



Improvements on Heuristic Algorithms for Solving Traveling Salesman Problem

Fidan Nuriyeva
Institute of Cybernetics
Azerbaijan National Academy of Sciences
Baku, Azerbaijan
nuriyevafidan@gmail.com

Gözde Kızılates
Department of Mathematics
Faculty of Science, Ege University
Izmir, Turkey
gozde.kizilates@gmail.com

Murat Erşen Berberler
Department of Computer Science
Faculty of Science, Dokuz Eylul University
Izmir, Turkey
murat.berberler@deu.edu.tr

Abstract: In this paper, four new heuristics are proposed in order to solve the traveling salesman problem. Comparisons are made between the results obtained from those heuristics. A new version of 2-opt and 3-opt methods are developed namely as 2-opt + 3-opt Shifting method. In addition, a new hybrid algorithm based on NN and Greedy algorithms is proposed. Computational experiments and comparisons are made on library problems for Hybrid, NN, and Greedy algorithms. Obtained results show the efficiency of the algorithms.

Keywords: Traveling salesman problem; heuristic algorithms; hyper-heuristic algorithms; hybrid algorithms

I. INTRODUCTION

The traveling salesman problem (TSP) is a well-known and important combinatorial optimization problem [11]. The goal is to find the shortest (least expensive) tour that visits each city (node) in a given list exactly once and then returns to the starting city. In other words, TSP can be considered as a graph problem in which vertices represent cities and distances between cities are represented by edges.

Formally, the TSP can be stated as follows: The distances between n cities are stored in a distance matrix D with elements d_{ij} where $i, j = 1, \dots, n$ and the diagonal elements d_{ii} are zero. A tour can be represented by a cyclic permutation π of $\{1, 2, \dots, n\}$ where π_i represents the city that follows city i on the tour. The traveling salesman problem is then the optimization problem of finding a permutation π that minimizes the length of the tour denoted

$$\text{by } \sum_{i=1}^n d_i \pi(i).$$

In this paper we shall concentrate on the symmetric TSP, in which the distances satisfy $d(i, j) = d(j, i)$ for $1 \leq i, j \leq n$.

There are many variations of TSP: Symmetric TSP, Asymmetric TSP, The MAX TSP, The MIN TSP, TSP with multiple visits (TSPM), TSP with a closed tour, TSP with an open tour [3]. There are many variations of the problem. In this work, we examine the classic symmetric TSP.

Solving TSP is an important part of many applications in different fields including vehicle routing, computer wiring, machine sequencing and scheduling, frequency assignment in communication networks as well as data analysis in psychology and clustering in biostatistics [12, 17]. For

example, data analysis applications in psychology ranging from profile smoothing to finding an order in developmental data are presented by [5]. Clustering and ordering using TSP solvers are currently becoming popular in biostatistics [2, 13]. For example, [18] described an application for ordering genes and [9] used a TSP solver for clustering proteins.

Given that the problem is NP-Hard, and hence the polynomial-time algorithms for finding optimal tours are unlikely to exist, much attention has been addressed to the question of efficient heuristic algorithms, fast algorithms that attempt only to find near-optimal tours.

The rest of this paper is organized as follows. Section 2 describes some approaches for solving the TSP. Section 3 presents basic tour constructing algorithms such as NN and Greedy. Section 4 presents our new tour constructing proposed heuristics. Section 5 presents a new version of 2-opt and 3-opt algorithms that we have proposed. Section 6 presents other improved algorithms. Finally, section 7 concludes the paper.

II. APPROACHES FOR SOLVING TSP

Although definition of the TSP is easy, it belongs to NP-hard [6]. There are a number of algorithms used to find optimal tours, because of this problem is NP-hard, none are feasible for large instances since they all grow exponentially. That's why heuristic algorithms are useful for this problem.

The following approaches are developed for solving TSP.

