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Application of Linear Programming Problem in Manuufacturing of Block

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Abstract: This research application of linear programming to block industry (A case study of Motland Block Industry Osogbo was aimed to maximixe the profit of block industry and to determine the inches of block they can specialize on data was obtained on the cost and profit of 4 inches,6inches and 9 inches. Data obtained were formulated as linear Programming Problem, From the analysis, it is observed that in order to make a profit of #588, Motland block industry should produce 11 units of product B (6 inches block) and 23 units of product C (9 inches block). The sensitivity analysis of the optimal table changes in relative to profit co-efficient of a basic and non basic variable shows that , at the optimality stage in as much as product A (4 inches block) is less than #24.33k, It is not economically profitable. More so, if product A's profit is increased to #29, the new optimal product is to produce 7 units of product A (4 inches block) in order to make a total profit of #612.42k .It is also clear that when the profit per unit price of all the three products is changed from $24x_1+20x_2+16x_3$ to $21x_1+26x_2+19x_3$, the total profit is increased from #612.42k to #713.5k As a result, Motland block industry is advised to maintain the current production mix. Any attempt to increase the resources will undermine its profit.

Keywords: Application, industry, maximization, profit and cost.

I. INTRODUCTION

Linear programming is the process of taking various linear inequality relating to some situations and finding the "best" value obtainable under those conditions[2]. Many economic activities and business are concerned with the problem of planning. If the supply of resources is unlimited, the need for linear programming would not arise. In this case, there is limited availability of resources and to make use of these resources, programming and planning problems could be formulated with sole aim of maximizing profit or minimizing the loss. The term programming refers to the process of determining a plan of action.

In real life, linear programming is a part of a very important area of mathematics called optimization techniques. The fields of study are used everyday in the organization and allocation of resources. This real life system can have dozens or hundreds of variables or more[3]. Mathematical formulation of linear programming problem can be done by graphical solution or simplex method artificial variable technique. The former is employed when dealing with a linear programming problem involving two decision variables, easy to understand and enhances automatic elimination of the redundant constraints from the system[1]. The latter is applicable when it is not possible to obtain the graphical solution to the linear programming of more than two variables.

Linear programming can be viewed as part of a great revolutionary development which has given mankind the ability to state general goals and lay out a part of a detailed decision to take in order to "best" achieve it goals when faced with practical situation of great complexity. Our tools for doing this are ways to formulate real-world problems in detailed mathematical terms (models), techniques for solving the models (algorithms) and engines for executing the step of algorithms[4][5].

II. METHODOLOGY

A. General Form Of Linear Programming Problem:

The maximization problem is of the form

$$\text{Max } z = c_1x_1 + c_2x_2 + c_3x_3 \ldots\ldots\ldots c_nx_n \qquad (1)$$

$$\begin{aligned} \text{Subject to } \; & a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots + a_{1n}x_n \leq b_1 \\ & a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots + a_{2n}x_n \leq b_2 \qquad (ii) \\ & a_{31}x_1 + a_{32}x_2 + \ldots\ldots\ldots + a_{3n}x_n \leq b_3 \\ & a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots + a_{mn}x_n \leq b_m \\ & x_1, \; x_2, \; \ldots\ldots x_n \geq 0 \ldots\ldots\ldots \end{aligned}$$

(iii)

Equation (1) is the objective function (ii) is the constraint (iii) is the non negative conditions.

In matrix form

$$\text{Max } z = C^T X$$

$$\text{s.t } Ax \leq b$$

$$X \geq 0$$

Where C= X=

$$\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ . \\ . \\ . \\ C_n \end{pmatrix} \qquad \begin{pmatrix} x_1, x_2, x_3, \ldots\ldots\ldots x_n \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \ldots\ldots\ldots a_{1n} \\ a_{21} & a_{22} & a_{23} \ldots\ldots\ldots a_{2n} \\ a_{31} & a_{32} & a_{33} \ldots\ldots\ldots a_{3n} \\ .. & .. & .. \ldots\ldots\ldots \\ .. & .. & .. \ldots\ldots\ldots \\ a_{n1} & a_{n2} & a_{n3} \ldots\ldots\ldots a_{nm} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ . \\ x_n \end{pmatrix} \qquad \begin{pmatrix} b_1 \\ b_2 \\ . \\ . \\ . \\ b_n \end{pmatrix}$$

III. LAYOUT OF LINEAR PROGRAMMING TECHNIQUE

Table:1

BV	CB	X_1	X_2	X_n	X_{n+1}	X_{n+2}	X_{m+n}	XB
X_{n+1}	0	a_{11}	a_{12}	a_{1n}	1	0	0	b_1
X_{n+2}	0	a_{21}	a_{22}	a_{22}	0	1	0	b_2
X_{m+n}	0	a_{31}	a_{32}	a_{32}	0	0	1	b_m
	c_j	c_1	c_2	c_n	0	o	0	
	z_j	z_1	z_2					
	$(z_j\text{-}c_j)\Delta_j$	Δ_1	Δ_2	Δ_n	Δ_{n+1}	Δ_{n+2}	Δ_{n+3}	

