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Maintainability and Availability of a complex semi-up system

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Abstract: The increasing complexity of present day equipment has brought into focus the important aspects of reliability, known as maintainability and availability. When any system fails, the ease with which it is brought back into operation reflects, in some measure, the maintainability character of the equipment. Similarly, if the system is capable of being repaired easily or if it has high maintainability factor, then the availability will also be high. In this paper we will touch upon various factors leading to the calculation of MTSF, Availability, Busy period of the repairman in repairing the failed units and expected profit of a system which is laid in semi-up state. A system is defined into a semi-up state wherein, some units of the equipment are in failed state yet the equipment is delivering its normal work but the system cannot be put in working condition for a longer period to avoid the complete failure. In other words, we can classify such systems into systems working with failures till the immediate repairing facility arrives. Such systems may be useful in remote areas where, arduous type of work is done on robust machines and repair facility arrives late. In recent years many papers on reliability have been written in order to predict, estimate or optimize the probability of survival, the mean life or more generally the life distribution of components or systems. Earlier, Murari et al [7] have done reliability analysis taking units in two different modes. Rander et-al [4] has evaluated the cost analysis of two dissimilar cold standby systems with preventive maintenance and replacement of standby units. In the past, Arora et-al [2] has done reliability analysis of two unit standby redundant system with constrained repair time. Gupta et-al [6] has worked on a cold standby system with arrival time of server and correlated failures and repairs. A pioneer work in this field was done by Gopalan [1] and Osaki [3] by performing analysis of warm standby system and parallel system with bivariate exponential life respectively. Earlier Pathak et al [8& 9] studied reliability parameters of a main unit with its supporting units and also compared the results with two different distributions. In all the papers authors have worked with only two kinds of states i.e. up state and down state. However, there are times and conditions wherein the states are neither up or completely down mode. We can refer such states in sleeping mode or more in reliability terms they can be stated in semi-up mode. In many papers, the working system is often assumed to be down when the main unit is not operating. In fact, it is not so. There is need to conceptualize systems in Industries that are complex in nature but they yield such results which are more suitable to industries in order to meet the ever increasing demands of society. In this paper an attempt has been made by authors by incorporating the concept of semi-up mode and tried to obtain the reliability parameters of working system using regenerative point technique.

***Keywords*:** Regenerative Point, MTSF, Availability, Busy Period

1. LITERATURE REVIEW

Murari, K and Muruthachalan, C[7] presents two different models of two unit system, where the system operates under different conditions of working and the two units are interlinked. In model I, the system works for a period of time with two units in series, then one unit is switched off leaving other to continue alone. In model II, the first period consists of one unit working and other in standby mode. These periods alternate repeated. The reliability of the system for each model is examined.

Rander, M.C., Kumar Ashok and Suresh K [4] evaluated cost analysis of two dissimilar cold standby systems with preventive maintenance and replacement of standby units under the assumption that the standby unit being of substandard quality and is not repairable. It is to be replaced on failure. In this paper, the failure time distributions are negative exponential while all the other distributions are general.

Arora , J. R. [2] This work considers a two-unit warm standby redundant system with repair. The The repair of a failed unit is constrained as follows : Associated with each failure of a unit, a random variable known as maximum repair time (MRT). If the repair of the failed unit is not completed within the MRT, the unit is rejected. Two types of failure situations for the system are considered. (1) no allowed down time and (2) some allowed down time. The expression for cumulative distribution function of time to system failure (TSF) is derived using markov renewal processes.

Gupta R, Tyagi, P. K. and Goel L. R. [6] Presents an analysis of two unit cold standby system with random arrival time of a server. The failure and repair time of each unit is assumed to be correlated and their joint density function is taken to be bivariate exponential. Using regenerative point technique various reliability characteristics of the system ave been obtained.

Gopalan, M.N. [1] discusses the probabilistic analysis with n-unit system keeping (n-1) units in warm standby configuration w ith a single repair facility. The failure time of the operating unit and of a standby unit are assumed to be exponentially distributed. Intially, a unit is switched on and other units are kept as warm standby. The system break down when the last operating unit fails. In this paper, the laplace-transform technique is used to solve integral equations.

Osaki [3] discusses a two unit parallel redundant system with a single repair facility in which the life time of two units obey a bivariate exponential distribution and the repair

time of the failed unit obeys an arbitrary distribution. Applying extended Markov renewal process, the quantities of interests in reliability theory are obtained.

Chandrasekar, P. and Nataraja R. [6] proposed to obtain a 100 percent limit for steady state availability of a two unit standby system, when the failure rate of an online unit is constant and the repair time of the failed unit has a Erlangian distribution.

Pathak, V.K. [8] & [9] considered a two unit system with single repair facility. If the working unit fails, it is immediately taken over by a standby unit and the repair on the failed unit is started immediately. Reliability analysis is done by calculating various characteristics such as mean time to system failure, availability and busy period of repairman using regenerative point technique. Comparative study is also done taking two types of distributions viz. Weibull and Erlangian.

2. SYSTEM DESCRIPTION ABOUT THE MODEL

The system consists of four units namely one main unit M and two types of associate units A & B. Unit A has one stand by unit with it. Here the main unit M dependent upon associate units A & B and the system is operable when the main unit and both associate units are in operable and the system is semi operable when the main unit is failed and both associate unit are in operable. associate units A & B are employed to rotate main unit M. As soon as a job arrives, all the units work with load. It is assumed that only one job is taken for processing at a time. There is a single repairman who repairs the failed units on first come first served basis. Using regenerative point technique several system characteristics such as transition probabilities, mean sojourn times, availability and busy period of the repairman are evaluated. In the end the expected profit is also calculated.

3. ASSUMPTIONS USED IN THE MODEL

a. The system consists of one main unit and two associate units with one of associate units has standby partner.
b. The main unit M works with the help of associate unit A and B.
c. There is a single repairman which repairs the failed units on priority basis.
d. After random period of time the whole system goes to preventive maintenance.
e. All units work as new after repair.
f. The failure rates of all the units are taken to be exponential whereas the repair time distributions are arbitrary.
g. Switching devices are perfect and instantaneous.

4. SYMBOLS AND NOTATIONS

p_{ij} = Transition probabilities from S_i to S_j

μ_i = Mean sojourn time at time t

E_0 =State of the system at epoch t=0

E=set of regenerative states

$S_0 - S_9$

$q_{ij}(t)$ = Probability density function of transition time from S_i to S_j

$Q_{ij}(t)$ = Cumulative distribution function of transition time from S_i to S_j

$\pi_i(t)$ = Cdf of time to system failure when starting from state $E_0 = S_i \in E$

$\mu_i(t)$ = Mean Sojourn time in the state $E_0 = S_i \in E$

$B_i(t)$ = Repairman is busy in the repair at time t / $E_0 = S_i \in E$

$r_1 / r_2 / r_3 / r_4$ =Constant repair rate of Main unit M /associate Unit A/ associate Unit B

$\alpha / \beta / \gamma$ =Failure rate of Main unit M / associate Unit A/ associate Unit B

$g_1 / g_2 / g_3$ =Probability density function of repair time of Main unit M /associate Unit A/ associate Unit B

$\overline{G}_1 / \overline{G}_2 / \overline{G}_3$ =Cumulative distribution function of repair time of Main unit M /associate Unit A/ associate Unit B

a(t) = Probability density function of preventive maintenance .

b(t) = Probability density function of preventive maintenance completion time.