A. Exact Approaches:

These approaches usually utilize the integer linear programming model of the TSP. "Branch & Bound" is one of the examples for this category [10]. One approach that comes to mind first is to try all possibilities. Other approach can be dynamic programming [4]. But these approaches are

expensive to calculate and take long time for the number of cities greater than 20 since TSP is an NP-hard problem [20].

B. Approximation Approaches:

Solving the TSP optimally takes too long; instead, one normally uses approximation algorithms, or heuristics. The difference is approximation algorithms give us a guarantee, which indicates how bad solutions we can get, normally specified as c times the optimal value.

Best-known approximation algorithms for TSP are Christofides Algorithm (guaranteed value $3/2$), Minimum-Spanning Tree (MST) based algorithms (guaranteed value 2), and others [14].

The best approximation algorithm stated is that of Sanjeev Arora [8]. The algorithm guarantees $(1+1/c)$ approximation for every $c > 1$. It is based on geometric partitioning and quad trees. Although theoretically c can be very large, it will have a negative effect on its running time ($O(n(\log_2[n])^{O(c)})$ for two-dimensional problem instances).

C. Heuristic Algorithms:

One of the algorithm types, which are used in computer science, is heuristic algorithm [8]. These algorithms are not exact and they do not perform it all the time or do not guarantee the best result but still they are useful to find a solution of the problem. In practice, heuristic algorithms are preferred to exact algorithms for solving NP-hard problems. We can categorize the heuristic algorithms for TSP: heuristics composing the tour, heuristics improving the tour and hybrid heuristic using both.

- a. **Heuristics Composing the Tour:** The characteristic of these algorithms does not try to improve the result when they find a solution. Algorithm stops at that point. The known heuristics composing the tour are; Nearest Neighbour, Greedy, Insertion heuristic, Christofides algorithm [7] and others.
- b. **Heuristics Improving the Tour:** They try to improve the tour. Examples for these algorithms are 2-opt, 3-opt, Lin-Kernighan, similar local optimization algorithms [1] and others.
- c. **Hybrid Approaches:** They use both composing and improving heuristics at the same time. Iterated Lin-Kernighan is an example for these algorithms. The best results are obtained by using hybrid approaches [19].

D. Metaheuristic Algorithms:

Metaheuristic algorithms are the techniques which try to improve iteratively the candidate solution (or solutions) found by a specific approach for hard optimization problems. Metaheuristic algorithms accept the heuristic approach for solving the problem as a black box and don't care about the details. They only try to optimize the functions used to solve the problem. These functions are named as goal functions or objective functions.

Tabu search, genetic algorithms, simulated annealing, artificial neural networks, ant colony algorithm and similar artificial intelligence approaches are the examples of the metaheuristic algorithms [19].

E. Hyperheuristic Algorithms:

Hyperheuristics are the algorithms searching the heuristic space for solving the hard optimization problems.

In this sense, a hyperheuristic decides which heuristic is more efficient to solve the problem instead of trying to solve the problem. This means that if there is more than one heuristic solution for a problem, deciding which one of these will be more successful is called as hyper heuristic.

The decision algorithm in situations where there is more than one heuristic applied to the problem is also called as hyperheuristic.

F. Distinguishing Metaheuristic and Hyperheuristic Algorithms:

The difference between metaheuristic and hyperheuristic is lie actually on the solution space of the problem. Both of these approaches search the solution heuristically but the solution spaces are different. Metaheuristics search on the solution space while hyper heuristics search on heuristic search space. In the literature, there are two different ways to do it. During the process, either one of the heuristic is chosen from the heuristic set applied in each step, new solution is accepted or refused, or a new heuristic is created (i.e. using genetic programming) using available components. By this way, metaheuristic algorithms are used as hyperheuristics. However, there are hyperheuristics, which are not metaheuristics, for example, reinforcement learning based hyperheuristics.

From this perspective, it is necessary to create new integrated algorithms, which are interactive with each other. The biggest reason of forming the artificial intelligence is to create successful algorithms and form new integrated algorithms.

III. BASIC HEURISTIC ALGORITHMS

Now we will mention basic heuristics that we will use and we suggested in our previous studies.

