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Expected Value Minimization Models of Project Scheduling Problem

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Abstract: Project scheduling problem is to determine the schedule of allocating resources so as to balance the total cost and the completion time. This paper mainly introduce project scheduling problem with uncertain variables. In order to meet different management requirements, we put forward the number of workers constraint based on the activity duration times and total cost constraints. And activity duration times and the number of workers are assumed to be uncertain variables in this paper. Expected value model is an important tool for the solving of project scheduling problem. We established single-objective programming model and multi-objective programming model to meet different needs, respectively.

Keywords: Project scheduling problem; Uncertain variable; Expected value model

I. INTRODUCTION

Uncertainty theory was founded by Liu [1] in 2007 and subsequently studied by many researchers. Nowadays uncertainty theory has become a branch of axiomatic mathematics for modeling human uncertainty. The uncertainty theory has become a new tool to describe subjective uncertainty and has a wide application both in theory and engineering. As an application of uncertainty theory, Liu [2] presented uncertain programming which is a type of mathematical programming involving uncertain variables, and applied uncertain programming to system reliability design, project scheduling problem, vehicle routing problem, facility location problem, and machine scheduling problem and so on Project scheduling problem is to determine the schedule of allocating resources so as to balance the. A typical project scheduling problem can be described as follows: there are many activities in a project.

There are tight-front relations among some projects because of the technical request. Activity can’t be processed before its all tight-front works are finished. The structure of entire project can be described by a directed acyclic network graph. The pitch point represents transformation from an activity to another activity in the graph, and the arc represents tight-front relations among activities. A feasible plan can be defined as follows: the schedule of each activity has been determined; also each activity satisfies tight-front relation and resource restraint.

In real world, uncertainty always exists in project scheduling problem and assumed as randomness in many works. But, much uncertainty may not be replaced by randomness. For instance, fuzzy set theory, which describes another uncertainty, was introduced by Zadeh [3]. Prade [4] first applied fuzzy set theory into the project scheduling problem in 1979. In 2004, Ke and Liu [5] built three fuzzy models and quickly applied fuzzy set theory into the project scheduling problem successfully. Furthermore, in 2007, Ke and Liu [6] presented random fuzzy models, for example, the duration time of each activity is stochastic and stochastic parameters are fuzzy variables.

Expected value operator for uncertain variables has become an important role in both theory and practice. Golenko-Ginzburg and Gonik [7] established an expected value model in solving a simple type of project scheduling problem, where expected completion time is to be minimized under some deterministic resource constraint. Yuhan Liu [8] shown that the expected value of monotone function of uncertain variable is just the Lebesgue-Stieltjes integral of the function with respect to its uncertainty distribution.

In this paper, we chiefly introduce Expected value Minimization models with uncertain variables to solve project scheduling problem. Based on the expected value model, we established single-objective programming model and multi-objective programming model in order to meet different management requirements, respectively.

II. THEORETICAL PREPARATION

Let Γ be a nonempty set, and L a σ -algebra over Γ . Each element $\Lambda \in$ L is called an event. In order to present an axiomatic definition of uncertain measure, it is necessary to assign to each event Λ a number M { Λ } which indicates the level that Λ will occur. In order to ensure that the number M { Λ } have certain mathematical properties, Liu [1] presented the following four axioms:

(a). (Normality) M { Γ } =1.

(b). (Monotonicity) M { Λ_1 } $\le$ M { Λ_2 } wherever $\Lambda_1 \subset \Lambda_2$.

(c). (Self-Duality) M { Λ } + M { Λ^c } =1for any event Λ .

(d). (Countable Subadditivity) for every countable sequence of events { Λ_i }, we have

$$\mathrm{M}\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \le \sum_{i=1}^{\infty} \mathrm{M}\{\Lambda_i\}$$

A. *Uncertain variable:*

Roughly speaking, an uncertain variable is a real valued function on an uncertainty space. Before we show the concept of uncertain variable, we first give the definition of an uncertainty space.

Definition 1(Liu [1]) Let ξ be a nonempty set, L a σ -algebra over Γ , and M an uncertain measure. Then the triplet (Γ , L, M) is called an uncertainty space.