$\overline{A}(t)$ = Cumulative distribution functions of preventive maintenance.

$\overline{B}(t)$ = Cumulative distribution functions of preventive maintenance completion time.

s = Symbol for Laplace -stieltjes transforms**.**

c = Symbol for Laplace-convolution.

5. SYMBOLS USED FOR STATES OF THE SYSTEM

$M_0 / M_g / M_{wr}$ -- Main unit 'M' under operation/ in good and non-operative mode / waiting for repair.

$A_0 / A_r / A_s / A_g / A_{wr}$ -- Associate Unit 'A' under operation/repair/standby/ good and non-operative mode/ waiting for repair.

$B_0 / B_r / B_g$ -- Associate Unit 'B' under operation/repair/good and non-operative mode

P.M. -- System under preventive maintenance.
S.D. -- System in shut down mode.

Up states:

$S_0 = (M_0, A_0, A_s, B_0); S_2 = (M_0, A_r, A_o, B_0);$

Semi Up states:

$S_1 = (M_r, A_0, A_s, B_0); S_4 = (M_{wr}, A_r, A_o, B_0)$

Downstates:

$S_3 = (M_g, A_g, A_s, B_r); S_5 = (M_{wr}, A_g, A_s, B_r);$

$S_6 = (M_g, A_r, A_{wr}, B_g); S_7 = (M_g, A_r, A_g, B_r);$

$S_8 = (S.D.); S_7 = (P.M.)$

6. TRANSITION PROBABILITIES

Simple probabilistic considerations yield the following non-zero transition probabilities:

1. $Q_{01}(t) = \int_0^t \alpha\, e^{-x_1 t}\, \overline{A}(t)dt$ 2. $Q_{02}(t) = \int_0^t \beta\, e^{-x_1 t}\, \overline{A}(t)dt$

3. $Q_{03}(t) = \int_0^t \gamma\, e^{-x_1 t}\, \overline{A}(t)dt$ 4. $Q_{10}(t) = \int_0^t e^{-(\beta+\gamma)t} g_1(t)dt$

5. $Q_{14}(t) = \int_0^t \beta e^{-(\beta+\gamma)t} \overline{G_1}(t)dt$ 6. $Q_{15}(t) = \int_0^t \gamma e^{-(\beta+\gamma)t} \overline{G_1}(t)dt$

7. $Q_{20}(t) = \int_0^t e^{-x_1 t} g_2(t)dt$ 8. $Q_{24}(t) = \int_0^t \alpha e^{-x_1 t} \overline{G_2}(t)dt$

9. $Q_{26}(t) = \int_0^t \beta e^{-x_1 t} \overline{G_2}(t)dt$ 10. $Q_{27}(t) = \int_0^t \gamma e^{-x_1 t} \overline{G_2}(t)dt$

11. $Q_{30}(t) = \int_0^t g_3(t)dt$ 12. $Q_{41}(t) = \int_0^t e^{-(\beta+\gamma)t} g_2(t)dt$

13. $Q_{48}(t) = \int_0^t (\beta+\gamma) e^{-(\beta+\gamma)t} \overline{G}_2(t)dt$ 14. $Q_{51}(t) = \int_0^t g_3(t)dt$

15. $Q_{62}(t) = \int_0^t g_2(t)dt$ 16. $Q_{72}(t) = \int_0^t g_3(t)dt$

17. $Q_{80}(t) = \int_0^t g_4(t)dt$ 18. $Q_{90}(t) = \int_0^t b(t)dt$

19. $Q_{09}(t) = \int_0^t a(t) e^{-x_1 t} dt$

Where $x_1 = \alpha + \beta + \gamma$, Now letting $t \to \infty$, we get $\underset{t\to\infty}{Lim}\, Q_{ij}(t) = p_{ij}$

20 . $p_{01} = \int_0^\infty \alpha\, e^{-x_1 t}\, \overline{A}(t)dt = \frac{\alpha}{x_1}[1 - a^*(x_1)]$, 21. $p_{02} = \int_0^\infty \beta\, e^{-x_1 t}\, \overline{A}(t)dt = \frac{\beta}{(x_1)}[1 - a^*(x_1)]$,

22. $p_{03} = \int_0^\infty \gamma\, e^{-x_1 t}\, \overline{A}(t)dt = \frac{\gamma}{x_1}[1 - a^*(x_1)]$, 23. $p_{10} = \int_0^\infty e^{-(\beta+\gamma)t} g_1(t)dt = g_1^*(\beta+\gamma)$

24. $p_{14} = \int_0^\infty \beta\, e^{-(\beta+\gamma)t} \overline{G_1}(t)dt = \frac{\beta}{(\beta+\gamma)}[1 - g_1^*(\beta+\gamma)]$,

25. $p_{15} = \int_0^\infty \gamma\, e^{-(\beta+\gamma)t} \overline{G_1}(t)dt = \frac{\gamma}{(\beta+\gamma)}[1 - g_1^*(\beta+\gamma)]$,

26. $p_{20} = \int_0^\infty e^{-x_1 t} g_2(t)dt = g_2^*(x_1)$, 27. $p_{24} = \int_0^\infty \alpha e^{-x_1 t} \overline{G}_2(t)dt = \frac{\alpha}{x_1}[1 - g_2^*(x_1)]$

28. $p_{26} = \int_0^\infty \beta e^{-x_1 t} \overline{G}_2(t)dt = \frac{\beta}{x_1}[1 - g_2^*(x_1)]$, 29. $p_{27} = \int_0^\infty \gamma e^{-x_1 t} \overline{G}_2(t)dt = \frac{\gamma}{x_1}[1 - g_2^*(x_1)]$

30. $p_{30} = \int_0^\infty g_3(t)dt = 1$, 31. $p_{41} = \int_0^\infty e^{-(\beta+\gamma)t} g_2(t)dt = g_2^*(\beta+\gamma)$

32. $p_{48} = \int_0^\infty (\beta+\gamma)e^{-(\beta+\gamma)t}\overline{G}_2(t)dt = 1 - g_2^*(\beta+\gamma)$, 33. $p_{51} = \int_0^\infty g_3(t)dt = 1$

34. $p_{62} = \int_0^\infty g_2(t)dt = 1$, 35. $p_{72} = \int_0^\infty g_3(t)dt = 1$

36. $p_{80}(t) = \int_0^\infty g_4(t)dt = 1$, 37. $p_{90}(t) = \int_0^\infty b(t)dt = 1$

38. $p_{30} = p_{51} = p_{62} = p_{72} = p_{80} = p_{90} = 1$ [6.1-6.38]

It is easy to see that

$p_{01} + p_{02} + p_{03} + p_{09} = 1$, $p_{10} + p_{14} + p_{15} = 1$, $p_{20} + p_{24} + p_{26} + p_{27} = 1$, $p_{41} + p_{48} = 1$ [6.39-6.42]

And mean sojourn time are given by

43. $\mu_0 = \frac{1}{(\alpha+\beta+\gamma)}[1 - a^*(\alpha+\beta+\gamma)]$, 44. $\mu_1 = \frac{1}{\beta+\gamma}[1 - g_1^*(\beta+\gamma)]$,