The solution is optimal when $(z_j - c_j) \geq 0$

IV. DATA PRESENTATION AND ANALYSIS

Presentation of Data

Data are collected on the 3 different sizes of blocks. Namely, 4 inches (4") block size (ii) 6 inches (6") block size (iii) 9 inches (9") block size. The table below gives data on the unit profit on each block size, the requirement for each block size and total available resources.

Table 1: The unit profit on each block

product	Profit per unit
4 inches	#24
6 inches	#20
9 inches	#16

Table 2: The requirement for each block size

Block	Bags of cement	Labour hour	Machine hour
4 inches	7	1	2
6 inches	5	$1^1/_2$	2
9 inches	2	3	3

Table 3: Total available resources

Resources	Total availability
Bags of cement	100
Labour hours	84
Machine hours	168

V. FORMULATION OF LINEAR PROGRAMMING PROBLEM

Let the number of unit of 4 inches block size be x_1
Let the number of unit of 6 inches block size be x_2
Let the number of unit of 9 inches block size be x_3
Objective function Max $Z = 24x_1 + 20x_2 + 16x_3$
Constraints:
$7x_1+5x_2 + 2x_3 \leq 100$
$x_1 +1½x_2+3x_3 \leq 84$
$2x_1+ 2x_2 + 3x_3 \leq 168$

Consequently, the linear programming problem becomes

Max $z = 24x_1 + 20x_2 + 16x_3$ → Objective function

S.t

$7x_1 + 5x_2 + 2x_3 \leq 100$

$1x_1 + 1½x_2 + 3x_3 \leq 84$ Constraints

$2x_1 + 2x_2 + 3x_3 \leq 168$

$x_1, x_2, x_3 > 0$

VI. SOLUTION TO LINEAR PROGRAMMING PROBLEM

Max $z = 24x_1 + 20x_2 + 16x3$
S.t $7x_1 + 5x_2 + 2x_3 \leq 100$
$1x_1 + {}^3/_2x_2 + 3x_3 \leq 84$
$2x_1 + 2x_2 + 3x_3 \leq 168$
$x1, x2, x3 > 0$

Multiply (iii) through by 2
$2x_1 + 3x_2 + 6x_3 \leq 168$
In the standard form

Max $z = 24x_1 + 20x_2 + 16x_3 + 0x_4 + 0x_5 + 0x_6$
S.t $7x_1 + 5x_2 + 2x_3 + x_4 + 0x_5 + 0x_6 = 100$
$2x_1 + 3x_2 + 6x_3 + 0x_4 + x_5 + 0x_6 = 168$
$2x_1 + 2x_2 + 3x_3 + 0x_4 + 0x_5 + x_6 = 168$
$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$

Where x_1, x_2, x_3, are initial non basic variable 4" block size, 6" block size and 9" block size respectively.

Table 4; The initial simple table with the addition of slack variable

BV	CB	X_1	X_2	X_3	X_4	X_5	X_6	XB	M.R
X_4	0	7	5	2	1	0	0	100	100/7
X_3	0	2	3	6	0	1	0	168	84
X_6	0	2	2	3	0	0	1	168	84
	c_j	24	20	16	0	0	0		
	z_j	0	0	0	0	0	0		
	$(z_j\text{-}c_J)$	-24	-20	-16	0	0	0		

Table 5; The Optimal solution

BV	CB	X_1	X_2	X_3	X_4	X_5	X_6	XB
X_2	20	$^{19}/12$	1	0	$^{1}/4$	$^{-1}/12$	0	11
X_3	16	$^{-11}/24$	0	1	$^{-1}/8$	$^{5}/24$	0	$^{45}/2$
X_6	0	$^{5}/24$	0	0	$^{-1}/8$	$^{-11}/24$	1	$^{2873}/38$
	cj	24	20	16	0	0	0	
	zj	$^{73}/3$	20	16	3	$^{5}/3$	0	
	zj –cj	$^{1}/3$	0	0	3	$^{5}/3$	0	Z_{max}= ₦580

Since (zj–cj) ≥ 0, i.e all are positive. The solution is optimal.

Hence, the optimal tableau shows that the optimal product mix is to produce 11 units of product B(6 inc) and 23units of product C(9 inch) for a total profit of ₦580.