A. Nearest Neighbour:

This is perhaps the simplest and most straightforward TSP heuristic. The key to this algorithm is to always visit the nearest city. The steps of this algorithm are as following:

- a. Select a random city.
- b. Find the nearest unvisited city and go there.
- c. Are there any unvisited cities left? If yes, go to step 2.
- d. Return to the first city.

We can obtain the best result out of this algorithm by starting the algorithm over again for each vertex and repeat it for n times.

B. Greedy Algorithm:

The Greedy heuristic gradually constructs a tour by repeatedly selecting the shortest edge and adding it to the tour as long as it does not create a cycle with less than N edges, or increase the degree of any node by more than 2 . We must not add the same edge twice of course. The steps of this algorithm are as following:

- a. Sort all edges.
- b. Select the shortest edge and add it to our tour if it does not violate any of the above constraints.
- c. Do we have n edges in our tour? If no, go to step 2.
- d. Terminate the algorithm.

IV. NEW HEURISTIC ALGORITHMS

Four new heuristic algorithms which consider the bad vertices (the vertex which has the maximal distance to other vertices) are proposed below. [15] and [16] give details on three of these algorithms. In these four algorithms, the ideas behind NN and Greedy algorithms are improved and new ideas are also considered. The vertices, which are further than the others, are prioritized. The smallest 2 edges are selected for these kind of vertices (On the contrary, in NN algorithm, nearest vertex is selected, so only one edge is selected). While selecting further vertices, difference between the biggest and the smallest edges are also considered. These edges are problematic when they are left to the end in other known algorithms. When we sort edges according to their importance, not only their lengths but also the vertices they belong to are considered.

A. Algorithms 1 (Feinting):

This algorithm is about finding the maximum element for each row in the adjacency matrix. The algorithm continues to add to the tour the minimum distance of the row in which the maximum element exists. This process is applied to each row. The aim of the algorithm is to prevent the worst situations. The steps of this algorithm are as following:

- Find the maximum distance for each row in adjacency matrix, and add it to MAX column.
- Select the maximum distance in the MAX column.
- In the same row in which this maximum distance exists, select the minimum distance, which does not contain a sub tour and add it to the tour.
- Increase the number of selected edges by one.
- If the number of selected edges is less than n then go to step 2.

We can demonstrate how the algorithm works in the following chart.

	c_1	c_2	c_3	...	c_l	...	c_n	MAX
c_1	0	a_{12}	a_{13}	a_{1n}	m_1
c_2	a_{21}	0	a_{23}	a_{2n}	m_2
c_3	a_{31}	a_{32}	0	a_{3n}	m_3
...	0
c_k	a_{k1}	a_{k2}	a_{k3}	...	a_{kl}	...	a_{kn}	m_k
...	0
c_n	a_{n1}	a_{n2}	a_{n3}	0	m_n

Figure 1. Figure, which shows how Algorithm 1 works.

$$\text{Here, } a_{ij} = \text{distance}(c_i, c_j), m_s = \max_j \{a_{sj}\},$$

$$m_k = \max_i \{m_i\}, a_{kl} = \min_j \{a_{kj}\},$$

$$s = \overline{1, n}, i, j = \overline{1, n}$$

B. Algorithm 2 (The Most Advantageous Vertex):

This algorithm is about finding the maximum and minimum distances for each row in the adjacency matrix. The algorithm continues to find the difference between the maximum distance and the distances of the correspondent minimum column, and to add this difference to distance column. The steps of this algorithm are as following:

- Find the maximum and minimum distances for each row in the adjacency matrix and add them to MAX and MIN columns.

- Subtract the distances in MIN column from the correspondent distances in MAX column, and then add the result to DIFFERENCE column.
- Find the maximum distance in DIFFERENCE column.
- In the same row in which this maximum distance exists, select the minimum distance, which does not contain a sub tour and add it to the tour.
- Increase the number of selected edges by one.
- If the number of selected edges is less than n then go to step 3.