45. $\mu_2 = \frac{1}{\alpha+\beta+\gamma}[1 - g_2^*(\alpha+\beta+\gamma)]$, 46. $\mu_3 = \int_0^\infty \overline{G}_3(t)dt$

47. $\mu_4 = \frac{1}{\beta+\gamma}[1 - g_2^*(\beta+\gamma)]$, 48. $\mu_5 = \int_0^\infty \overline{G}_3(t)dt$

49. $\mu_6 = \int_0^\infty \overline{G}_2(t)dt$, 50. $\mu_7 = \int_0^\infty \overline{G}_3(t)dt$

51. $\mu_8 = \int_0^\infty \overline{G}_4(t)dt$, 52. $\mu_9 = \int_0^\infty \overline{B}(t)dt$

[6.43-6.52]

We note that the Laplace-stieltjes transform of $Q_{ij}(t)$ is equal to Laplace transform of $q_{ij}(t)$

i.e. $\tilde{Q}_{ij}(s) = \int_0^\infty e^{-st}Q_{ij}(t)dt = L\{Q_{ij}(t)\} = q_{ij}^*(s)$

[6.53]

54. $\tilde{Q}_{01}(s) = \int_0^\infty \alpha e^{-(s+\alpha+\beta+\gamma)t}\overline{A}(t)dt = \frac{\alpha}{s+\alpha+\beta+\gamma}[1 - a^*(s+\alpha+\beta+\gamma)]$

55. $\tilde{Q}_{02}(s) = \int_0^\infty \beta e^{-(s+\alpha+\beta+\gamma)t}\overline{A}(t)dt = \frac{\beta}{s+\alpha+\beta+\gamma}[1 - a^*(s+\alpha+\beta+\gamma)]$

56. $\tilde{Q}_{03}(s) = \int_0^\infty \gamma e^{-(s+\alpha+\beta+\gamma)t}\overline{A}(t)dt = \frac{\gamma}{s+\alpha+\beta+\gamma}[1 - a^*(s+\alpha+\beta+\gamma)]$

57. $\tilde{Q}_{09}(s) = \int_0^\infty e^{-(s+\alpha+\beta+\gamma)t}a(t)dt = a^*(s+\alpha+\beta+\gamma)$

58. $\tilde{Q}_{10}(s) = \int_0^\infty e^{-st}e^{-(\beta+\gamma)t}g_1(t)dt = g_1^*(s+\beta+\gamma)$

59. $\tilde{Q}_{14}(s) = \int_0^\infty \beta e^{-st}e^{-(\beta+\gamma)t}\overline{G}_1(t)dt = \frac{\beta}{s+\beta+\gamma}[1 - g_1^*(s+\beta+\gamma)]$

60. $\tilde{Q}_{15}(s) = \int_0^\infty \gamma e^{-st}e^{-(\beta+\gamma)t}\overline{G}_1(t)dt = \frac{\gamma}{s+\beta+\gamma}[1 - g_1^*(s+\beta+\gamma)]$

61. $\tilde{Q}_{20}(s) = \int_0^\infty e^{-(s+\alpha+\beta+\gamma)t} g_2(t)dt = g_2^*(s+\alpha+\beta+\gamma)$

62. $\tilde{Q}_{24}(s) = \int_0^\infty \alpha\, e^{-(s+\alpha+\beta+\gamma)t} \overline{G}_2(t)dt = \frac{\alpha}{s+\alpha+\beta+\gamma}[1-g_2^*(s+\alpha+\beta+\gamma)]$

63. $\tilde{Q}_{26}(s) = \int_0^\infty \beta\, e^{-(s+\alpha+\beta+\gamma)t} \overline{G}_2(t)dt = \frac{\beta}{s+\alpha+\beta+\gamma}[1-g_2^*(s+\alpha+\beta+\gamma)]$

64. $\tilde{Q}_{27}(s) = \int_0^\infty \gamma\, e^{-(s+\alpha+\beta+\gamma)t} \overline{G}_2(t)dt = \frac{\gamma}{s+\alpha+\beta+\gamma}[1-g_2^*(s+\alpha+\beta+\gamma)]$

65. $\tilde{Q}_{30}(s) = \int_0^\infty e^{-st} g_3(t)dt = g_3^*(s)$

66. $\tilde{Q}_{41}(s) = \int_0^\infty e^{-(s+\beta+\gamma)t} g_2(t)dt = g_2^*(s+\beta+\gamma)$

67. $\tilde{Q}_{46}(s) = \int_0^\infty (\beta+\gamma) e^{-(s+\beta+\gamma)t} \overline{G}_2(t)dt = \frac{(\beta+\gamma)}{(s+\beta+\gamma)}[1-g_2^*(s+\beta+\gamma)]$

68. $\tilde{Q}_{51}(s) = \int_0^\infty e^{-st} g_3(t)dt = g_3^*(s)$

69. $\tilde{Q}_{62}(s) = \int_0^\infty e^{-st} g_2(t)dt = g_2^*(s)$

70. $\tilde{Q}_{72}(s) = \int_0^\infty e^{-st} g_3(t)dt = g_3^*(s)$

71. $\tilde{Q}_{80}(s) = \int_0^\infty g_4(t)dt = g_4^*(s)$

72. $\tilde{Q}_{90}(s) = \int_0^\infty e^{-st} b(t)dt = b^*(s)$

[6.54-6.72]

We define m_{ij} as follows:-

$$m_{ij} = -[\frac{d}{ds}\tilde{Q}_{ij}(s)]_{s=0} = -Q_{ij}'(0) \quad [6.73]$$

It can to show that

$$m_{01}+m_{02}+m_{03}+m_{09} = \mu_0; m_{10}+m_{14}+m_{15} = \mu_1; m_{20}+m_{24}+m_{26}+m_{27} = \mu_2; m_{41}+m_{48} = \mu_4 \quad [6.74\text{-}6.77]$$

7. MEAN TIME TO SYSTEM FAILURE

Time to system failure can be regarded as the first passage time to the failed state. To obtain it we regard the down state as absorbing. Using the argument as for the regenerative process, we obtain the following recursive relations.

$\pi_0(t) = Q_{01}(t)$ s $\pi_1(t) + Q_{02}(t)$ s $\pi_2(t) + Q_{03}(t) + Q_{09}(t)$

$\pi_1(t) = Q_{10}(t)$ s $\pi_0(t) + Q_{14}(t)$ s $\pi_4(t) + Q_{15}(t)$

$\pi_2(t) = Q_{20}(t)$ s $\pi_0(t) + Q_{24}(t)$ s $\pi_4(t) + Q_{26}(t) + Q_{27}(t)$

$\pi_4(t) = Q_{41}(t)$ s $\pi_1(t) + Q_{48}(t)$ [7.1-7.4]

Taking Laplace -stieltjes transform of above equations and writing in matrix form.

