



Global Hyperchaos Synchronization of Hyperchaotic Chen and Lorenz Systems by Active Nonlinear Control

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Abstract: This paper investigates the global exponential synchronization of hyperchaotic systems, viz. identical hyperchaotic Chen systems (Yan and Yu, 2008), identical hyperchaotic Lorenz systems (Wang, Zheng and Liu, 2007), and synchronization of hyperchaotic Chen and Lorenz systems. Active nonlinear feedback control is the method used to achieve the synchronization of the hyperchaotic systems addressed in this paper. Our theorems on global exponential synchronization for hyperchaotic Chen and Lorenz systems are established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the nonlinear feedback control method is effective and convenient to synchronize identical and different hyperchaotic Chen and Lorenz systems. Numerical simulations are also given to illustrate and validate the synchronization results for hyperchaotic Chen and Lorenz systems.

Keywords: Chaos Synchronization, Nonlinear Control, Hyperchaotic Chen System, Hyperchaotic Lorenz System, Active Control.

I. INTRODUCTION

Chaotic systems are dynamical systems that are highly sensitive to initial conditions. This sensitivity is popularly referred to as the *butterfly effect* [1].

Chaos synchronization problem was first described by Fujisaka and Yemada [2] in 1983. This problem did not receive great attention until Pecora and Carroll [3-4] published their results on chaos synchronization in early 1990s. From then on, chaos synchronization has been extensively and intensively studied in the last three decades [3-22]. Chaos theory has been explored in a variety of fields including physical [5], chemical [6], ecological [7] systems, secure communications [8-10] etc.

Synchronization of chaotic systems is a phenomenon that may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem.

In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

Since the seminal work by Carroll and Pecora [3-4], a variety of impressive approaches have been proposed for the synchronization for the chaotic systems such as PC method [3-4], the sampled-data feedback synchronization method [10-11], OGY method [12], time-delay feedback approach [13], backstepping design method [14], adaptive design method [15-19], sliding mode control method [20], Lyapunov stability theory method [21], hyperchaos [22], etc.

Hyperchaotic system is usually defined as a chaotic system with at least two positive Lyapunov exponents, implying that its dynamics are expanded in several different directions simultaneously. For a continuous dynamical system to exhibit hyperchaotic behaviour, the system must be at least four-dimensional. Hyperchaotic systems have more complex dynamical behaviour, which can be used to improve the security of a chaotic communication system.

This paper has been organized as follows. In Section II, we give the problem statement and our methodology. In Section III, we discuss the chaos synchronization of two identical hyperchaotic Chen systems ([23], 2008). In Section IV, we discuss the chaos synchronization of two identical hyperchaotic Lorenz systems ([24], 2007). In Section V, we discuss the heterogeneous synchronization of hyperchaotic Chen and Lorenz systems. In Section VI, we present the conclusions of this paper.

II. PROBLEM STATEMENT AND OUR METHODOLOGY

Consider the hyperchaotic system described by

$$\dot{x} = Ax + f(x) \quad (1)$$

where $x \in \mathbf{R}^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f: \mathbf{R}^n \rightarrow \mathbf{R}^n$ is the nonlinear part of the system. We consider the system (1) as the *master* or *drive* system.

As the *slave* or *response* system, we consider the following hyperchaotic system described by

$$\dot{y} = By + g(y) + u \quad (2)$$

where $y \in \mathbf{R}^n$ is the state vector of the response system, B is the $n \times n$ matrix of the system parameters, $g: \mathbf{R}^n \rightarrow \mathbf{R}^n$ is the nonlinear part of the response system and $u \in \mathbf{R}^n$ is the controller of the response system.

If $A = B$ and $f = g$, then x and y are the states of two *identical* hyperchaotic systems. If $A \neq B$ and $f \neq g$, then x and y are the states of two *different* hyperchaotic systems.

In the nonlinear feedback control approach, we design a feedback controller u , which synchronizes the states of the master system (1) and the slave system (2) for all initial conditions $x(0), z(0) \in \mathbf{R}^n$.