We can demonstrate how the algorithm works in the following chart.

	c_1	c_2	c_3	...	c_l	...	c_n	MAX	MIN	DIFFERENCE
c_1	0	a_{12}	a_{13}	a_{1n}	m_1	n_1	d_1
c_2	a_{21}	0	a_{23}	a_{2n}	m_2	n_2	d_2
c_3	a_{31}	a_{32}	0	a_{3n}	m_3	n_3	d_3
...	0
c_k	a_{k1}	a_{k2}	a_{k3}	...	a_{kl}	...	a_{kn}	m_k	n_k	d_k
...	0
c_n	a_{n1}	a_{n2}	a_{n3}	0	m_n	n_n	d_n

Figure 2. Figure, which shows how Algorithm 2 works.

$$\text{Here, } a_{ij} = \text{distance}(c_i, c_j),$$

$$m_s = \max_j \{a_{sj}\}, n_s = \min_j \{a_{sj}\}, d_s = m_s - n_s,$$

$$d_k = \max_i \{d_i\}, a_{kl} = \min_j \{a_{kj}\}, s = \overline{1, n}, i, j = \overline{1, n}$$

C. Algorithm 3 (The Farthest Vertex):

This algorithm is about finding the sums of each row in the adjacent matrix. The algorithm continues to add to the tour the minimum two distances of each row which includes the maximum distance. This process is applied to each row.

The steps of this algorithm are as following:

- Find the sums for each row in the adjacency matrix and add them to SUM column.
- Find the maximum sum in SUM column.
- In the same row in which this maximum sum exists, select the two minimum distances, which do not contain a sub, tour and add them to the tour.
- Delete the row and column which correspondence to the maximum sum.
- Increase the number of selected vertex by one.
- If the number of selected vertex is less than n then go to step 2.

We can demonstrate how the algorithm works in the following chart.

	c_1	c_2	c_3	...	c_l	...	c_n	SUM
c_1	0	a_{12}	a_{13}	a_{1n}	s_1
c_2	a_{21}	0	a_{23}	a_{2n}	s_2
c_3	a_{31}	a_{32}	0	a_{3n}	s_3
...	0
c_k	a_{k1}	a_{k2}	a_{k3}	...	a_{kl}	...	a_{kn}	s_k
...	0
c_n	a_{n1}	a_{n2}	a_{n3}	0	s_n

Figure 3. Figure, which shows how Algorithm 3 works.

Here, $a_{ij} = \text{distance}(c_i, c_j)$,

$$s_i = \sum_{j=1}^n a_{ij}, s_k = \max_i \{s_i\}, a_{kl} = \min_j \{a_{kj}\}, i = \overline{1, n}$$

D. Computational Experiments for Heuristic Algorithms:

In the table below, these three algorithms are compared with NN and Greedy algorithms on library problems [21-23]. The results on the third row shows the best result found when applying NN algorithm starting from each vertices (n times).

