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Global Chaos Synchronization of Liu and Lü Systems by Nonlinear Control

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Abstract: This paper investigates the global exponential synchronization of chaotic systems, *viz.* identical Liu systems (Liu *et al.*, 2004), identical Lű systems (Lű and Chen, 2002) and synchronization of Liu and Lű systems. Nonlinear feedback control is the method used to achieve the synchronization of the chaotic systems addressed in this paper and our theorems on global exponential synchronization for T and Cai systems are established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the nonlinear feedback control method is effective and convenient to synchronize identical and different T and Cai systems. Numerical simulations are also given to illustrate and validate the synchronization results for Liu and Lű systems.

Keywords: Chaos Synchronization, Nonlinear Control, Liu System, Lű System, Feedback Control.

I. INTRODUCTION

Chaotic systems are dynamical systems that are highly sensitive to initial conditions. This sensitivity is popularly referred to as the *butterfly effect* [1].

Chaos synchronization problem was first described by Fujisaka and Yemada [2] in 1983. This problem did not receive great attention until Pecora and Carroll [3-4] published their results on chaos synchronization in early 1990s. From then on, chaos synchronization has been extensively and intensively studied in the last three decades [3-22], Chaos theory has been explored in a variety of fields including physical [5], chemical [6], ecological [7] systems, secure communications [8-10] etc.

Synchronization of chaotic systems is a phenomenon that may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem.

In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

Since the seminal work by Carroll and Pecora [3-4], a variety of impressive approaches have been proposed for the synchronization for the chaotic systems such as PC method [3-4], the sampled-data feedback synchronization method [10-11], OGY method [12], time-delay feedback approach [13], backstepping design method [14], adaptive design method [15-19], sliding mode control method [20], Lyapunov stability theory method [21], hyperchaos [22], etc.

This paper has been organized as follows. In Section II, we give the problem statement and our methodology. In Section III, we discuss the chaos synchronization of two identical Liu systems ([23], 2004). In Section IV, we discuss the chaos synchronization of two identical Lű systems ([24], 2002). In Section V, we discuss the heterogeneous synchronization of Liu and Lű systems. In Section VI, we present the conclusions of this paper.

II. PROBLEM STATEMENT AND OUR METHODOLOGY

Consider the chaotic system described by the dynamics

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in \boldsymbol{R}^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : \boldsymbol{R}^n \to \boldsymbol{R}^n$ is the nonlinear part of the system. We consider the system (1) as the *master* or *drive* system.

As the *slave* or *response* system, we consider the following chaotic system described by the dynamics

$$\dot{y} = By + g(y) + u \tag{2}$$

where $y \in \boldsymbol{R}^n$ is the state vector of the response system, B is the $n \times n$ matrix of the system parameters, $g : \boldsymbol{R}^n \to \boldsymbol{R}^n$ is the nonlinear part of the response system and $u \in \boldsymbol{R}^n$ is the controller of the response system.

If $A = B$ and $f = g$, then x and y are the states of two *identical* chaotic systems. If $A \neq B$ and $f \neq g$, then x and y are the states of two *different* chaotic systems.

In the nonlinear feedback control approach, we design a feedback controller u, which synchronizes the states of the master system (1) and the slave system (2) for all initial conditions $x(0), z(0) \in \boldsymbol{R}^n$.

If we define the *synchronization error* as

$$e = y - x, \quad (3)$$

then the synchronization error dynamics is obtained as

$$\dot{e} = By - Ax + g(y) - f(x) + u \quad (4)$$

Thus, the global synchronization problem is essentially to find a feedback controller u so as to stabilize the error dynamics (4) for all initial conditions $e(0) \in \boldsymbol{R}^n$, i.e.

$$\lim_{t\to\infty} \|e(t)\| = 0 \quad (5)$$

for all initial conditions $e(0) \in \boldsymbol{R}^n$.

We use Lyapunov stability theory as our methodology. We take as a candidate Lyapunov function

$$V(e) = e^T Pe, \quad (6)$$

where P is a positive definite matrix. Note that $V : \boldsymbol{R}^n \to \boldsymbol{R}^n$ is a positive definite function by construction. We assume that the parameters of the master and slave systems are known and that the states of both systems (1) and (2) are measurable.