If we define the *synchronization error* as

$$e = y - x, \quad (3)$$

then the synchronization error dynamics is obtained as

$$\dot{e} = By - Ax + g(y) - f(x) + u \quad (4)$$

Thus, the global synchronization problem is essentially to find a feedback controller u so as to stabilize the error dynamics (4) for all initial conditions $e(0) \in \mathbf{R}^n$, i.e.

$$\lim_{t \rightarrow \infty} \|e(t)\| = 0 \quad (5)$$

for all initial conditions $e(0) \in \mathbf{R}^n$.

We use Lyapunov stability theory as our methodology. We take as a candidate Lyapunov function

$$V(e) = e^T P e, \quad (6)$$

where P is a positive definite matrix. Note that $V: \mathbf{R}^n \rightarrow \mathbf{R}^n$ is a positive definite function by construction. We assume that the parameters of the master and slave systems are known and that the states of both systems (1) and (2) are measurable.

If we find a feedback controller u so that

$$\dot{V}(e) = -e^T Q e, \quad (7)$$

where Q is a positive definite matrix, then $\dot{V}: \mathbf{R}^n \rightarrow \mathbf{R}^n$ is a negative definite function.

Thus, by Lyapunov stability theory [26], the error dynamics (4) is globally exponentially stable and hence the condition (5) will be satisfied for all initial conditions $e(0) \in \mathbf{R}^n$. Then the states of the master system (1) and slave system (2) are globally exponentially synchronized.

III. SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC CHEN SYSTEMS

In this section, we apply the nonlinear control technique for the synchronization of two identical hyperchaotic Chen systems ([23], 2008).

Thus, the master system is described by the hyperchaotic Chen dynamics as

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= (c - a)x_1 - x_1x_3 + cx_2 \\ \dot{x}_3 &= -bx_3 + x_1x_2 - x_2x_3 + x_1x_3 - x_4 \\ \dot{x}_4 &= -dx_4 + x_2x_3 - x_1x_3 \end{aligned} \quad (8)$$

where $x_i (i = 1, 2, 3, 4)$ are the states and a, b, c, d are positive parameters.

The slave system is also described by the hyperchaotic Chen dynamics as

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= (c - a)y_1 - y_1y_3 + cy_2 + u_2 \\ \dot{y}_3 &= -by_3 + y_1y_2 - y_2y_3 + y_1y_3 - y_4 + u_3 \\ \dot{y}_4 &= -dy_4 + y_2y_3 - y_1y_3 + u_4 \end{aligned} \quad (9)$$

where $y_i (i = 1, 2, 3, 4)$ are the states and

$$u = [u_1 \quad u_2 \quad u_3 \quad u_4]^T$$

is the nonlinear controller to be designed.

The hyperchaotic Chen system (8) is a new 4-D hyperchaotic system derived from the Chen system by Yan and Yu ([23], 2008).

The four-dimensional system (8) is hyperchaotic when

$$a = 37, \quad b = 3, \quad c = 26 \quad \text{and} \quad d = 38.$$

Figure 1 depicts the portrait of the hyperchaotic Chen system (8).

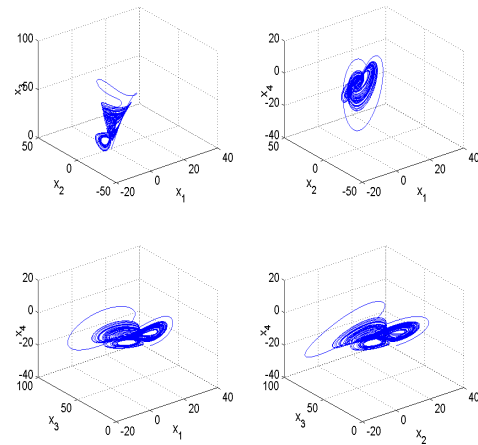


Figure 1. Portrait of the Hyperchaotic Chen System (8)