Table I. Computational Experiments for Heuristic Algorithms

G	Optimal	NN Time(s)	Greedy Time(s)	Algorithm1 Time(s)	Algorithm2 Time(s)	Algorithm3 Time(s)
eil51	429.983	505.774 0.016	481.518 0.125	443.070 0.031	495.628 0.031	440.746 0.000
berlin52	7544.365	8182.192 0.000	9954.062 0.281	9047.211 0.031	9413.732 0.047	8618.198 0.002
st70	678.597	761.689 0.000	746.044 0.485	785.284 0.094	811.974 0.094	727.778 0.011
eil76	545.387	612.656 0.016	617.131 0.672	588.074 0.140	606.117 0.140	581.407 0.010
rat99	1211	1369.535 0.016	1528.308 1.875	1311.904 0.266	1273.747 0.282	1316.432 0.030
kroA100	21236.951	24698.497 0.016	24197.285 1.937	26135.302 0.360	24697.677 0.391	24093.242 0.020
kroB100	22141	25882.973 0.016	25815.214 2.469	24700.544 0.406	23651.697 0.406	23419.490 0.031
kroC100	20750.762	23566.403 0.015	25313.671 2.610	23962.861 0.391	24879.757 0.391	23512.300 0.030
kroD100	21294.290	24855.799 0.016	24631.533 2.359	24783.197 0.422	23201.380 0.390	24758.054 0.010
kroE100	22068	24907.022 0.016	24420.355 2.609	26036.072 0.375	25499.724 0.406	24822.113 0.010
rd100	7910.396	9427.333 0.015	8702.605 2.922	9866.781 0.406	8945.544 0.375	9384.955 0.030
eil101	642.309	736.368 0.015	789.112 2.609	712.461 0.329	694.685 0.359	704.361 0.010
lin105	14382.995	16939.441 0.015	16479.785 3.187	19679.294 0.360	17744.411 0.344	18354.693 0.030
pr107	44303	46678.154 0.016	48261.816 2.109	56635.995 0.453	47060.739 0.438	54003.941 0.010
ch130	6110.860	7198.741 0.016	7142.045 7.688	6963.303 0.875	6636.392 0.953	6873.837 0.055
kroA150	26524	31482.020 0.047	31442.994 11.094	31027.270 1.469	28444.581 1.453	30216.728 0.082
kroB150	26130	31320.340 0.047	31519.083 11.156	31824.589 1.547	30815.938 1.454	29631.101 0.064
rat195	2323	2628.561 0.109	2957.176 29.719	2763.001 3.563	2637.019 3.532	2554.672 0.085
kroA200	29368	34547.691 0.125	37650.812 45	35195.046 5.187	35792.822 5.172	33629.972 0.112

V. NEW HEURISTIC ALGORITHMS

Once a tour has been generated by some tour construction heuristic, we might wish to improve that solution. There are several ways to do this, but the most common ones are the 2-opt and 3-opt local searches. The 2-opt algorithm basically removes two edges from the tour, and reconnects these two paths which are formed by removing these two edges. There is only one way to reconnect the two paths so that we still have a valid tour (Figure 4, 5). We do this only if the new tour will be shorter. This process of removing and reconnecting the tour continues until no 2-opt improvement is found. The tour we obtain at the end of this process is now 2-optimal. The 3-opt algorithm works in a similar fashion, but instead of removing two edges we remove three. This means that we have two ways of reconnecting the three paths into a valid tour (Figure 6). A 3-opt move can actually be seen as two or three 2-opt moves. We finish our search when no more 3-opt moves can improve the tour. If a tour is 3-optimal it is also 2-optimal. If we look at the tour as a permutation of all the cities, a 2-opt

move will result in reversing a segment of the permutation. A 3-opt move can be seen as two or three segment reversals [3].

A. 2-opt and Shifting:

The 2-opt code is as following:

```
for (i = 1; i <= n - 3; i++)
```

```
  for (j = i + 2; j <= n - 1; j++)
```

```
    if
```

```
      (d[a[i]][a[i+1]]+d[a[j]][a[j+1]]>d[a[i]][a[j]] +
```

```
      d[a[i+1]][a[j+1]])
```

```
        swap(a[i+1], a[j])
```

The code above improves the solution by shifting the vertexes when their indexes in solution vector are proper, that is when their position are between the 1st and the (n-1)th vertex. If, however, the shifting vertex j is in the nth position in the solution vector, then the if part of the algorithm does not run and the algorithm does improve anything. In order to prevent this handicap, we shift the elements of the solution vector 1 unit to the left. Therefore, since the nth vertex is now (n-1)th, the if part of the algorithm runs properly and the algorithm improves its solution by shifting these two

vertices. This whole process is explained in an example below:

Table II. Distance matrix

	1	2	3	4	5	6
1	0	1	3	5	3	1
2	1	0	1	3	5	3
3	3	1	0	1	3	5
4	5	3	1	0	1	3
5	3	5	3	1	0	1
6	1	3	5	3	1	0

For example, let us say that the optimum result of the traveling salesman problem is 6 and the solution vector is $[v_1v_2v_3v_4v_5v_6]$ (6). The length of the tour of the given tours is computed by giving attention to the distance matrix in Table II. If we apply the 2-opt algorithm to a vector like $[v_1v_2v_3v_4v_6v_5]$ (10) whose tour cost is 10, then the algorithm will not make any shifts between the vertices. Yet, if the elements of solution vector are switched 1 unit to the left, then the 2-opt algorithm will find the optimum result: $[v_3v_4v_5v_6v_1v_2]$ (6).