If we we find a feedback controller u so that

$$\dot{V}(e) = -e^T Qe, \quad (7)$$

where Q is a positive definite matrix, then $\dot{V} : \boldsymbol{R}^n \to \boldsymbol{R}^n$ is a negative definite function.

Thus, by Lyapunov stability theory [25], the error dynamics (4) is globally exponentially stable and hence the condition (5) will be satisfied for all initial conditions $e(0) \in \boldsymbol{R}^n$. Then the states of the master system (1) and slave system (2) will be globally exponentially synchronized.

III. Synchronization of Identical Liu Systems

In this section, we apply the nonlinear control technique for the synchronization of two identical Liu systems ([23], 2004).

Thus, the master system is taken as the Liu dynamics described by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= bx_1 - x_1x_3 \\ \dot{x}_3 &= -cx_3 + dx_1^2 \end{aligned} \quad (8)$$

where x_1, x_2, x_3 are states of the system and $a > 0,\ b > 0,\ c > 0,\ d > 0$ are parameters of the system.

The slave system is also taken as the Liu dynamics described by

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= by_1 - y_1y_3 + u_2 \\ \dot{y}_3 &= -cy_3 + dy_1^2 + u_3 \end{aligned} \quad (9)$$

where y_1, y_2, y_3 are states of the system and

$$u = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T$$

is the nonlinear controller to be designed.

The Liu system (8) is one of the paradigms of 3-D chaotic systems derived by Liu et al. ([23], 2004). The Liu system (8) is chaotic when

$$a = 10,\ b = 40,\ c = 2.5 \text{ and } d = 4.$$

Figure 1 illustrates the chaotic portrait of the Liu system (8).

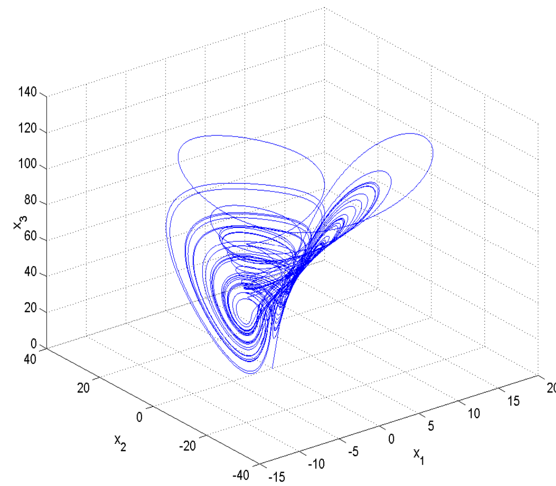

Figure 1. Chaotic Portrait of the Liu System (8)

The synchronization error e is defined by

$$e_i = y_i - x_i, \qquad (i = 1, 2, 3) \quad (10)$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + u_1 \\ \dot{e}_2 &= be_1 - (y_1y_3 - x_1x_3) + u_2 \\ \dot{e}_3 &= -ce_3 + d\left(y_1^2 - x_1^2\right) + u_3 \end{aligned} \quad (11)$$

In order to find the synchronizing controller, we first let

$$\begin{aligned} u_2 &= u_{2a} + u_{2b} \\ u_3 &= u_{3a} + u_{3b} \end{aligned} \quad (12)$$

where

$$\begin{aligned} u_{2b} &= y_1y_3 - x_1x_3 \\ u_{3b} &= -d\left(y_1^2 - x_1^2\right) \end{aligned} \quad (13)$$

Substituting (12) and (13) into (11), we obtain

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + u_1 \\ \dot{e}_2 &= be_1 + u_{2a} \\ \dot{e}_3 &= -ce_3 + u_{3a} \end{aligned} \quad (14)$$

Next, we consider the candidate Lyapunov function

$$V(e) = \frac{1}{2}e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \quad (15)$$

which is a positive definite function on $\boldsymbol{R}^3$.