The synchronization error e is defined by

$$e_i = y_i - x_i, \quad (i = 1, 2, 3, 4) \quad (10)$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + u_1 \\ \dot{e}_2 &= (c - a)e_1 + ce_2 - (y_1y_3 - x_1x_3) + u_2 \\ \dot{e}_3 &= -be_3 - e_4 + (y_1y_2 - x_1x_2) \\ &\quad - (y_2y_3 - x_2x_3) + (y_1y_3 - x_1x_3) + u_3 \\ \dot{e}_4 &= -de_4 + (y_2y_3 - x_2x_3) - (y_1y_3 - x_1x_3) + u_4 \end{aligned} \quad (11)$$

We choose the nonlinear controller as

$$\begin{aligned} u_1 &= 0 \\ u_2 &= -ce_1 - (c + 1)e_2 + y_1y_3 - x_1x_3 \\ u_3 &= e_4 - (y_1y_2 - x_1x_2) + (y_2y_3 - x_2x_3) \\ &\quad - (y_1y_3 - x_1x_3) \\ u_4 &= -(y_2y_3 - x_2x_3) + (y_1y_3 - x_1x_3) \end{aligned} \quad (12)$$

Substituting (12) into (11), we obtain the error dynamics as

$$\begin{aligned}\dot{e}_1 &= a(e_2 - e_1) \\ \dot{e}_2 &= -ae_1 - e_2 \\ \dot{e}_3 &= -be_3 \\ \dot{e}_4 &= -de_4\end{aligned}\quad (14)$$

Next, we consider the candidate Lyapunov function

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2} (e_1^2 + e_2^2 + e_3^2 + e_4^2) \quad (15)$$

which is a Lyapunov function on \mathbf{R}^4 .

Differentiating V along the trajectories of (14), we get

$$\dot{V}(e) = -ae_1^2 - e_2^2 - be_3^2 - de_4^2 \quad (16)$$

which is a negative definite function on \mathbf{R}^4 .

Hence, by Lyapunov stability theory [25], the error dynamics (14) is globally exponentially stable.

Thus, we have proved the following result.

Theorem 1. The identical hyperchaotic Chen systems (8) and (9) are exponentially and globally synchronized for any initial conditions with the nonlinear controller u defined by (12). ■

Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems using MATLAB with time-step equal to 10^{-6} .

For the hyperchaotic Chen system (8), the parameter values are taken as those which result in the hyperchaotic behaviour of the system, viz. $a = 37$, $b = 3$, $c = 26$ and $d = 38$.

The initial values of the master system (8) are taken as

$$x_1(0) = 8, x_2(0) = 12, x_3(0) = 4, x_4(0) = 7$$

while the initial values of the slave system (9) are taken as

$$y_1(0) = 12, y_2(0) = 3, y_3(0) = 8, y_4(0) = 2.$$

Figure 2 shows that synchronization between the states of the master system (8) and the slave system (9) occur in 4 seconds.

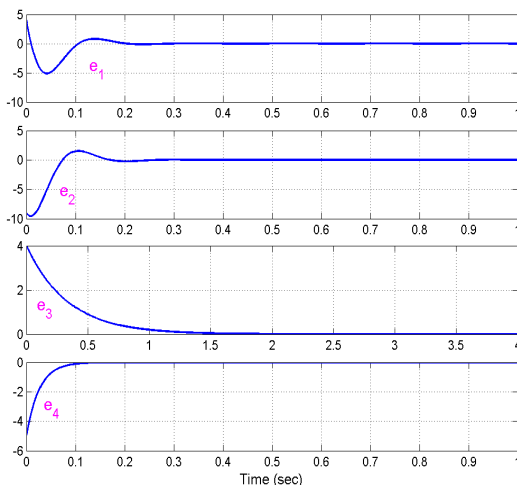


Figure 2. Synchronization of the States of (8) and (9)

IV. SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC LORENZ SYSTEMS

In this section, we apply the nonlinear control technique for the synchronization of two identical hyperchaotic Lorenz systems ([24], 2007).