We get
$$\begin{vmatrix} 1 & -\tilde{Q}_{01} & -\tilde{Q}_{02} & 0 \\ -\tilde{Q}_{10} & 1 & 0 & -\tilde{Q}_{14} \\ -\tilde{Q}_{20} & 0 & 1 & -\tilde{Q}_{24} \\ 0 & -\tilde{Q}_{41} & 0 & 1 \end{vmatrix} \begin{bmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \\ \pi_4 \end{bmatrix} = \begin{bmatrix} \tilde{Q}_{03} + \tilde{Q}_{09} \\ \tilde{Q}_{15} \\ \tilde{Q}_{26} + \tilde{Q}_{27} \\ \tilde{Q}_{48} \end{bmatrix}$$

$$D_1(s) = \begin{vmatrix} 1 & -\tilde{Q}_{01} & -\tilde{Q}_{02} & 0 \\ -\tilde{Q}_{10} & 1 & 0 & -\tilde{Q}_{14} \\ -\tilde{Q}_{20} & 0 & 1 & -\tilde{Q}_{24} \\ 0 & -\tilde{Q}_{41} & 0 & 1 \end{vmatrix}$$

$$D_1(s) = 1 - \tilde{Q}_{14}\tilde{Q}_{41} - \tilde{Q}_{01}\tilde{Q}_{10} - \tilde{Q}_{02}\tilde{Q}_{20} - \tilde{Q}_{02}\tilde{Q}_{10}\tilde{Q}_{41}\tilde{Q}_{24} + \tilde{Q}_{02}\tilde{Q}_{20}\tilde{Q}_{14}\tilde{Q}_{41} \quad [7.5]$$

And $N_1(s) = \begin{vmatrix} \tilde{Q}_{03} + \tilde{Q}_{09} & -\tilde{Q}_{01} & -\tilde{Q}_{02} & 0 \\ \tilde{Q}_{15} & 1 & 0 & -\tilde{Q}_{14} \\ \tilde{Q}_{26} + \tilde{Q}_{27} & 0 & 1 & -\tilde{Q}_{24} \\ \tilde{Q}_{48} & -\tilde{Q}_{41} & 0 & 1 \end{vmatrix}$

$$N_1(s) = (\tilde{Q}_{03} + \tilde{Q}_{09})(1 - \tilde{Q}_{14}\tilde{Q}_{41}) + \tilde{Q}_{01}(\tilde{Q}_{15} + \tilde{Q}_{14}\tilde{Q}_{48}) - \tilde{Q}_{02}\{\tilde{Q}_{15}\tilde{Q}_{24}\tilde{Q}_{41} - \tilde{Q}_{24}\tilde{Q}_{48} - \tilde{Q}_{26} - \tilde{Q}_{27} + \tilde{Q}_{14}\tilde{Q}_{41}(\tilde{Q}_{26} + \tilde{Q}_{27})\} \quad [7.6]$$

Now letting $s \to 0$ we get

$$D_1(0) = 1 - p_{01}p_{10} - p_{02}p_{20} - p_{14}p_{14} - p_{02}p_{10}p_{24}p_{41} - p_{02}p_{20}p_{14}p_{41} \quad [7.7]$$

The mean time to system failure when the system starts from the state S_0 is given by

$$\text{MTSF} = E(T) = -[\frac{d}{ds}\tilde{\pi}_0(s)]_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)} \quad [7.8]$$

To obtain the numerator of the above equation, we collect the coefficients of relevant of m_{ij} in $D_1'(0) - N_1'(0)$.

Coeff. of ($m_{01} = m_{02} = m_{03} = m_{09}$) $= 1 - p_{14}p_{41}$

Coeff. of ($m_{10} = m_{14} = m_{15}$) $= p_{01} + p_{02}p_{24}p_{41}$

Coeff. of ($m_{20} = m_{24} = m_{26} = m_{27}$) $= p_{02}(1 - p_{14}p_{14})$

Coeff. of ($m_{41} = m_{48}$) $= p_{01}p_{14} + p_{02}p_{24}$ [7.9-7.12]

From equation [7.8]

$$\text{MTSF} = E(T) = -[\frac{d}{ds}\tilde{\pi}_0(s)]_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)}$$

$$= \frac{\mu_0(1 - p_{14}p_{14}) + \mu_1(p_{01} + p_{02}p_{24}p_{41}) + \mu_2 p_{02}(1 - p_{14}p_{14}) + \mu_4(p_{02}p_{24} + p_{01}p_{14})}{1 - p_{01}p_{10} - p_{02}p_{20} - p_{02}p_{01}p_{24}p_{41} - p_{02}p_{20}p_{14}p_{41}} \quad [7.13]$$

8. AVAILABILITY ANALYSIS

Let $M_i(t)(i = 0,1,2,4)$ denote the probability that system is initially in regenerative state $S_i \in E$ is up at time t without passing through any other regenerative state or returning to itself through one or more non regenerative states .i.e. either it continues to remain in regenerative S_i or a non regenerative state including itself . By probabilistic arguments, we have the following recursive relations

$$M_0(t) = e^{-(\alpha+\beta+\gamma)t}\overline{A}(t), \quad M_1(t) = e^{-(\beta+\gamma)t}\overline{G_1}(t), \quad M_2(t) = e^{-(\alpha+\beta+\gamma)t}\overline{G_2}(t), \; M_4(t) = e^{-(\beta+\gamma)t}\overline{G_4}(t)$$

[8.1-8.4]

Recursive relations giving point wise availability $A_i(t)$ given as follows:

$$A_0(t) = M_0(t) + \sum_{i=1,2,3,9} q_{0i}(t) \boxed{c} A_i(t) \quad ; \qquad A_1(t) = M_1(t) + \sum_{i=0,4.5} q_{1i}(t) \boxed{c} A_i(t) ;$$

$$A_2(t) = M_2(t) + \sum_{i=0,4,6,7} q_{2i}(t) \boxed{c} A_i(t) \quad ; \qquad A_3(t) = q_{30}(t) \boxed{c} A_0(t) ;$$

$$A_4(t) = M_4(t) + \sum_{i=1,8} q_{4i}(t) \boxed{c} A_i(t) \quad ; \qquad A_5(t) = q_{51}(t) \boxed{c} A_1(t) ;$$

$$A_6(t) = q_{62}(t) \boxed{c} A_2(t) \quad ; \qquad A_7(t) = q_{72}(t) \boxed{c} A_2(t) ;$$

$$A_8(t) = q_{80}(t) \boxed{c} A_0(t) \quad ; \qquad A_9(t) = q_{90}(t) \boxed{c} A_0(t) ;$$

[8.5-8.14]

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

$$q_{10X10}[A_0^*, A_1^*, A_2^*, A_3^*, A_4^*, A_5^*, A_6^*, A_7^*, A_8^*, A_9^*]^{/} = [M_0^*, M_1^*, M_2^*, 0, M_4^*, 0, 0, 0, 0, 0]^{/} \qquad [8.15]$$

Where q_{10X10} =

$$\begin{vmatrix}
1 & -q_{01}^* & -q_{02}^* & -q_{03}^* & 0 & 0 & 0 & 0 & 0 & -q_{09}^* \\
-q_{10}^* & 1 & 0 & 0 & -q_{14}^* & -q_{15}^* & 0 & 0 & 0 & 0 \\
-q_{20}^* & 0 & 1 & 0 & -q_{24}^* & 0 & -q_{26}^* & -q_{27}^* & 0 & 0 \\
-q_{30}^* & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -q_{41}^* & 0 & 0 & 1 & 0 & 0 & 0 & -q_{48}^* & 0 \\
0 & -q_{51}^* & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -q_{62}^* & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -q_{72}^* & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
-q_{80}^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
-q_{90}^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{vmatrix}$$