A simple calculation gives

$$\dot{V}(e) = -ae_1^2 - ce_3^2 + (a+b)e_1e_2 + e_1u_1 + e_2u_{2a} + e_3u_{3a} \tag{16}$$

Therefore, we choose

$$\begin{aligned} u_1 &= -(a+b)e_2 \\ u_{2a} &= -e_2 \\ u_{3a} &= 0 \end{aligned} \tag{17}$$

Substituting (17) into (14), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -ae_1 - be_2 \\ \dot{e}_2 &= be_1 - e_2 \\ \dot{e}_3 &= -ce_3 \end{aligned} \tag{18}$$

Substituting (17) into (16), we obtain

$$\dot{V}(e) = -ae_1^2 - e_2^2 - ce_3^2 \tag{19}$$

which is a negative definite function on $\boldsymbol{R}^3$ since a and c are positive constants.

Hence, by Lyapunov stability theory [25], the error dynamics (18) is globally exponentially stable.

Combining (12), (13) and (17), the synchronizing nonlinear controller u is obtained as

$$\begin{aligned} u_1 &= -(a+b)e_2 \\ u_2 &= -e_2 + y_1y_3 - x_1x_3 \\ u_3 &= -d\left(y_1^2 - x_1^2\right) \end{aligned} \tag{20}$$

Thus, we have proved the following result.

Theorem 1. The identical Liu systems (8) and (9) are exponentially and globally synchronized for any initial conditions with the nonlinear controller u defined by (20). ■

Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the system using MATLAB.

For the Liu system (8), the parameter values are taken as those which result in the chaotic behaviour of the system, *viz.* $a = 10, b = 40, c = 2.5, d = 4.$

The initial values of the master system (8) are taken as

$$x_1(0) = 5,\ x_2(0) = 12,\ x_3(0) = 2$$

while the initial values of the slave system (9) are taken as

$$y_1(0) = 24,\ y_2(0) = 4,\ y_3(0) = 15$$

Figure 2 shows that synchronization error between the states of the master system (8) and the slave system (9) converges to zero exponentially in 2 seconds.

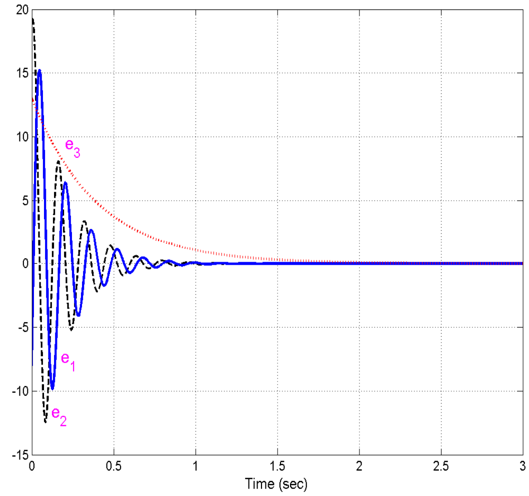

Figure 2. Synchronization of the States of (8) and (9)

IV. SYNCHRONIZATION OF IDENTICAL LŰ SYSTEMS

In this section, we apply the nonlinear control technique for the synchronization of two identical Lű systems ([24], 2002).

Thus, the master system is taken as the Lű dynamics described by

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) \\ \dot{x}_2 &= -x_1x_3 + \gamma x_2 \\ \dot{x}_3 &= x_1x_2 - \beta x_3 \end{aligned} \tag{21}$$

where x_1, x_2, x_3 are states of the system and $\alpha > 0,\ \beta > 0,\ \gamma > 0$ are parameters of the system.

The slave system is also taken as the Lű dynamics described by

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= -y_1y_3 + \gamma y_2 + u_2 \\ \dot{y}_3 &= y_1y_2 - \beta y_3 + u_3 \end{aligned} \tag{22}$$

where y_1, y_2, y_3 are states of the system and

$$u = [u_1 \quad u_2 \quad u_3]^T$$

is the nonlinear controller to be designed.

