduced order system controller is easy as compared to original physical system controller. Reduced order Controller will take less space than Original system controller and also more economical in comparison.

II. PROBLEM STATEMENT

Let mathematical model of original upper order interval system in frequency domain be[12][19];

$$G_O(s) = \frac{[\underline{a}_0,\bar{a}_0]+[\underline{a}_1,\bar{a}_1]s+\cdots+[\underline{a}_{n-1},\bar{a}_{n-1}]s^{n-1}}{[\underline{b}_0,\bar{b}_0]+[\underline{b}_1,\bar{b}_1]s+\cdots+[\underline{b}_{n-1},\bar{b}_{n-1}]s^{n-1}+[\underline{b}_n,\bar{b}_n]s^n} \quad (1)$$

where:

$[\underline{a_i},\overline{a_i}]$ for i= 0,1,2,..n-1 are interval parameters of numerator of original model.

$[\underline{b_i},\overline{b_i}]$ for i= 0,1,2,..n. are interval parameters of denominator polynomials.

$G_O(s)$ is transfer function of original upper order system.

and the reduced order mathematical model in frequency domain is considered as[12][19]

$$G_R(s) = \frac{[\underline{c}_0,\bar{c}_0]+[\underline{c}_1,\bar{c}_1]s+..+[\underline{c}_{k-1},\bar{c}_{k-1}]s^{k-1}}{[\underline{d}_0,\bar{d}_0]+[\underline{d}_1,\bar{d}_1]s+..+[\underline{d}_{k-1},\bar{d}_{k-1}]s^{k-1}+[\underline{d}_k,\bar{d}_k]s^k} \quad (2)$$

where:

$[\underline{c_j},\overline{c_j}]$ for j= 0,1,2,..,k-1 are interval parameters of numerator of reduced order model.

$[\underline{d_j},\overline{d_j}]$ for j= 0,1,2,..,k are bounded parameters of denominator of lower mathematical model.

$G_R(s)$ is frequency domain transfer function of lower order model.

The pre proposed interval arithmetic [12] given below

Let [l,h] and [L,H] be two bounded range.

Addition:

[l',h']+[L',H']=[l'+H',h'+L'];(3)

Substraction:

[l',h'] - [L',H']=[l'-H',h'-L'](4)

Multiplication:

[l',h']×[L',H']=[Min,Max(l'L',l'H',h'L',h'H')(5)

Division:

$$\frac{[l',h']}{[L',H']} = [l',H'] \times \left[\frac{1}{L'},\frac{1}{H'}\right], \text{ provide } [L,H] \neq 0 \quad (6)$$

III. ROUTH-PADÉ APPROXIMATION

In this method of MOR, The numerator is compact by maintaining the preliminary time moments and the Markov

parameter of the interval system of unique model, and the denominator is compacted by using the Routh chart.
Let the reduced order model $G_R(s)$ is of order k is required and transfer function is of the form:

$$G_R(s) = \frac{N_{k'}(s)}{D_{k'}(s)} \quad (7)$$

Where

$$N_{k'}(s) = [\underline{c}_0, \bar{c}_0] + [\underline{c}_1, \bar{c}_1]s + .. + [\underline{c}_{k'-1}, \bar{c}_{k'-1}]s^{k'-1} \quad (8)$$

$$D_{k'}(s) = [\underline{d}_0, \bar{d}_0] + [\underline{d}_1, \bar{d}_1]s + .. + [\underline{d}_{k'-1}, \bar{d}_{k'-1}]s^{k'-1} + [\underline{d}_{k'}, \bar{d}_{k'}]s^{k'} \quad (9)$$

and $[\underline{c}_i, \bar{c}_i]$ for i=0 to 'k'-1' and $[\underline{d}_j, \bar{d}_j]$ for j=0 to 'k'' are unknown interval parameters of numerator and denominator respectively.