Then the concept of uncertain variable was proposed by Liu [1]:

Definition 2 (Liu [1]) an uncertain variable is a measurable function ξ from an uncertainty space (Γ , L, M) to the set of real numbers, i.e., for any Borel set B of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma \,|\, \xi(\gamma) \in B\}$$

is an event.

B. *Expected Value:*

Expected value is the average value of uncertain variable in the sense of uncertain measure, and represents the size of uncertain variable.

Definition (Liu [1]) let ξ be an uncertain variable, then the expected value of is defined by

$$E(\xi) = \int_0^{+\infty} \mathrm{M}\{\xi \ge r\}\mathrm{d}r - \int_{-\infty}^{0} \mathrm{M}\{\xi \le r\}\mathrm{d}r$$

provided that at least one of the two integrals is finite.

III. PROBLEM DESCRIPTION

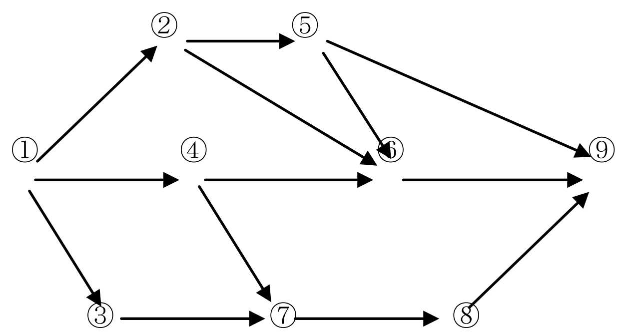

Figure 1: A Project Graph

Let $G=(V,A)$ represents a project, where $V=\{1,2,\cdots,n+1\}$ is the set of nodes standing for the milestones and $A=\{(i,j) \,|\, i,j \in V\}$ is the set of arcs representing the activities of the project.

In a real world application, a large engineering project is a big complex system. In order to establish the corresponding mathematical models, we must give some simplifications and assumptions to meet different management requirements:

a. ***Assumption (1):*** all of the costs are obtained by loans with some given interest rate;

b. ***Assumption (2):*** each activity can start only if the loan needed is allocated and all the foregoing activities are finished;

c. ***Assumption (3):*** each activity should be processed without interruption;

d. ***Assumption (4):*** duration time of each activity is assumed to a continuous and an uncertain variable;

e. ***Assumption (5):*** each man-power needed for each activity is an uncertain variable; a part of duration time of each activity is inversely proportional to the number of workers;

f. ***Assumption (6):*** the cost needed for each activity is only considered to workers' wages and loans with some given interest rate.

In order to model the project scheduling problem, we introduce the following mathematical signs and symbols:

$\xi=\{\xi_{ij} \,|\, (i,j) \in A\}$, duration times of activities represented by (i,j) in A , and ξ_{ij} are uncertain variables;

$\eta = \{\eta_{ij} | (i,j) \in A\}$, numbers of workers for activities represented by (i,j) in A, and η_{ij} are uncertain variables;

$x = (x_1, x_2, \cdots, x_n)$, decision vector and x_i represents the allocating time of all the loans needed for activities represented by (i,j) in A;

$y = (y_1, y_2, \cdots, y_n)$, decision vector and y_i represents the allocating number of workers needed for activities represented by (i,j) in A;

$T_i(x, y, \xi, \eta)$: starting times of all activities represented by (i,j) in A;

$M(t)$: number of workers needed for the project at the time point denoted as t;

$f_{ij}(y, \xi, \eta)$: the duration time of activity (i,j) in A;

$d_{ij}(y, \xi, \eta)$: the cost needed for activity (i,j) in A;

c_{ij}: the fixed cost needed for activity (i,j) in A;

r: the interest rate;

e_{ij}: the coefficient of wages needed for activity (i,j) in A (Yuan per person in unit time);

$\lambda_i \in [0\ 1]$: the coefficients about the irrelevant part between the duration time and the number of workers of activity (i,j) in A;

According to the assumption (4), the duration time of activity (i,j) in A can be calculated by

$$f_{ij}(y, \xi, \eta) = \left[(1-\lambda_i)\frac{\eta_{ij}}{y_i} + \lambda_i\right]\xi_{ij} \tag{1}$$

Obviously, $f_{ij}(y, \xi, \eta)$ is an uncertain variable.