Hence, by performing a sensitively analysis, it is possible to obtain adequate information regarding alternative production schedules in the neighborhood of the optimal solution. One of the reasons for the expensive use of linear programming is its ability to provide sensitivity analysis along with the optimal solution.

The sensitivity of the current optimal solution can be obtained by studying how the optimal table changes in the relative profit co-efficient of non basic variable X_1.

The present optimal,

$CB = (x_2\ x_3\ x_6)$

$CB = (x_2\ x_3\ x_6) = (20\ 16\ 0)$

$Z_1 – C_1 =$

$= {}^{73}/3 - C_1 \geq 0$

$= {}^{73}/3 \geq C_1$

$= C_1 \leq {}^{73}/3$

This implies that as long as the unit profit of product A (4” block) is less that #24.33k, it is not economical to produce product A.

Suppose the unit profit on product A (4” block) is increased to #29, then $Z_1 – C_1 = {}^{-14}/3$ the optimal tableau become non optimal and the new tables are given below.

Table 6; The unit profit on product A (4” block) is increased to #29

BV	CB	X_1	X_2	X_3	X_4	X_5	X_6	XB	M.R
X_2	20	$^{19}/12$	1	0	$^{1}/4$	$^{-1}/12$	0	11	$^{132}/19$
X_3	16	$^{-11}/2$	0	1	$^{-1}/8$	$^{5}/24$	0	$^{45}/2$	$^{-540}/11$
X_6	0	$^{5}/24$	0	0	$^{-1}/8$	$^{-11}/24$	1	$^{2873}/38$	$^{34476}/95$
	c_j	29	20	16	0	0	0		
	z_j	$^{73}/3$	20	16	3	$^{5}/3$	0		
	zj –cj	$^{-14}/3$	0	0	3	$^{5}/3$	0		

Table 7; The optimal table when the unit profit on product A (4” block) is increased to #29

BV	CB	X_1	X_2	X_3	X_4	X_5	X_6	XB
X_2	29	1	$^{12}/19$	0	$^{3}/19$	$^{-1}/19$	0	$^{132}/19$
X_3	16	0	$^{11}/24$	1	$^{-1}/19$	$^{7}/38$	0	$^{488}/19$
X_6	0	0	$^{-5}/38$	0	$^{-3}/19$	$^{-17}/38$	1	$^{1409}/19$
	cj	29	20	16	0	0	0	
	zj	29	$^{1462}/57$	16	$^{71}/19$	$^{27}/19$	0	
	zj - cj	0	$^{322}/57$	0	$^{71}/19$	$^{27}/19$	0	Z_{max}=#612.42k

Suppose we want to determine the effect of changes on the unit profit of product B (C_2). It is observed that when C_2 decrease below a certain level, it may not be profitable to include product A in the optimal product mix. Hence if C_2 is increased it is possible that it may change the optimal product mix at some levels Such that product A becomes so profitable that the product mix may include only product B. Meanwhile, there is an upper and lower limit on the variation of C_2 within which the optimal solution is not affected.

To determine the range on C_2, there is an observation that a change in C_2 leads to changes in the profit vector of the basic variable CB as long as (Zj – Cj) ≥ 0, the optimal table is still optimal.

We can therefore express the values of (Zj – Cj), (Z_4 – C_4) and (Z_5 – C_5) as function of C2

From the calculation

$(Z_1 – C_1) \geq 0$ as long as $C2 \geq {}^{200}/19$

$(Z_4 – C_4) \geq 0$ as long as $C2 \geq 8$

$(Z_5 – C_5) \geq 0$ as long as $C2 \geq 40$

The optimal table (solution) remains optimal as long as C_2 is greater or equal to $^{200}/19$ (11) and any value below (11) will render the solution non – optimal.

When the value of C_2 goes beyond the range provided by sensitivity analysis, that table will no longer be optimal, as one of the non basic +ve (Zj – Cj) will become negative. Consequently a new simplex table must be drawn to determine the new optimal solution.

VII. CONCLUSION

From the analysis, it is observed that in order to make a profit of #588, Motland block industry should produce 11 units of product B (6 inches block) and 23 units of product C (9 inches block).

The sensitivity analysis of the optimal table changes in relative to profit co-efficient of a basic and non basic variable shows that , at the optimality stage in as much as product A (4 inches block) is less than #24.33k, It is not economically profitable. More so, if product A’s profit is increased to #29, the new optimal product is to produce 7 units of product A (4 inches block) in order to make a total profit of #612.42k

Further more, it is clear that the optimal table remains optimal as long as C_2 is greater or equal to 200/19 i.e 11 units and any value below 11 units will render the solution non optimal.

It is also clear that when the profit per unit price of all the three products is changed from $24x_1+20x_2+16x_3$ to $21x_1+26x_2+19x_3$, the total profit is increased from #612.42k to #713.5k

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