Therefore
$$D_2(s)=\begin{vmatrix} 1 & -q_{01}^{*} & -q_{02}^{*} & -q_{03}^{*} & 0 & 0 & 0 & 0 & 0 & -q_{09}^{*} \\ -q_{10}^{*} & 1 & 0 & 0 & -q_{14}^{*} & -q_{15}^{*} & 0 & 0 & 0 & 0 \\ -q_{20}^{*} & 0 & 1 & 0 & -q_{24}^{*} & 0 & -q_{26}^{*} & -q_{27}^{*} & 0 & 0 \\ -q_{30}^{*} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -q_{41}^{*} & 0 & 0 & 1 & 0 & 0 & 0 & -q_{48}^{*} & 0 \\ 0 & -q_{51}^{*} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -q_{62}^{*} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -q_{72}^{*} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -q_{80}^{*} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -q_{90}^{*} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\begin{aligned} D_2(s) &= (1-q_{26}^{*}q_{62}^{*}-q_{27}^{*}q_{72}^{*})-q_{14}^{*}q_{41}^{*}(1-q_{26}^{*}q_{62}^{*}-q_{27}^{*}q_{72}^{*})-q_{15}^{*}q_{51}^{*}(1-q_{26}^{*}q_{62}^{*}-q_{27}^{*}q_{72}^{*}) \\ &\quad -q_{01}^{*}q_{10}^{*}(1-q_{26}^{*}q_{62}^{*}-q_{27}^{*}q_{72}^{*})-q_{01}^{*}q_{14}^{*}q_{48}^{*}q_{80}^{*}(1-q_{26}^{*}q_{62}^{*}-q_{27}^{*}q_{72}^{*}) \\ &\quad -q_{02}^{*}\{q_{10}^{*}q_{24}^{*}q_{41}^{*}-q_{14}^{*}q_{41}^{*}q_{20}^{*}+q_{15}^{*}q_{51}^{*}(-q_{20}^{*}-q_{24}^{*}q_{48}^{*}q_{80}^{*})-1(-q_{20}^{*}-q_{24}^{*}q_{48}^{*}q_{80}^{*})\} \\ &\quad -q_{03}^{*}q_{30}^{*}\{(1-q_{26}^{*}q_{62}^{*}-q_{27}^{*}q_{72}^{*})-q_{14}^{*}q_{41}^{*}(1-q_{26}^{*}q_{62}^{*}-q_{27}^{*}q_{72}^{*})-q_{15}^{*}q_{51}^{*}(1-q_{26}^{*}q_{62}^{*}-q_{27}^{*}q_{72}^{*})\} \\ &\quad -q_{07}^{*}q_{70}^{*}\{(1-q_{26}^{*}q_{62}^{*}-q_{27}^{*}q_{72}^{*})-q_{14}^{*}q_{41}^{*}(1-q_{26}^{*}q_{62}^{*}-q_{27}^{*}q_{72}^{*})-q_{15}^{*}q_{51}^{*}(1-q_{26}^{*}q_{62}^{*}-q_{27}^{*}q_{72}^{*})\} \end{aligned}$$

If $s\to 0$ we get $D_2(0)=0$ which is true [8.16]

Now
$$N_2(s)=\begin{vmatrix} M_0^{*} & -q_{01}^{*} & -q_{02}^{*} & -q_{03}^{*} & 0 & 0 & 0 & 0 & 0 & -q_{09}^{*} \\ M_1^{*} & 1 & 0 & 0 & -q_{14}^{*} & -q_{15}^{*} & 0 & 0 & 0 & 0 \\ M_2^{*} & 0 & 1 & 0 & -q_{24}^{*} & 0 & -q_{26}^{*} & -q_{27}^{*} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ M_4^{*} & -q_{41}^{*} & 0 & 0 & 1 & 0 & 0 & 0 & -q_{48}^{*} & 0 \\ 0 & -q_{51}^{*} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -q_{62}^{*} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -q_{72}^{*} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

Solving this Determinant, we get

$$\begin{aligned} N_2(s) &= M_0^{*}(1-q_{26}^{*}q_{62}^{*}-q_{27}^{*}q_{72}^{*})(1-q_{14}^{*}q_{41}^{*}-q_{15}^{*}q_{51}^{*})+M_1^{*}q_{01}^{*}(1-q_{26}^{*}q_{62}^{*}-q_{27}^{*}q_{72}^{*}) \\ &\quad +M_2^{*}q_{02}^{*}(1-q_{14}^{*}q_{41}^{*}-q_{15}^{*}q_{51}^{*})++M_4^{*}q_{01}^{*}q_{14}^{*}(1-q_{26}^{*}q_{62}^{*}-q_{27}^{*}q_{72}^{*}) \end{aligned}$$ [8.17]

If $s\to 0$ we get

$$\begin{aligned} N_2(0) &= \mu_0(p_{20}+p_{24})(1-p_{14}p_{14}-p_{15})+\mu_1 p_{01}(p_{20}+p_{24}) \\ &\quad +\mu_2 p_{02}(1-p_{14}p_{14}-p_{15})+\mu_4 p_{01}p_{14}(p_{20}+p_{24}) \end{aligned}$$ [8.18]

Let $(p_{20}+p_{24})(1-p_{14}p_{14}-p_{15})=L_0$, $p_{01}(p_{20}+p_{24})=L_1$,

$p_{02}(1-p_{14}p_{14}-p_{15})=L_2$, $p_{01}p_{14}(p_{02}+p_{24})=L_4$ [8.19 - 8.22]

To find the value of $D_2^{'}(0)$ we collect the coefficient m_{ij} in $D_2(s)$ we get

Coeff. of $(m_{01} = m_{02} = m_{03} = m_{09}) = (p_{20} + p_{24})(1 - p_{14}p_{14} - p_{15}) = L_0^{'}$

Coeff. of $(m_{10} = m_{14} = m_{15}) = p_{01}(p_{20} + p_{24}) + p_{02}p_{24}p_{41} = L_1^{'}$

Coeff. of $(m_{20} = m_{24} = m_{26} = m_{27}) = p_{02}(1 - p_{14}p_{14} - p_{15}) = L_2^{'}$

Coeff.of $(m_{30}) = p_{01}p_{03}(p_{20} + p_{24}) = L_3^{'}$

Coeff.f $(m_{41} = m_{48}) = p_{02}p_{24}(1 - p_{15}) + p_{01}p_{14}(p_{02} + p_{24}) = L_4^{'}$

Coeff. of $(m_{51}) = p_{01}p_{15}(p_{20} + p_{24}) + p_{02}p_{24}p_{15}p_{41} = L_5^{'}$

$\text{Coeff. of } (m_{62}) = p_{02}p_{26}(1 - p_{41} + p_{51}p_{41} - p_{15}) = L_6^{'}$

$\text{Coeff. of } (m_{72}) = p_{02}p_{27}(1 - p_{41} + p_{51}p_{41} - p_{15}) = L_7^{'}$