The Lű system (21) is one of the paradigms of 3-D chaotic systems derived by Lű and Chen ([24], 2002). The Lű system (21) is chaotic when

$$\alpha = 36,\ \beta = 3 \quad \text{and} \quad \gamma = 20.$$

Figure 3 illustrates the chaotic portrait of the Lű system (21).

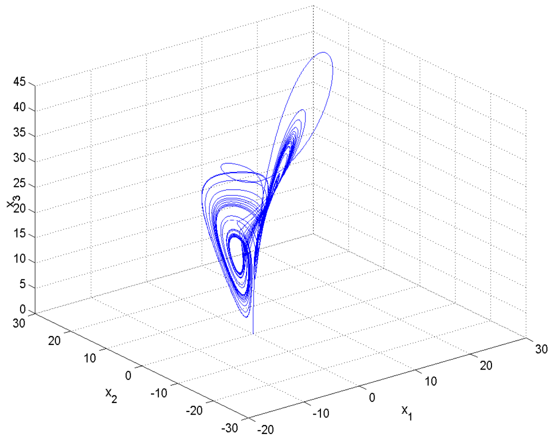

Figure 3. Chaotic Portrait of the Lű System (21)

The synchronization error e is defined by

$$e_i = y_i - x_i, \qquad (i = 1, 2, 3) \tag{23}$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= \alpha(e_2 - e_1) + u_1 \\ \dot{e}_2 &= \gamma e_2 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -\beta e_3 + y_1 y_2 - x_1 x_2 + u_3 \end{aligned} \tag{24}$$

In order to find the synchronizing controller, we first let

$$\begin{aligned} u_2 &= u_{2a} + u_{2b} \\ u_3 &= u_{3a} + u_{3b} \end{aligned} \tag{25}$$

where

$$\begin{aligned} u_{2b} &= y_1 y_3 - x_1 x_3 \\ u_{3b} &= -y_1 y_2 + x_1 x_2 \end{aligned} \tag{26}$$

Substituting (25) and (26) into (24), we obtain

$$\begin{aligned} \dot{e}_1 &= \alpha(e_2 - e_1) + u_1 \\ \dot{e}_2 &= \gamma e_2 + u_{2a} \\ \dot{e}_3 &= -\beta e_3 + u_{3a} \end{aligned} \tag{27}$$

Next, we consider the candidate Lyapunov function

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{28}$$

which is a positive definite function on $\boldsymbol{R}^3$.

A simple calculation gives

$$\begin{aligned} \dot{V}(e) = -\alpha e_1^2 + \gamma e_2^2 - \beta e_3^2 + \alpha e_1 e_2 + e_1 u_1 \\ + e_2 u_{2a} + e_3 u_{3a} \end{aligned} \tag{29}$$

Therefore, we choose

$$\begin{aligned} u_1 &= -\alpha e_2 \\ u_{2a} &= -(\gamma + 1) e_2 \\ u_{3a} &= 0 \end{aligned} \tag{30}$$

Substituting (30) into (27), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -\alpha e_1 \\ \dot{e}_2 &= -e_2 \\ \dot{e}_3 &= -\beta e_3 \end{aligned} \tag{31}$$

Substituting (30) into (29), we obtain

$$\dot{V}(e) = -\alpha e_1^2 - e_2^2 - \beta e_3^2 \tag{32}$$

which is a negative definite function on $\boldsymbol{R}^3$ since α and β are positive constants.

Hence, by Lyapunov stability theory [25], the error dynamics (31) is globally exponentially stable.

Combining (25), (26) and (30), the synchronizing nonlinear controller u is obtained as

$$\begin{aligned} u_1 &= -\alpha e_2 \\ u_2 &= -(\gamma + 1) e_2 + y_1 y_3 - x_1 x_3 \\ u_3 &= -y_1 y_2 + x_1 x_2 \end{aligned} \tag{33}$$

Thus, we have proved the following result.

Theorem 2. The identical Lű systems (21) and (22) are exponentially and globally synchronized for any initial conditions with the nonlinear controller u defined by (33). ■

Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the system using MATLAB.