Order Reduction of Numerator:

For determining the reduced order model $N_k(s)$ of numerator polynomial we have to keep the preliminary time moments and the Markov parameter. Reduced model has time moment and Markov parameter contribution as:

$$N_{k'}(s) = N_{k't}(s) + N_{k'm}(s) \quad (10)$$

Where:
$N_{k't}(s)$ is the time moment contribution,
$N_{k't}(s)$is the Markov parameter contribution, and
K'= t+m (k' is reduced model order).
If the higher order model having two (even) dominant poles, then it can be reduced up to second order model with only one Markov parameter. i.e ; k'=2, m=1 and t=1
If a system model has an odd number of dominant poles, then higher number of Markov parameters will be obtained, i.e , If the system has 3 (odd) dominant poles, i.e.,
K'=3, then m=2 and t=1.
All the time extra figure of Markov parameters should be retained in the compact order model.

$$N_{r'}(s) = T_1 + T_2 s + \cdots + T_t s^{k'-m+1} + M_m s^{k'-m} + \cdots + M_2 s^{k'-2} + M_1 s^{k'-1} \quad (11)$$

Time moment for the compressed model is deliberated from the equation.

$$T_1(s) = (\underline{d_0}, \overline{d_0}) \frac{(\underline{a_{t-1}}, \overline{a_{t-1}})}{(\underline{b_0}, \overline{b_0})} \quad (12)$$

Markov parameter can be determined of reduced order model by Equation

$$[\underline{M_m}, \overline{M_m}] = \frac{1}{(\underline{A_n}, \overline{A_n})} \sum_{i=1}^{m} (\underline{a_{n-1}}, \overline{a_{n-1}})(\underline{d_{k-(m-i)}}, \overline{d_{k-(m-i)}}) - \sum_{j=0}^{m-1} (\underline{M_i}, \overline{M_i})(\underline{b_{n-(m-1i}}, \overline{b_{n-(m-i)}}) \quad (13)$$

Initial Markov parameter is taken as $(\underline{M_0}, \overline{M_0}) = (0,0)$

Order Reduction of Denominator:

The devisor of compressed order model is constructed by the Routh steadiness arrey scheme [15-21]. For denominator Routh chart is given as:

$[\underline{b}_{11}, \overline{b}_{11}][\underline{b}_{12}, \overline{b}_{12}][\underline{b}_{13}, \overline{b}_{13}][\underline{b}_{14}, \overline{b}_{14}]$ ….
$[\underline{b}_{21}, \overline{b}_{21}][\underline{b}_{22}, \overline{b}_{22}][\underline{b}_{23}, \overline{b}_{23}][\underline{b}_{24}, \overline{b}_{24}]$ …..
$[\underline{b}_{31}, \overline{b}_{31}][\underline{b}_{32}, \overline{b}_{32}][\underline{b}_{33}, \overline{b}_{33}]$ ….
$[\underline{b}_{41}, \overline{b}_{41}][\underline{b}_{42}, \overline{b}_{42}][\underline{b}_{43}, \overline{b}_{43}]$ ….
: :
$[\underline{b}_{(n-1)1}, \overline{b}_{(n-1)1}][\underline{b}_{(n-1)2}, \overline{b}_{(n-1)2}]$
$[\underline{b}_{n1}, \overline{b}_{n1}]$
$[\underline{b}_{(n+1)1}, \overline{b}_{(n+1)1}]$ (14)

From this routh array we may find the value of devisor of compressed order structure ($D_{K'}(s)$) as

$$D_{k'}(s) = b_{(n+1-r'),1}s^{r'} + b_{(n+2-r'),1}s^{r'-1} + b_{(n+1-r'),2}s^{r'-2} + .. \quad (15)$$

Example 1.
Consider interval system[20]transfer function of 3rd order as

$$G(s) = \frac{[2,3]s^2 + [17.5,18.5]s + [15,16]}{[2,3]s^3 + [17,18]s^2 + [35,36]s + [20.5,21.5]} \equiv \frac{N(s)}{D(s)} \quad (16)$$

Suppose, a first order model (k=1) of the form

$$G_R(s) = \frac{[\underline{c}_0, \bar{c}_0]}{[\underline{d}_0, \bar{d}_0] + [\underline{d}_1, \bar{d}_1]s} \equiv \frac{\widehat{c_0}}{\widehat{d_1}s + \hat{d}_0} \equiv \frac{N_k(s)}{D_k(s)}$$ is desired.