Thus, the master system is described by the hyperchaotic Lorenz dynamics as

$$\begin{aligned}\dot{x}_1 &= l(x_2 - x_1) \\ \dot{x}_2 &= mx_1 - x_2 - x_1x_3 + x_4 \\ \dot{x}_3 &= x_1x_2 - nx_3 \\ \dot{x}_4 &= -px_1\end{aligned}\quad (17)$$

where $x_i (i = 1, 2, 3, 4)$ are the states and l, m, n, p are positive parameters.

The slave system is also described by the hyperchaotic Lorenz dynamics as

$$\begin{aligned}\dot{y}_1 &= l(y_2 - y_1) + u_1 \\ \dot{y}_2 &= my_1 - y_2 - y_1y_3 + y_4 + u_2 \\ \dot{y}_3 &= y_1y_2 - ny_3 + u_3 \\ \dot{y}_4 &= -py_1 + u_4\end{aligned}\quad (18)$$

where $y_i (i = 1, 2, 3, 4)$ are the states and

$$u = [u_1 \ u_2 \ u_3 \ u_4]^T$$

is the nonlinear controller to be designed.

The hyperchaotic Lorenz system (17) is a new 4-D hyperchaotic system derived from the Lorenz system by Wang, Zheng and Liu ([24], 2007).

The four-dimensional system (17) is hyperchaotic when

$$l = 10, m = 28, n = 28/3 \text{ and } p = 5.$$

Figure 3 depicts the portrait of the hyperchaotic Lorenz system (17).

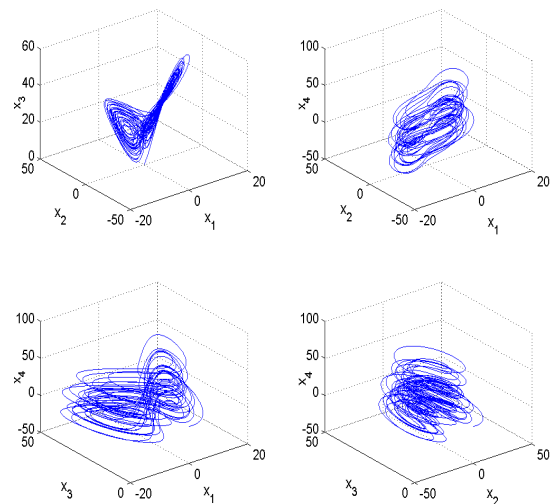


Figure 3. Portrait of the Hyperchaotic Lorenz System (17)

The synchronization error e is defined by

$$e_i = y_i - x_i, \quad (i = 1, 2, 3, 4) \quad (19)$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= l(e_2 - e_1) + u_1 \\ \dot{e}_2 &= me_1 - e_2 + e_4 - (y_1 y_3 - x_1 x_3) + u_2 \\ \dot{e}_3 &= -ne_3 + y_1 y_2 - x_1 x_2 + u_3 \\ \dot{e}_4 &= -pe_1 + u_4 \end{aligned} \quad (20)$$

We choose the nonlinear controller as

$$\begin{aligned} u_1 &= 0 \\ u_2 &= -(l+m)e_1 - e_4 + y_1 y_3 - x_1 x_3 \\ u_3 &= -(y_1 y_2 - x_1 x_2) \\ u_4 &= pe_1 - e_4 \end{aligned} \quad (21)$$

Substituting (21) into (20), we obtain the error dynamics as

$$\begin{aligned} \dot{e}_1 &= l(e_2 - e_1) \\ \dot{e}_2 &= -le_1 - e_2 \\ \dot{e}_3 &= -ne_3 \\ \dot{e}_4 &= -e_4 \end{aligned} \quad (22)$$

Next, we consider the candidate Lyapunov function

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2} (e_1^2 + e_2^2 + e_3^2 + e_4^2) \quad (23)$$

which is a Lyapunov function on \mathbf{R}^4 .

Differentiating V along the trajectories of (22), we get

$$\dot{V}(e) = -le_1^2 - e_2^2 - ne_3^2 - e_4^2 \quad (24)$$

which is a negative definite function on \mathbf{R}^4 .

Hence, by Lyapunov stability theory [25], the error dynamics (22) is globally exponentially stable.