According to the assumption (6), the costs needed for activity (i,j) in A can be calculated by

$$d_{ij}(y, \xi, \eta) = c_{ij} + f_{ij}(y, \xi, \eta) e_{ij} y_i \tag{2}$$

According to the assumption (2), we have $T_i(x, y, \xi, \eta) \ge x_i$ and

$$T_i(x, y, \xi, \eta) \ge \max_{(k,i)\in A}\{T_k(x, y, \xi, \eta) + f_{ij}(y, \xi, \eta)\}$$

Therefore, the starting times of all activities represented by (i,j) in A, can be calculated by

$$\begin{cases} T_i(x, y, \xi, \eta) = x_i \vee \max_{(k,i)\in A}\{T_k(x, y, \xi, \eta) + f_{ki}(y, \xi, \eta)\} \\ T_1(x, y, \xi, \eta) = x_1 \end{cases} \tag{3}$$

Then the completion time of the total project can be calculated by

$$T(x, y, \xi, \eta) = \max_{(k,n+1)\in A}\{T_k(x, y, \xi, \eta) + f_{k(n+1)}(y, \xi, \eta)\} \tag{4}$$

The total cost of the project can be calculated by

$$C(x, y, \xi, \eta) = \sum_{(i,j)\in A} d_{ij}(x, y, \xi, \eta)(1+r)^{\lceil T(x,y,\xi,\eta) - x_i \rceil} \tag{5}$$

Obviously, we can get

$$M(t, x, y, \xi, \eta) = \sum_{\substack{t\in[T_i, T_i + f_{ij}) \\ (i,j)\in A}} y_i \tag{6}$$

The maximal numbers of workers needed for the project can be denoted as

$$M(x, y, \xi, \eta) = \max_{x_1 \le t \le T} \sum_{\substack{t\in[T_i, T_i + f_{ij}) \\ (i,j)\in A}} y_i \tag{7}$$

Where $T_i = T_i(x, y, \xi, \eta)$, and $f_{ij} = f_{ij}(y, \xi, \eta)$.

As these basic formulas have been given in the above section, we can establish different hybrid programming models to meet different management requirements.

IV. MODELS ESTABLISHMENT

Expected value model (EVM), which is to optimize some expected objectives with some expected constraints, is a widely used method in solving practical problems with uncertain factors.

A. Single-objective Programming Models with Uncertain Variables:

If we want to minimization the expected cost of the project under the expected completion time and the number of workers constraint, we may construct the following expected value minimization model with uncertain variables as following:

Model 1: expected value minimization model

$$\begin{cases} \min\ E[C(x,y,\xi,\eta)] \\ \text{subject to :} \\ \quad E[T(x,y,\xi,\eta)] \le T_0 \\ \quad E[M(x,y,\xi,\eta)] \le M_0 \\ \quad x \ge 0, \quad \text{integer vector} \\ \quad y \ge 0, \quad \text{integer vector} \end{cases}$$

Where T_0 is the due date of the project, and M_0 is the man-power restraint.

B. Multi-objective Programming Models with Uncertain Variables:

If we consider both minimal cost and minimal numbers of workers of the total project, we can build the corresponding multi-objective programming models with uncertain variables.

Model 2: expected value multi-objective programming

$$\begin{cases} \min\{E[C(x,y,\xi,\eta)], E[M(x,y,\xi,\eta)]\} \\ \text{subject to :} \\ \quad E[T(x,y,\xi,\eta)] \le T_0 \\ \quad x \ge 0, \quad \text{integer vector} \\ \quad y \ge 0, \quad \text{integer vector} \end{cases}$$

Where T_0 is the due date of the project.

V. CONCLUSION

Considering in the process of practical application, a large engineering project is a large complex system. In this paper, we attempted to solve project scheduling problem with expected value model. And we embark from the actual. On the basis of the time constraints, allocating number of workers is also an important factor which cannot be ignored. We tend to minimize the total cost with completion time and the number of workers limits, and minimize the total cost and the numbers of workers with completion time limit. That is also the innovation of this paper. We set up corresponding mathematical models, so as to adapt to different management needs. In the further study, we can put the above models to the practical problems to meet the needs of reality.

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