The Routh stability table for D(s) is given as
Table -I

[2,3]	[35,36]
[17,18]	[20.5, 21.5]
[29.47, 35.71]	
[16.92, 26.05]	

(17)

The $D_{k'}(s)$ obtaines from Table-I, using equ. (15)

$$D_{1'}(s) = [29.47,35.71]s + [16.92,26.05] \quad (18)$$

The first time moment of high-order and reduced order continuous interval system are,

T_1=[0.6976,0.7805] for original order interval structure (19)

$$\widehat{T}_1 = \frac{\hat{c}_0}{\hat{d}_0} \text{ for abridged order model} \quad (20)$$

By harmonizing the initial time moments, the initial order model is

$$G_1(s) = \frac{[11.8033,20.3320]}{[29.47,35.71]s + [16.92,26.05]} \quad (21)$$

Example 2: Let second order reduced model is consider as

$$G_R(s) = \frac{[\underline{c}_0, \bar{c}_0]}{[\underline{d}_0, \bar{d}_0] + [\underline{d}_1, \bar{d}_1]s} \equiv \frac{\hat{c}_1 s + \widehat{c_0}}{\hat{d}_2 s^2 + \hat{d}_1 s + \hat{d}_0} \equiv \frac{N_k(s)}{D_k(s)} \quad (22)$$

For the equation (16) of example 1.
The denominator $D_k(s)$ obtaines from Table-I, using (15) is

$$D_{2'}(s) = [17,18]s^2 + [29.47,35.71]s + [20.5, 21.5] \quad (23)$$

The first time moment of high-order continuous interval system and decreased order continuous system with parametric uncertainty are given in (19) and (20), and the first Markov parameter of high-order continuous system with parametric uncertainty and decreased order continuous system with parametric uncertainty are calculated as,

$$M_1 = [0.6667, 1.5000] \quad (24)$$

$$\widehat{M}_1 = \frac{\hat{c}_1}{\hat{d}_2} \quad (25)$$

By matching the first time moment of the system to that of the model and the first Markov parameter of the system to that of the model, the second-order model derived is

$$G_2(s) = \frac{[11.2200,27]s + [14.3008,16.7807]}{[17,18]s^2 + [29.47,35.71]s + [20.5,21.5]} \quad (26)$$

IV. STABILITY THEORY

Stability of any system is main concern in physical modeling. In case of Interval system, the stability is suggested by Kharitonov polynomials stability [25-31]. If Kharitonov polynomials are stable, then compressed order interval scheme is also stable[27][28].

Kharitonov polynomial is given as

$K^{1'}(s) = K^{even}_{min} + K^{odd}_{min}$

$K^{2'}(s) = K^{even}_{min} + K^{odd}_{max}$

$K^{3'}(s) = K^{even}_{max} + K^{odd}_{min}$

$K^{4'}(s) = K^{even}_{max} + K^{odd}_{max}$ (26)

Where,

$K^{even}_{min} = \underline{a_0} + \overline{a_2}s^2 + \underline{a_4}s^4 + \overline{a_6}s^6 + ..,$

$K^{even}_{max} = \overline{a_0} + \underline{a_2}s^2 + \overline{a_4}s^4 + \underline{a_6}s^6 + ..,$

$K^{odd}_{min} = \underline{a_1}s + \overline{a_3}s^3 + \underline{a_5}s^5 + \overline{a_7}s^7 + ..,$

$K^{odd}_{max} = \overline{a_1}s + \underline{a_3}s^3 + \overline{a_5}s^5 + \underline{a_7}s^7 + ..,$

and thus the Kharitonovpolynomials are written as:

$K^{1'}(s)=\underline{a_0} + \underline{a_1}s + \overline{a_2}s^2 + \overline{a_3}s^3 + \underline{a_4}s^4 + \underline{a_5}s^5 + \overline{a_6}s^6 + ..$,(27)