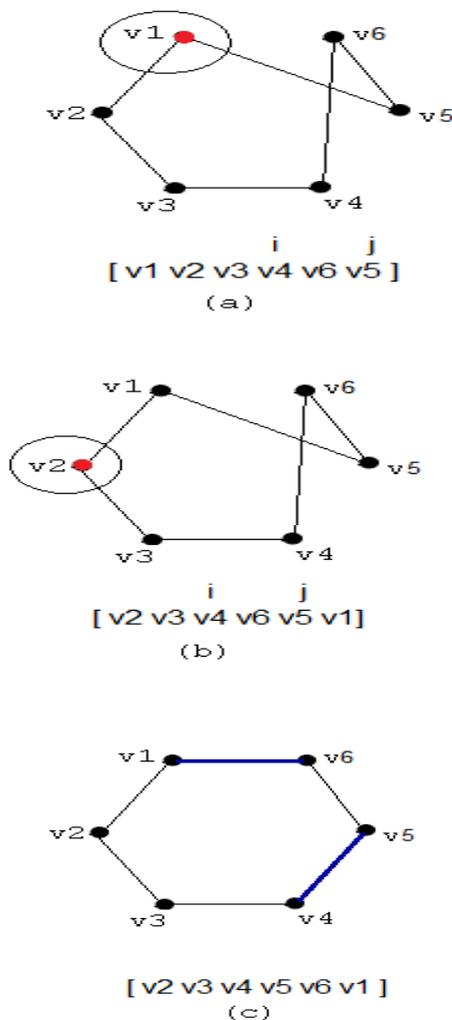


Figure 4. The demonstration of the shifting operation in a 2-opt application on an example

In the same way, if we apply the 2-opt algorithm to the vector, $[v_1v_4v_5v_2v_3v_6]$ (18), the algorithm improves this vector three times and the solution vector obtained at the end of the process is $[v_1v_3v_2v_4v_5v_6]$ (10). Even if the elements

of this solution vector are shifted 1 unit to the left, there will be no improve. Yet if we apply the 2-opt algorithm after shifting the elements of this solution vector 2 units to the left, then we can obtain the optimum result for the problem $[v_1v_2v_3v_4v_5v_6]$ (6).

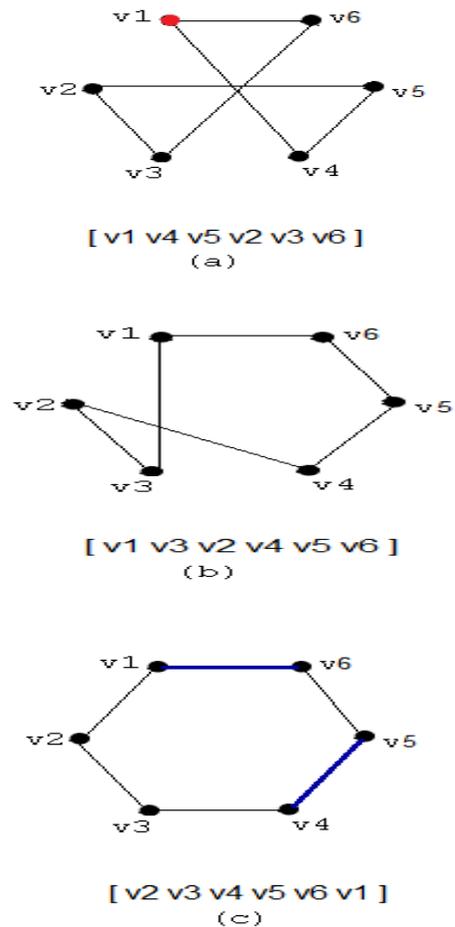


Figure 5. The demonstration of the double shifting operation in a 2-opt application on an example