Coeff. of $(m_{80}) = p_{01}p_{14}p_{48}(p_{20} + p_{24}) + p_{02}p_{24}p_{48} = L_8^{'}$

Coeff. of $(m_{90}) = p_{09}(p_{20} + p_{24})(1 - p_{14}p_{14} - p_{15}) = L_9^{'}$

[8.23-8.32]

Thus the solution for the steady-state availability is given by

$$A_0^{*}(\infty) = \lim_{t\to\infty} A_0^{*}(t) = \lim_{S\to 0} sA_0^{*}(s) = \frac{N_2(0)}{D_2^{'}(0)} = \frac{\mu_0 L_0 + \mu_1 L_1 + \mu_2 L_2 + \mu_4 L_4}{\sum_{i=0,1,2,3,4,5,6,7,8,9} \mu_i L_i^{'}}$$

[8.33]

9. BUSY PERIOD ANALYSIS

(a) Let $W_i(t)$ $(i = 1,2,3,4,5,6,7)$ denote the probability that the repairman is busy initially with repair in regenerative state S_i and remains busy at epoch t without transiting to any other state or returning to itself through one or more regenerative states.
By probabilistic arguments we have

$W_1(t) = \overline{G_1}(t), W_2(t) = \overline{G_2}(t), W_3(t) = \overline{G_3}(t), W_4(t) = \overline{G_4}(t), W_5(t) = \overline{G_5}(t), W_6(t) = \overline{G_6}(t), W_7(t) = \overline{G_7}(t)$

[9.1-9.7]

Developing similar recursive relations as in availability, we have

$B_0(t) = \sum_{i=1,2,3,9} q_{0i}(t) \boxed{c} B_i(t)$; $B_1(t) = W_1(t) + \sum_{i=0,4,5} q_{1i}(t) \boxed{c} B_i(t)$;

$B_2(t) = W_2(t) + \sum_{i=0,4,6,7} q_{2i}(t) \boxed{c} B_i(t)$; $B_3(t) = W_3(t) + q_{30}(t) \boxed{c} B_0(t)$;

$B_4(t) = W_4(t) + \sum_{i=1,8} q_{4i}(t) \boxed{c} B_i(t)$; $B_5(t) = W_5(t) + q_{51}(t) \boxed{c} B_1(t)$;

$B_6(t) = W_6(t) + q_{62}(t) \boxed{c} B_0(t)$; $B_7(t) = W_7(t) + q_{72}(t) \boxed{c} B_2(t)$

$B_8(t) = q_{80}(t) \boxed{c} B_0(t)$; $B_9(t) = q_{90}(t) \boxed{c} B_0(t)$

[9.8-9.17]

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

$q_{10X10}[B_0^{*}, B_1^{*}, B_2^{*}, B_3^{*}, B_4^{*}, B_5^{*}, B_6^{*}, B_7^{*} B_8^{*}, B_9^{*}]^{/} = [0, W_1^{*}, W_2^{*}, W_3^{*}, W_4^{*}, W_5^{*}, W_6^{*}, W_7^{*}, 0, 0]^{/}$

[9.18]

Where q_{10x10} is denoted by [8.15] and therefore $D_2^{'}(s)$ is obtained as in the expression of availability.

$$\text{Now } N_3(s) = \begin{vmatrix} 0 & -q_{01}^{*} & -q_{02}^{*} & -q_{03}^{*} & 0 & 0 & 0 & 0 & 0 & -q_{09}^{*} \\ W_1^{*} & 1 & 0 & 0 & -q_{14}^{*} & -q_{15}^{*} & 0 & 0 & 0 & 0 \\ W_2^{*} & 0 & 1 & 0 & -q_{24}^{*} & 0 & -q_{26}^{*} & -q_{27}^{*} & 0 & 0 \\ W_3^{*} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ W_4^{*} & -q_{41}^{*} & 0 & 0 & 1 & 0 & 0 & 0 & -q_{48}^{*} & 0 \\ W_5^{*} & -q_{51}^{*} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ W_6^{*} & 0 & -q_{62}^{*} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ W_7^{*} & 0 & -q_{72}^{*} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

Solving this Determinant, In the long run, we get the value of this determinant after putting $s \to 0$ is

$$\begin{aligned} N_3(0) &= \mu_1 p_{01}(1-p_{26}-p_{27}) + \mu_2 p_{02}(1-p_{14}p_{14}-p_{15}) + \mu_3 p_{03}(1-p_{14}p_{14}-p_{15}) \\ &+ \mu_4 p_{01}p_{14}(1-p_{26}-p_{27}) + \mu_5 p_{01}p_{15}(1-p_{26}-p_{27}) + \mu_2 p_{02}p_{26}(1-p_{14}p_{14}-p_{15}) \\ &+ \mu_2 p_{02}p_{27}(1-p_{14}p_{14}-p_{15}) \\ &= \mu_1 L_1 + \mu_2 L_2 + \mu_3 L_3 + +\mu_4 L_4 + \mu_5 L_5 + \mu_6 L_6 + \mu_7 L_7 = \sum_{i=1,2,3,4,5,6,7} \mu_i L_i \end{aligned}$$

[9.19]

Where $p_{01}(p_{20}+p_{24}) = L_1$, $p_{02}(1-p_{14}p_{14}-p_{15}) = L_2$, $p_{03}(1-p_{14}p_{14}-p_{15}) = L_3$,
$p_{01}p_{14}(p_{20}+p_{24}) = L_4$, $p_{01}p_{15}(p_{20}+p_{24}) = L_5$, $p_{02}p_{26}(1-p_{14}p_{14}-p_{15}) = L_6$,
$p_{02}p_{27}(1-p_{14}p_{14}-p_{15}) = L_7$ [9.20–9.26]

Thus the fraction of time for which the repairman is busy with repair of the failed unit is given by:

$$B_0^{1^{*}}(\infty) = \lim_{t\to\infty} B_0^{1^{*}}(t) = \lim_{s\to 0} B_0^{1^{*}}(s) = \frac{N_3(0)}{D_2^{/}(0)} = \frac{\sum_{i=1,2,3,4,5,6,7} \mu_i L_i}{\sum_{i=0,1,2,3,4,5,6,7,8,9} \mu_i L_i^{'}} \quad [9.27]$$

(b)Busy period of the Repairman in preventive maintenance in time (0, t], By probabilistic arguments we have

$$W_9(t) = \overline{B}(t) \quad [9.28]$$

Similarly developing similar recursive relations as in 9(a), we have

$$B_0(t) = \sum_{i=1,2,3,9} q_{0i}(t) \boxed{c} B_i(t) \quad ; \quad B_1(t) = \sum_{i=0,4,5} q_{1i}(t) \boxed{c} B_i(t) \; ;$$

$$B_2(t) = \sum_{i=0,4,6,7} q_{2i}(t) \boxed{c} B_i(t) \quad ; \quad B_3(t) = q_{30}(t) \boxed{c} B_0(t) \quad ;$$

$$B_4(t) = \sum_{i=1,8} q_{4i}(t) \boxed{c} B_i(t) \quad ; \quad B_5(t) = q_{51}(t) \boxed{c} B_1(t) \quad ;$$

$$B_6(t) = q_{62}(t) \boxed{c} B_0(t) \quad ; \quad B_7(t) = q_{72}(t) \boxed{c} B_2(t) \quad ;$$

$$B_8(t) = q_{80}(t) \boxed{c} B_0(t) \quad ; \quad B_9(t) = W_9(t) + q_{90}(t) \boxed{c} B_0(t)$$

[9.29-9.38]

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

$$q_{10X10}[B_0^*, B_1^*, B_2^*, B_3^*, B_4^*, B_5^*, B_6^*, B_7^* B_8^*, B_9^*]' = [0,0,0,0,0,0,0,0,0, W_9^*]'$$

[9.39]

Where q_{10x10} is denoted by [8.15] and therefore $D_2'(s)$ is obtained as in the expression of availability.