$K^{2'}(s)=\overline{a_0} + \overline{a_1}s + \underline{a_2}s^2 + \underline{a_3}s^3 + \overline{a_4}s^4 + \overline{a_5}s^5 + \underline{a_6}s^6 + ..$,(28)

$K^{3'}(s)=\overline{a_0} + \underline{a_1}s + \underline{a_2}s^2 + \overline{a_3}s^3 + \overline{a_4}s^4 + \underline{a_5}s^5 + \underline{a_6}s^6 + ..$,(29)

$K^{4'}(s)=\underline{a_0} + \overline{a_1}s + \overline{a_2}s^2 + \underline{a_3}s^3 + \underline{a_4}s^4 + \overline{a_5}s^5 + \overline{a_6}s^6 + ..$,(30)

Using the above algorithm, Kharitonov polynomial for both numerator and denominator is written for both original and reduced order system. Using Routh Stability Theorem, the stability of both the original and reduced order Interval System is checked.

Kharitonov polynomials for the numerator of equation (16) are,

$K^{1N}(s) = 15 + 18.5s + 3s^2$ (31)

$K^{2N}(s) = 16 + 17.5s + 2s^2$ (32)

$K^{3N}(s) = 16 + 18.5s + 3s^2$ (33)

$K^{4N}(s) = 15 + 17.5s + 3s^2$ (34)

The Kharitonov polynomials of the denominator polynomial of equation (16) are,

$K^{1D}(s) = 20.5 + 36s + 18s^2 + 2s^3$ (35)

$K^{2D}(s) = 21.5 + 35s + 17s^2 + 3s^3$ (36)

$K^{3D}(s) = 21.5 + 36s + 17s^2 + 2s^3$ (37)

$K^{4D}(s) = 20.5 + 35s + 18s^2 + 3s^3$ (38)

The Kharitonov polynomials of the reduced order denominator polynomial of equation (21) are,

$K_1^{1D}(s) = 16.92 + 35.71s$ (39)

$K_1^{2D}(s) = 26.05 + 29.47s$ (40)

$K_1^{3D}(s) = 26.05 + 35.71s$ (41)

$K_1^{4D}(s) = 16.92 + 29.47s$ (42)

The Kharitonov polynomials of the reduced order numerator polynomial of equation (21) are,

$K_1^{1N}(s) = 14.30$ (43)

$K_1^{1N}(s) = 16.78$ (44)

$K_1^{1N}(s) = 16.78$ (45)

$K_1^{1N}(s) = 14.30$ (46)

The Kharitonov polynomials of the reduced order denominator polynomial of equation (26)

$K_2^{1D}(s) = 20.5 + 35.71s + 18s^2$ (47)

$K_2^{2D}(s) = 21.5 + 29.47s + 17s^2$ (48)

$K_2^{3D}(s) = 21.5 + 35.71s + 17s^2$ (49)

$K_2^{4D}(s) = 20.5 + 29.47s + 18s^2$ (50)

The Kharitonov polynomials of the reduced order numerator polynomial of equation (26)

$K_2^{1N}(s) = 14.3008 + 27s$ (51)

$K_2^{1N}(s) = 16.7807 + 11.3339s$ (52)

$K_2^{1N}(s) = 16.7807 + 27s$ (53)

$K_2^{1N}(s) = 14.3008 + 11.3339s$ (54)

V. RESULTS AND DISCUSSIONS

Fig.1 shows the Kharitonovrectangle for original 3rd order interval system. It has been plotted in ‘MATLAB 2015’between real part and imaginary part of Kharitonov polynomials of ‘equation 35’ to ‘equation 38’ for various frequencies. In Fig.1 the encirclement of the origin is anticlockwise hence the original interval system is stable.

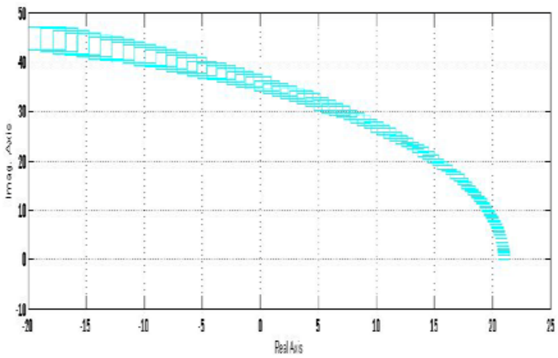

Fig. 1. Kharitonov Rectangle for Higher order system(HOIS)Equ.(16).