For the Lű system (21), the parameter values are taken as those which result in the chaotic behaviour of the system, *viz.* $\alpha = 36,\ \beta = 3,\ \gamma = 20.$

The initial values of the master system (21) are taken as

$$x_1(0) = 10,\ x_2(0) = 4,\ x_3(0) = 8$$

while the initial values of the slave system (22) are taken as

$$y_1(0) = 2,\ y_2(0) = 12,\ y_3(0) = 16$$

Figure 4 shows that synchronization error between the states of the master system (21) and the slave system (22) converges to zero exponentially in 6 seconds.

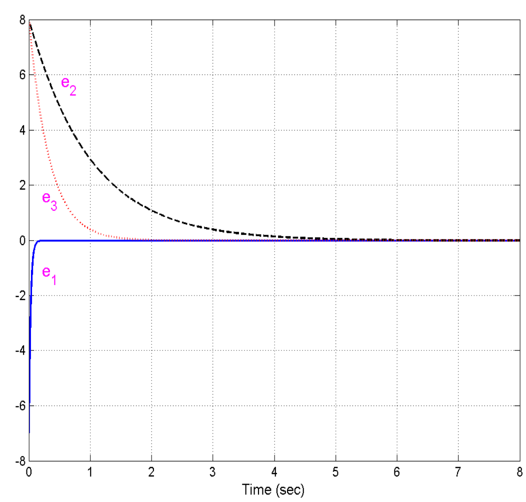

Figure 4. Synchronization of the States of (21) and (22)

V. SYNCHRONIZATION OF LIU AND LŰ SYSTEMS

In this section, we apply the nonlinear control technique for the synchronization of non-identical Liu and Lű chaotic systems. As the master system, we consider the Liu system ([23], 2004) described by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= bx_1 - x_1 x_3 \\ \dot{x}_3 &= -cx_3 + dx_1^2 \end{aligned} \tag{34}$$

where x_1, x_2, x_3 are states of the system and $a > 0,\ b > 0,\ c > 0,\ d > 0$ are parameters of the system.

As the slave system, we consider the Lű system ([24], 2002) described by

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= -y_1 y_3 + \gamma y_2 + u_2 \\ \dot{y}_3 &= y_1 y_2 - \beta y_3 + u_3 \end{aligned} \tag{35}$$

where all the parameters α, β, γ, h are positive real constants and $u = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T$ is the nonlinear control to be designed.

The synchronization error e is defined by

$$e_i = y_i - x_i, \qquad (i = 1, 2, 3) \tag{36}$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= \alpha(e_2 - e_1) + (\alpha - a)(x_2 - x_1) + u_1 \\ \dot{e}_2 &= \gamma e_2 - bx_1 + \gamma x_2 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -\beta e_3 + (c - \beta)x_3 - dx_1^2 + y_1 y_2 + u_3 \end{aligned} \tag{37}$$

In order to find the synchronizing controller, we first let

$$\begin{aligned} u_1 &= u_{1a} + u_{1b} \\ u_2 &= u_{2a} + u_{2b} \\ u_3 &= u_{3a} + u_{3b} \end{aligned} \tag{38}$$

where

$$\begin{aligned} u_{1b} &= (a - \alpha)(x_2 - x_1) \\ u_{2b} &= bx_1 - \gamma x_2 + y_1 y_3 - x_1 x_3 \\ u_{3b} &= (\beta - c)x_3 + dx_1^2 - y_1 y_2 \end{aligned} \tag{39}$$

Substituting (38) and (39) into (37), we obtain

$$\begin{aligned} \dot{e}_1 &= \alpha(e_2 - e_1) + u_{1a} \\ \dot{e}_2 &= \gamma e_2 + u_{2a} \\ \dot{e}_3 &= -\beta e_3 + u_{3a} \end{aligned} \tag{40}$$

Next, we consider the candidate Lyapunov function

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{41}$$

which is a positive definite function on $\boldsymbol{R}^3$.