Thus, we have proved the following result.

Theorem 2. The identical hyperchaotic Lorenz systems (17) and (18) are exponentially and globally synchronized for any initial conditions with the nonlinear controller u defined by (21). ■

Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems using MATLAB with time-step equal to 10^{-6} .

For the hyperchaotic Lorenz system (17), the parameter values are taken as those which result in the hyperchaotic behaviour of the system, viz. $l = 10$, $m = 28$, $n = 28/3$ and $p = 5$.

The initial values of the master system (17) are taken as

$$x_1(0) = 5, x_2(0) = 2, x_3(0) = 14, x_4(0) = 9$$

while the initial values of the slave system (18) are taken as

$$y_1(0) = 10, y_2(0) = 12, y_3(0) = 6, y_4(0) = 1.$$

Figure 4 shows that synchronization between the states of the master system (17) and the slave system (18) occur in 4 seconds.

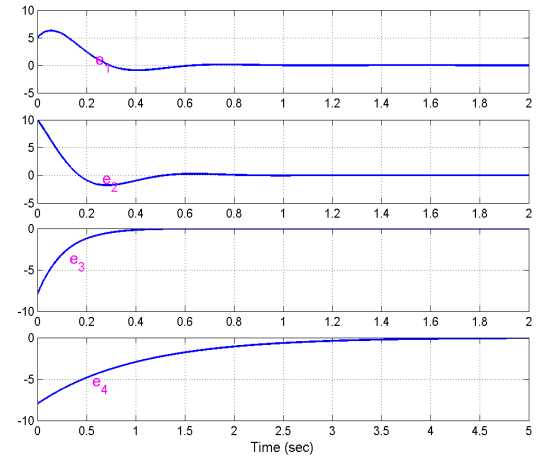


Figure 4. Synchronization of the States of (17) and (18)

V. SYNCHRONIZATION OF HYPERCHAOTIC CHEN AND LORENZ SYSTEMS

In this section, we apply the nonlinear control technique for the synchronization of non-identical hyperchaotic Chen and Lorenz systems. As the master system, we consider the hyperchaotic Chen system ([23], 2008) described by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= (c - a)x_1 - x_1 x_3 + cx_2 \\ \dot{x}_3 &= -bx_3 + x_1 x_2 - x_2 x_3 + x_1 x_3 - x_4 \\ \dot{x}_4 &= -dx_4 + x_2 x_3 - x_1 x_3 \end{aligned} \quad (25)$$

As the slave system, we consider the hyperchaotic Lorenz system ([24], 2007) described by

$$\begin{aligned} \dot{y}_1 &= l(y_2 - y_1) + u_1 \\ \dot{y}_2 &= my_1 - y_2 - y_1 y_3 + y_4 + u_2 \\ \dot{y}_3 &= y_1 y_2 - ny_3 + u_3 \\ \dot{y}_4 &= -py_1 + u_4 \end{aligned} \quad (26)$$

where all the parameters are positive real constants and

$$u = [u_1 \ u_2 \ u_3 \ u_4]^T$$

is the nonlinear controller to be designed.

The synchronization error e is defined by

$$e_i = y_i - x_i, \quad (i = 1, 2, 3, 4) \quad (27)$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= l(e_2 - e_1) + (l - a)(x_2 - x_1) + u_1 \\ \dot{e}_2 &= me_1 - e_2 + e_4 + (m - c + a)x_1 - (c + 1)x_2 \\ &\quad + x_4 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -ne_3 + (b - n)x_3 + x_4 + y_1 y_2 - x_1 x_2 \\ &\quad + x_2 x_3 - x_1 x_3 + u_3 \\ \dot{e}_4 &= -pe_1 - px_1 + dx_4 - x_2 x_3 + x_1 x_3 + u_4 \end{aligned} \quad (28)$$