Now $N_4(s) =$

$$\begin{vmatrix} 0 & -q_{01}^* & -q_{02}^* & -q_{03}^* & 0 & 0 & 0 & 0 & 0 & -q_{09}^* \\ 0 & 1 & 0 & 0 & -q_{14}^* & -q_{15}^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -q_{24}^* & 0 & -q_{26}^* & -q_{27}^* & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -q_{41}^* & 0 & 0 & 1 & 0 & 0 & 0 & -q_{48}^* & 0 \\ 0 & -q_{51}^* & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -q_{62}^* & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -q_{72}^* & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ W_9^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

Solving this Determinant, In the long run, we get the value of this determinant after putting $s \to 0$ is

$$N_4(0) = \mu_9 p_{09}(p_{20} + p_{24})(1 - p_{14}p_{14} - p_{15}p_{51}) = \mu_9 L_9$$

[9.40]

Thus the fraction of time for which the system is under preventive maintenance is given by:

$$B_0^{2^*}(\infty) = \lim_{t\to\infty} B_0^{2^*}(t) = \lim_{s\to 0} sB_0^{2^*}(s) = \frac{N_4(0)}{D_2'(0)} = \frac{\mu_9 L_9}{\sum_{i=0,1,2,3,4,5,6,7,8,9} \mu_i L_i'}$$

[9.41]

(c)Busy period of the Repairman in Shut Down repair in time (0, t], By probabilistic arguments we have

$$W_8(t) = \overline{G}_8(t)$$

[9.42]

Similarly developing similar recursive relations as in 9(b), we have

$$B_0(t) = \sum_{i=1,2,3,9} q_{0i}(t) \boxed{c} B_i(t) \quad ; \quad B_1(t) = \sum_{i=0,4,5} q_{1i}(t) \boxed{c} B_i(t) \; ;$$

$$B_2(t) = \sum_{i=0,4,6,7} q_{2i}(t) \boxed{c} B_i(t) \quad ; \quad B_3(t) = q_{30}(t) \boxed{c} B_0(t) \quad ;$$

$$B_4(t) = \sum_{i=1,8} q_{4i}(t) \boxed{c} B_i(t) \quad ; \quad B_5(t) = q_{51}(t) \boxed{c} B_1(t) \quad ;$$

$$B_6(t) = q_{62}(t) \boxed{c} B_0(t) \quad ; \quad B_7(t) = q_{72}(t) \boxed{c} B_2(t) \quad ;$$

$$B_8(t) = W_8(t) + q_{80}(t) \boxed{c} B_0(t) \quad ; \quad B_9(t) = q_{90}(t) \boxed{c} B_0(t)$$

[9.43-9.52]

Taking Laplace stieltjes transformation of above equations; and writing in matrix form, we get

$$q_{10X10}[B_0^*, B_1^*, B_2^*, B_3^*, B_4^*, B_5^*, B_6^*, B_7^* B_8^*, B_9^*]' = [0,0,0,0,0,0,0,0, W_8^*, 0]'$$

Where q_{10x10} is denoted by [8.15] and therefore $D_2'(s)$ is obtained as in the expression of availability.

Now $N_5(s) = \begin{vmatrix} 0 & -q_{01}^{*} & -q_{02}^{*} & -q_{03}^{*} & 0 & 0 & 0 & 0 & 0 & -q_{09}^{*} \\ 0 & 1 & 0 & 0 & -q_{14}^{*} & -q_{15}^{*} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -q_{24}^{*} & 0 & -q_{26}^{*} & -q_{27}^{*} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -q_{41}^{*} & 0 & 0 & 1 & 0 & 0 & 0 & -q_{48}^{*} & 0 \\ 0 & -q_{51}^{*} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -q_{62}^{*} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -q_{72}^{*} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ W_8^{*} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$

In the long run, we get the value of this determinant after putting $s \to 0$ is

$$N_5(0) = \mu_8 p_{01} p_{14} p_{48} (p_{20} + p_{24}) = \mu_8 L_8 \qquad [9.53]$$

Thus the fraction of time for which the system is under shut down is given by:

$$B_0^{3^*}(\infty) = \lim_{t\to\infty} B_0^{3^*}(t) = \lim_{s\to 0} sB_0^{3^*}(s) = \frac{N_5(0)}{D_2'(0)} = \frac{\mu_8 L_8}{\sum_{i=0,1,2,3,4,5,6,7,8,9} \mu_i L_i'} \qquad [9.54]$$

10. PARTICULAR CASES

When all repair time distributions are n-phase Erlangian distributions i.e.

Density function $g_i(t) = \frac{nr_i (nr_i t)^{n-1} e^{-nr_i t}}{n-1!}$ And Survival function $\overline{G}_j(t) = \sum_{j=0}^{n-1} \frac{(nr_i t)^j e^{-nr_i t}}{j!}$ [10.1-10.2]

And other distributions are negative exponential

$a(t) = \theta e^{-\theta t}, b(t) = \eta e^{-\eta t}, \overline{A}(t) = e^{-\theta t}, \overline{B}(t) = e^{-\eta t}$

For n=1 $g_i(t) = r_i e^{-r_i t}$, $\overline{G}_i(t) = e^{-r_i t}$ If i=1, 2, 3, 4

$g_1(t) = r_1 e^{-r_1 t}, g_2(t) = r_2 e^{-r_2 t}, g_3(t) = r_3 e^{-r_3 t}, g_4(t) = r_4 e^{-r_4 t}$

$\overline{G}_1(t) = e^{-r_1 t}$, $\overline{G}_2(t) = e^{-r_2 t}$, $\overline{G}_3(t) = e^{-r_3 t}$, $\overline{G}_4(t) = e^{-r_4 t}$ [10.3-10.14]