A simple calculation gives

$$\begin{aligned} \dot{V}(e) = & -\alpha e_1^2 + \gamma e_2^2 - \beta e_3^2 + \alpha e_1 e_2 + e_1 u_{1a} \\ & + e_2 u_{2a} + e_3 u_{3a} \end{aligned} \tag{42}$$

Therefore, we choose

$$\begin{aligned} u_{1a} &= -\alpha e_2 \\ u_{2a} &= -(\gamma + 1)e_2 \\ u_{3a} &= 0 \end{aligned} \tag{43}$$

Substituting (43) into (40), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= -\alpha e_1 \\ \dot{e}_2 &= -e_2 \\ \dot{e}_3 &= -\beta e_3 \end{aligned} \tag{44}$$

Substituting (43) into (42), we obtain

$$\dot{V}(e) = -\alpha e_1^2 - e_2^2 - \beta e_3^2 \tag{45}$$

which is a negative definite function on $\boldsymbol{R}^3$ since α and β are positive constants.

Hence, by Lyapunov stability theory [25], the error dynamics (44) is globally exponentially stable.

Combining (38), (39) and (43), the synchronizing nonlinear controller u is obtained as

$$\begin{aligned} u_1 &= -\alpha e_2 + (a - \alpha)(x_2 - x_1) \\ u_2 &= -(\gamma + 1)e_2 + bx_1 - \gamma x_2 + y_1 y_3 - x_1 x_3 \\ u_3 &= (\beta - c)x_3 + dx_1^2 - y_1 y_2 \end{aligned} \tag{46}$$

Thus, we have proved the following result.

Theorem 3. The non-identical Liu system (34) and Lű system (35) are exponentially and globally synchronized for any initial conditions with the nonlinear controller u defined by (46). ■

Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the system using MATLAB.

For the Liu system (34), the parameter values are taken as those which result in the chaotic behaviour of the system, *viz.* $a = 10, b = 40, c = 2.5, d = 4.$

For the Lű system (35), the parameter values are taken as those which result in the chaotic behaviour of the system, *viz.* $\alpha = 36,\ \beta = 3,\ \gamma = 20.$

The initial values of the master system (34) are taken as

$$x_1(0) = 15,\ x_2(0) = 8,\ x_3(0) = 4$$

while the initial values of the slave system (35) are taken as

$$y_1(0) = 5,\ y_2(0) = 12,\ y_3(0) = 18$$

Figure 5 shows that synchronization error between the states of the master system (34) and the slave system (35) converges to zero exponentially in 6 seconds.

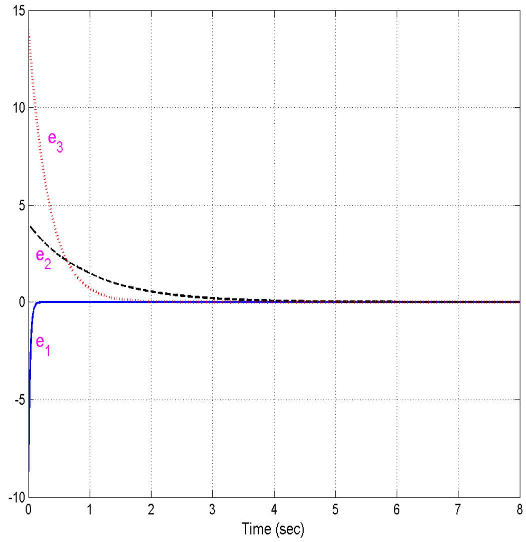

Figure 5. Synchronization of the States of (34) and (35)

VI. CONCLUSIONS

In this paper, we have used nonlinear control method based on Lyapunov stability theory to achieve global chaos synchronization for the following three cases:

(A) Identical Liu systems (Liu et al., 2004)

(B) Identical Lű systems (Lű and Chen, 2002)

(C) Non-Identical Liu and Lű Systems.

Numerical simulations are also given to validate the proposed synchronization approach for the global chaos synchronization of the chaotic systems. Since the Lyapunov exponents are not required for these calculations, the nonlinear control method is very effective and convenient to achieve global chaos synchronization for the three cases of chaotic systems discussed in this paper.

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