B. 3-opt and Shifting:

The 3-opt code is as following:

```

for(i=1;i<=n-5;i++)
    for(j=i+2;j<=n-3;j++)
        for(k=j+2;k<=n-1;k++)
            if(d[a[i]][a[i+1]]+d[a[j]][a[j+1]]
                +d[a[k]][a[k+1]]>d[a[i]][a[j+1]]
                +d[a[j]][a[k+1]]+d[a[k]][a[i+1]])
                {
                    swap(a[i+1], a[j+1])
                    swap(a[j], a[k])
                }
    
```

As in 2-opt algorithm, if the position of shifting vertex is n in the solution vector, then the algorithm will skip the if part and will not improve the solution. However, when the elements of the solution vector are shifted to the left, then we again obtain the optimum result. Also in this method, there are examples in which shifting the elements of the solution vector 1 unit to the left is not enough so there should be shifting more than 1 unit to the left.

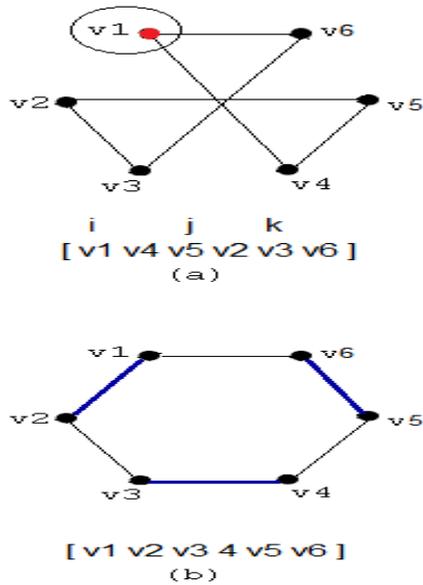


Figure 6. The demonstration of the shifting operation in a 3-opt application on an example

C. Computational Experiments for 3-opt + 2-opt:

In Table III, the second column shows the results of 2-opt and 3-opt applications with shifting operation, and the third column shows the results of 2-opt and 3-opt applications without shifting operation. As it can be seen in the table, the results of the applications with shifting operation give better results than the applications without shifting.

Table III. The comparison of the 2-opt and 3-opt applications with and without shifting operation

G	3-opt + 2-opt (S+)	3-opt + 2-opt (S-)
eil51	429.484	456.265
berlin52	7544.365	7993.064
kroA100	21285.443	22146.600
rd100	8101.042	8206.614
lin105	14382.995	14670.207
ch130	6250.213	6409.888
u159	43786.312	45147.919

VI. OTHER IMPROVED ALGORITHMS

Below, a hybrid of NN and greedy algorithms and an improved version of vertex ranking based on learning are presented.

A. Hybrid Heuristic Algorithm:

The algorithm that we propose is a hybrid of the traditional NN and Greedy heuristic algorithm. We start the algorithm with NN for each vertex and repeat it for n times. Each time the algorithm is applied, we give a “priority” to the edge according to the result of the solution. Let the “priority” of the selected edges in the first solution be 1 all the others be 0. Suppose that the length of the first tour is D_1

Then, we add $\frac{D_1}{D_i}$ (Here, D_i is the length of the tour, which

is found at step i) to the “priorities” of the selected edges. Thus, each edge has a “priority” after n steps. Then, we sort the edges in descending order by “priority” and solve the problem with greedy algorithm. Let the length of the tour be

D_{n+1} , and we add $\frac{D_1}{D_{n+1}}$ to the “priorities” of the selected

edges. At the next steps, the edges are sorted in descending order by their updated “priorities”, and then, we solve the problem with the greedy algorithm. This process continues until there is no change on the sorting anymore. The result of the algorithm is the best solution found during this process. The steps of the algorithm are as follows:

- Solve the problem n times by NN algorithm starting with different vertices at each time. Find the “priorities” of edges. Assign the best solution, as a record solution.
- Sort the edges in descending order by “priority”. Then, solve the problem by the greedy algorithm. If the solution is better (smaller) than the record solution update the record.
- If there is no any change in the sorting then, stops the algorithm otherwise go to Step 2.