Also

$$p_{01} = \frac{\alpha}{x_1 + \theta}, p_{02} = \frac{\beta}{x_1 + \theta}, p_{03} = \frac{\gamma}{x_1 + \theta}, p_{09} = \frac{\theta}{x_1 + \theta}, \; p_{10} = \frac{r_1}{\beta + \gamma + r_1}, p_{14} = \frac{\beta}{\beta + \gamma + r_1}, p_{15} = \frac{\gamma}{\beta + \gamma + r_1},$$

$$p_{20} = \frac{r_2}{x_1 + r_2}, p_{24} = \frac{\alpha}{x_1 + r_2}, p_{26} = \frac{\beta}{x_1 + r_2}, p_{27} = \frac{\gamma}{x_1 + r_2}, \; p_{41} = \frac{r_2}{\beta + \gamma + r_2}, p_{46} = \frac{\gamma}{\beta + \gamma + r_2},$$

$$p_{30} = p_{51} = p_{62} = p_{72} = p_{80} = p_{90} = 1$$

$$\mu_0 = \frac{1}{x_1 + \theta}, \mu_1 = \frac{1}{\beta + \gamma + r_1}, \mu_2 = \frac{1}{x_1 + r_2}, \mu_4 = \frac{1}{\beta + \gamma + r_2},$$

$$\mu_3 = \frac{1}{r_3}, \mu_5 = \frac{1}{r_3}, \mu_6 = \frac{1}{r_2}, \mu_7 = \frac{1}{r_3}, \mu_8 = \frac{1}{r_4}, \mu_9 = \frac{1}{\eta}$$

where $x_1 = \alpha + \beta + \gamma$ [10.15-10.38]

MTSF= $\frac{\mu_0(1 - p_{14}p_{14}) + \mu_1(p_{01} + p_{02}p_{24}p_{41}) + \mu_2 p_{02}(1 - p_{14}p_{14}) + \mu_4(p_{02}p_{24} + p_{01}p_{14})}{1 - p_{01}p_{10} - p_{02}p_{20} - p_{02}p_{01}p_{24}p_{41} - p_{02}p_{20}p_{14}p_{41}}$,

$$A_0(\infty) = \frac{\mu_0 L_0 + \mu_1 L_1 + \mu_2 L_2 + \mu_4 L_4}{\sum_{i=0,1,2,3,4,5,6,7,8,9} \mu_i L_i'},$$

$$B_0^{1^*}(\infty) = \frac{\sum \mu_i L_i}{\sum_{i=0,1,2,3,4,5,6,7,8,9} \mu_i L_i'}, B_0^{2^*}(\infty) = \frac{\mu_9 L_9}{\sum_{i=0,1,2,3,4,5,6,7.8.9} \mu_i L_i'}, B_0^{3^*}(\infty) = \frac{\mu_8 L_8}{\sum_{i=0,1,2,3,4,5,6,7.8.9} \mu_i L_i'} \quad [10.39\text{-}10.43]$$

11. PROFIT ANALYSIS

The profit analysis of the system can be carried out by considering the expected busy period of the repairman in repair of the unit in (0,t].

Therefore, G(t) = Expected total revenue earned by the system in (0,t] -Expected repair cost of the failed units -Expected repair cost of the repairman in preventive maintenance -Expected repair cost of the Repairman in shut down

$$= C_1 \mu_{up}(t) - C_2 \mu_{b1}(t) - C_3 \mu_{b2}(t) - C_4 \mu_{b3}(t)$$

$$= C_1 A_0 - C_2 B^1{}_0 - C_3 B^2{}_0 - C_4 B^3{}_0 \quad [11.1]$$

Where $\mu_{up}(t) = \int_0^t A_0(t)\,dt$; $\mu_{b1}(t) = \int_0^t B_0^1(t)\,dt$; $\mu_{b2}(t) = \int_0^t B_0^2(t)\,dt$; $\mu_{b3}(t) = \int_0^t B_0{}^3(t)dt$ [11.2-11.5]

C_1 is the revenue per unit time and C_2, C_3, C_4 are the cost per unit time for which the system is under simple repair, preventive maintenance and shut down repair respectively.

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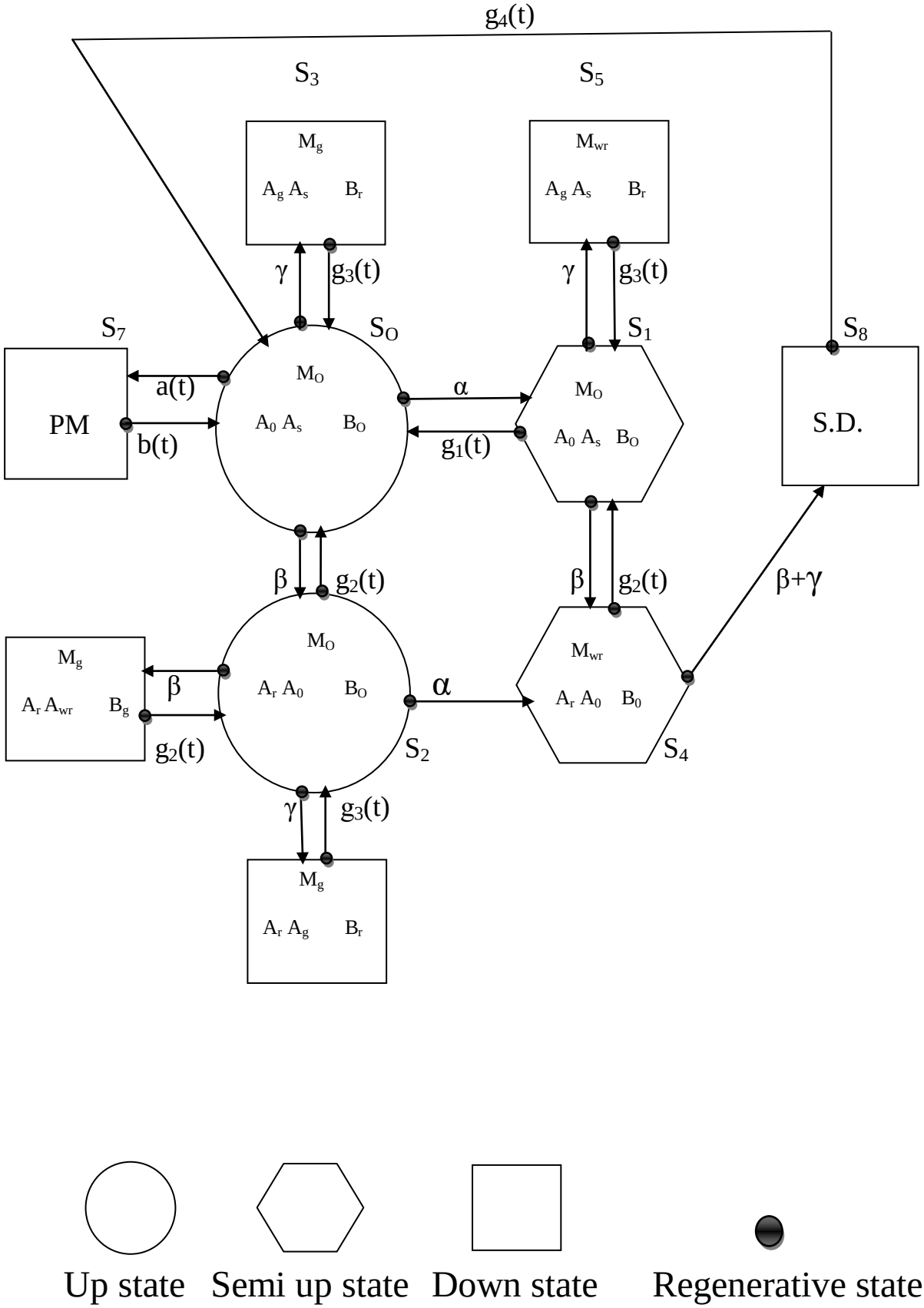
Figure 1: state transition diagram



Up state Semi up state Down state Regenerative state