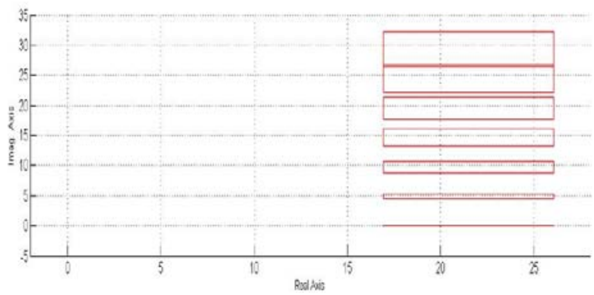

Fig. 2. Kharitonov Rectangle for lower 1st order system Equ.(21)

In the above plot for frequency 0<w<0.5 Kharitonov rectangles are plotted for ‘equation 39’ to ‘equation 42’. As per Kharitonov the rectangle encirclement the origin in anticlockwise direction thus model is stable

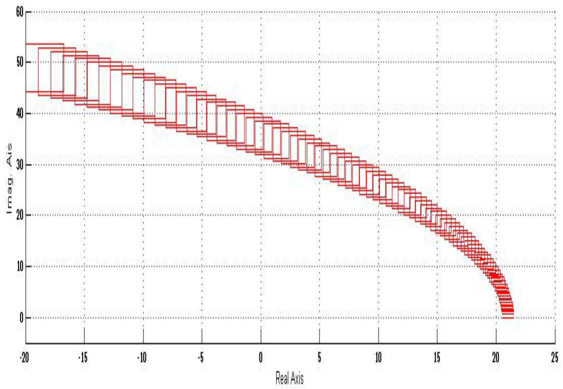

Fig. 3. Kharitonov plot for Equ.(26)

In the above plot for frequency 0<w<0.5 Kharitonov rectangles are plotted of 'equation 47' to 'equation 50'. As per Kharitonov the rectangle encirclement of the origin in anticlockwise direction thus lower order model is stable.

For all the models the step response are plotted, shown in Fig.4 and Fig.5.

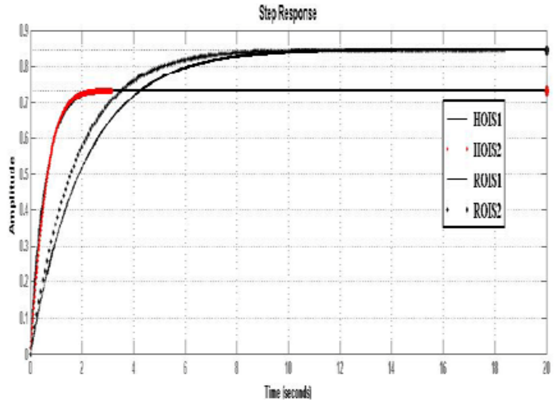

Fig. 4. Step response of HOIS Equ.(16) & ROIS Equ.(21).

Fig. 4, shows the behavior of original 3rd order system withparametric uncertainty and the decreased 1st order system with parametric uncertainty for step input. For plotting step response, two systems HOIS1 & HOIS2 for higher order interval system are constructed using the Kharitonov polynomials of equ. 16 and two systems ROIS1 & ROIS2 for reduced 1st order interval system are constructed using the Kharitonov polynomials ofequ. 21. From the response it is clear that the both the systems are stable as the steady state response for both the system is constant.