We choose the nonlinear controller as

$$\begin{aligned} u_1 &= (a-l)(x_2 - x_1) \\ u_2 &= -(l+m)e_1 - e_4 - (m-c+a)x_1 + (c+1)x_2 \\ &\quad - x_4 + y_1y_3 - x_1x_3 \end{aligned} \quad (29)$$

$$u_3 = (n-b)x_3 - x_4 - y_1y_2 + x_1x_2 - x_2x_3 + x_1x_3$$

$$u_4 = pe_1 - e_4 + px_1 - dx_4 + x_2x_3 - x_1x_3$$

Substituting (29) into (28), we obtain the error dynamics as

$$\begin{aligned} \dot{e}_1 &= l(e_2 - e_1) \\ \dot{e}_2 &= -le_1 - e_2 \\ \dot{e}_3 &= -ne_3 \\ \dot{e}_4 &= -e_4 \end{aligned} \quad (30)$$

Next, we consider the candidate Lyapunov function

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2} (e_1^2 + e_2^2 + e_3^2 + e_4^2) \quad (31)$$

which is a Lyapunov function on \mathbf{R}^4 .

Differentiating V along the trajectories of (31), we get

$$\dot{V}(e) = -le_1^2 - e_2^2 - ne_3^2 - e_4^2 \quad (32)$$

which is a negative definite function on \mathbf{R}^4 .

Hence, by Lyapunov stability theory [25], the error dynamics (30) is globally exponentially stable.

Thus, we have proved the following result.

Theorem 3. The non-identical hyperchaotic Chen system (25) and hyperchaotic Lorenz system (26) are exponentially and globally synchronized for any initial conditions with the nonlinear controller u defined by (29). ■

Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems using MATLAB with time-step equal to 10^{-6} .

For the hyperchaotic Chen system (25) and hyperchaotic Lorenz system (26), the parameter values are taken as those which result in the hyperchaotic behaviour of the systems, viz. $a=37$, $b=3$, $c=26$, $d=38$, $l=10$, $m=28$, $n=28/3$ and $p=5$.

The initial values of the master system (25) are taken as

$$x_1(0)=9, x_2(0)=6, x_3(0)=10, x_4(0)=4$$

while the initial values of the slave system (26) are taken as

$$y_1(0)=2, y_2(0)=15, y_3(0)=4, y_4(0)=10.$$

Figure 5 shows that synchronization between the states of the master system (25) and the slave system (26) occur in 5 seconds.

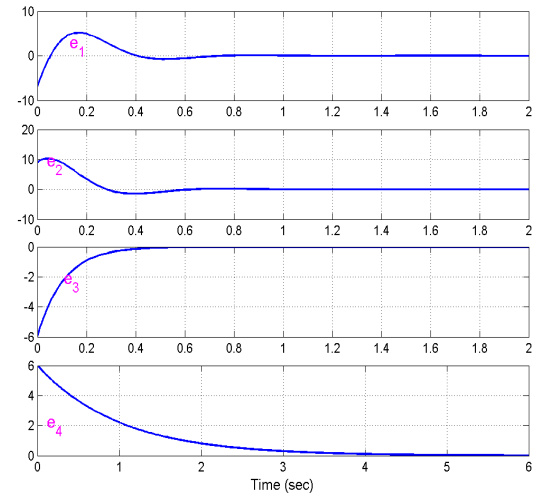


Figure 5. Synchronization of the States of (25) and (26)

VI. CONCLUSIONS

Since the hyperchaotic systems have more complex dynamical behaviours, they can be used to improve the security of a chaotic communication system. In this paper, we have used nonlinear control method based on Lyapunov stability theory to achieve global chaos synchronization for the following three cases of hyperchaotic systems:

- (A) Identical Hyperchaotic Chen systems (2008).
- (B) Identical Hyperchaotic Lorenz systems (2007).
- (C) Non-Identical Hyperchaotic Chen and Lorenz systems.

Simulation results are also given to illustrate and validate the proposed synchronization approach for the global chaos synchronization of the hyperchaotic systems. Since the Lyapunov exponents are not required for these calculations, the nonlinear control method is very effective and convenient to achieve global chaos synchronization for the three cases of hyperchaotic systems discussed in this paper.

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