B. Hyperheuristic Algorithm (Vertex Ranking):

Finding the initial solution:

For each vertex, find the row sum in the adjacency matrix and assign it to array of sums.

- For each vertex, find the first shortest edge in the adjacency matrix and increase their importance values an array of importance’s by in this order.
- For each edge, set the priority value in the array of priorities as the biggest value of its vertices importance values in the array of importance’s.
- Sort the edges decreasingly according to their importance values. If there are same importance values, sort them decreasingly according to their priority values. If priorities are same too, then sort them increasingly according to their lengths.
- Add the vertex, which will not create a sub tour, to the solution from the array of importances in order.

Improving the initial solution:

- After finding the initial solution, add 1 to the importance value of the first edge, which is not added to the solution and go to step 4.

If the new solution is worse than the best solution found so far, subtract n from the importance value of the last vertex which is added 1 to its importance value before and add 1 to the importance value of the first edge, which is not added to the solution and go to step 4 again. In other case where the new solution is not worse than the best solution found so far, add 1 to the importance value of the first edge, which is not added to the solution and go to step 4 again. This process is repeated two times. n is the total vertex number of the graph.

The principles of this algorithm can be explained as follows:

In the proposed algorithm, a new algorithm is created using the algorithms mentioned in section 3. Edges are given a priority in the third step of the algorithm according their distances. In the fourth step, each edge is given an

importance value according to its priority value. Edges are sorted according to their importance values in decreasing order. An initial solution is found using a greedy algorithm in step 5. In step 6, initial solution is tried to improve using a learning based iterative approach.

Table IV. Computational experiments for the heuristics, which improve the tour

G	Optimal	2opt+3opt Time(s)	Hyperheuristic Time(s)	Hybrid Time(s)
eil51	429.983	429.983*	444.415	505.774
berlin52	7544.365	7544.365*	8396.818	8182.192
st70	678.597	688.280	722.679	761.689
eil76	545.387	562.331	566.774	612.655
rat99	1211	1247.094	1252.272	1369.534
kroA100	21236.951	21285.443	21936.297	24698.497
kroB100	22141	22585.399	23665.102	25882.973
kroC100	20750.762	20786.896	21954.034	23566.403
kroD100	21294.290	21733.785	23733.511	24855.799
kroE100	22068	22331.660	23102.137	24907.022
rd100	7910.396	8101.042	8544.428	9427.333
eil101	642.309	661.138	678.246	736.368
lin105	14382.995	14382.995*	15730.244	16939.441
pr107	44303	44576.123	44324.838	46678.154
ch130	6110.860	6250.213	6337.452	7198.741
kroA150	26524	27229.789	27968.152	31482.020
kroB150	26130	26802.108	28295.964	31320.340
rat195	2323	2473.668	2472.163	2618.045
kroA200	29368	30876.078	31137.834	34547.691

C. Computational Experiments for the Heuristics which Improve the Tour:

Table IV shows the result of computational experiments of these 3 algorithms conducted on library problems [21–23]. The best results are marked on the table. As it is shown in the table, best results are usually obtained by “2-opt + 3-opt + shifting”. In Table IV, * sign shows the optimum result.

VII. CONCLUSION

In this study, we have proposed four new heuristics to solve traveling salesman problem. We have offered a number of different NN versions. We have found better solution than NN and greedy by combined them. In addition, we consider the tour-improved techniques; 2opt, 3opt and we have improved their performance by some modification. When we consider the study of old and newly proposed algorithms and the results of computational experiments, it is seen that the best solution is obtained by applying “2opt + 3opt + shifting” on best result found by applying NN algorithm n times, for each vertices as starting vertex. For further study, the

different combinations of the heuristics will be investigated in order to improve the results.

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