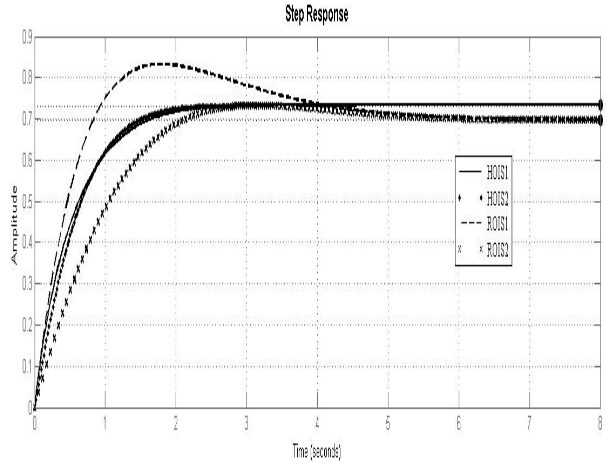

Fig. 5. Step response of HOIS Equ.(16) & ROIS Equ.(26).

In fig. 5, characteristic of 3rd order system withparametric uncertainty is shown for step input and the decreased2st order system with parametric uncertainty for step input is shown.For plotting step response, two systems HOIS1 & HOIS2 for higher order interval system are constructed using the Kharitonov polynomials of equ. 16 and two systems ROIS1 & ROIS2 for reduced 2st order interval system are constructed using the Kharitonov polynomials of equ. 26. From the response it is clear that the original system and reduced order system both are stable as the steady state response for both the system is constant.

To study the effect of impulse on higher order system and lower order (1st , 2rd order) interval system the impulse response are plotted. Impulse response shown in Fig.6 & Fig.7. In Fig.6 the characteristic of original interval system and 1st order system is shown for impulse input. For plotting the response previous systems are used of equation 16 & 21.

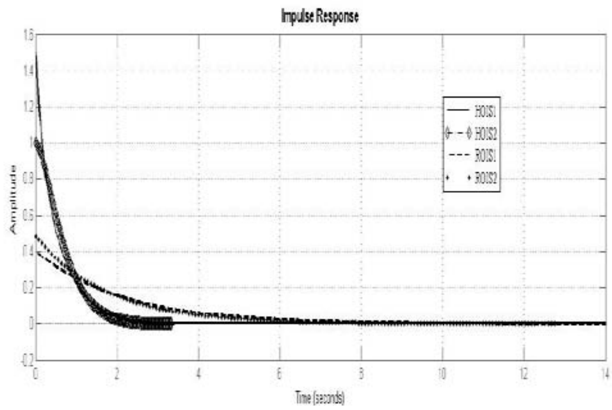

Fig. 6. Impulse response of HOIS equ.(16) & ROIS equ.(21).

From the response it is clear that the system will survive if the sudden disturbances are came in existence.

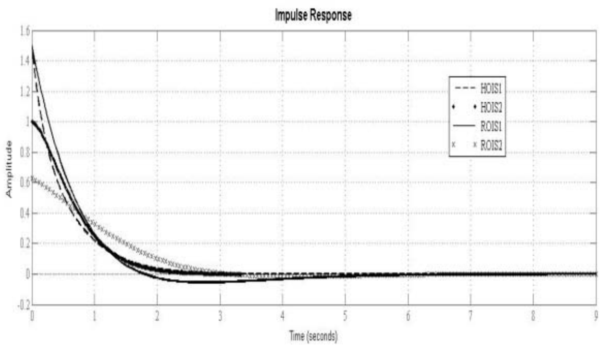

Fig.7. Impulse response of HOIS Equ.(16) & ROIS Equ.(26).

In Fig.7. the characteristic of equ.16 is shown for impulse input and characteristic of equ.26 is shown for impulse input. From the response it is clear that the system will survive if the sudden disturbances are came in existence.

The conventional method which is used for determining the stability of any interval system depends upon the stability of all the four Kharitonov polynomials of any system. If any one of these four polynomial set are unstable then the hole system is unstable.

VI. CONCLUSIONS

In this note it is satisfactorily explained that how to use Routh Padé MOR method for reducing 3rd order interval system. The paper also encourages the graphical stability study of interval model. For this the 3rd order model with parametric uncertainty system and decreased1st& 2nd order system with parametric uncertainty stability analysis is satisfactorily analyzed. The conventional method for determining interval system's stability is time consuming compared to graphical analysis method. For the complete analysis Y. Shamash [20] example is considered.

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