



## Performance Evaluation of Combinatorial Optimization Problems Using Meta Heuristics Algorithms

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**Abstract-** In real life scheduling, variations of the standard traveling salesman problem is very often encountered. The aim of this work is to present a new heuristic method for solving three such special instances with a common approach. The proposed algorithm uses a variant of the threshold accepting method, enhanced with intense local search, while the candidate solutions are produced through an insertion heuristic scheme. The main characteristic of the algorithm is that it does not require modifications and parameter tuning in order to cope with the three different problems. Computational results on a variety of real life and artificial problems are presented at the end of this work and prove the efficiency and the ascendancy of the proposed method over other algorithms found in the literature.

### I. INTRODUCTION

One of the most notorious as well as well studied problem in the field of combinatorial optimization is the traveling salesman problem (TSP) [15]. The problem is easy and straightforward to state, but its solution has obstructed researchers.

In its most simplified form as exhibited by Lawler et al. [16] and Reinelt [19], a number of cities and the distances between them are given and the task is to find the minimum-length closed tour that visits each city once and returns to its starting point. It can be viewed as a graph-theory problem if the cities are identified with the nodes of a graph, and the links between the cities are associated with arcs. Nevertheless, due to physical or technical constraints, real world problems can hardly ever be described by this simple form.

When the distance between any two cities  $i$  and  $j$  is equal to the distance between  $j$  and  $i$  the problem becomes a symmetric TSP (STSP). If these distances are different, the problem becomes more general and is called asymmetric TSP (ATSP).

Furthermore, precedence constraints among cities or nodes, introduce the sequential ordering problem (SOP). According to Escudero [23], it consists of finding a minimum weight Hamiltonian cycle (tour) on a directed graph with weights on the arcs and the nodes, subject to precedence constraints among nodes, where nodes can be considered as cities or jobs to be completed. In other words, the objective is to find a city or job sequence that minimizes the total tour or makes pan, subject to the precedence constraints.

Another family of problems originating from TSP is the asymmetric traveling salesman problems with time windows (ATSPTW). In ATSPTW problems the arcs of the graph correspond to job transitions that is, the set-up times needed to start processing a job (node) after the previous job has been completed. For every job, the following information is given: a processing time  $p$ , an earliest time  $r$  and a latest time  $d$  to start processing the job. The interval  $[r, d]$  is called time window. The problem is to find the minimum cost path that visits all nodes and satisfies the time windows restrictions, i.e., a node sequence with minimal total cost

such that, for every node the starting time lies within the time window [6,22].

The TSP has practical applications and is representative of a large class of important scientific and engineering problems, like VLSI routing, vehicle routing, mixed Chinese postman problems, integrated circuit boardchip insertion problems [11,15], workshop scheduling and computer wiring [16,23], etc. However, asymmetric models are relevant to a wider range of applications and are more general than symmetric models. Instances of such problems are the tilted drilling machine problem encountered by Johnson et al. [17], pay phone collection problems and no-wait flow shop problems [23,19], vehicle routing (distribution of goods and services), robotic motion planning, tape drive reading, computer wiring [15], code optimization, table compression [17,18] and ATS problem of routing in circuit switched telecommunication networks [10].

Variations of ATSP also cope with a large range of applications: Multi-ATSP [15] is used in scheduling two (non-identical) machines with sequence dependent set-up times and also in hot rolling production scheduling [14]. Schneider [12] expanded the ATSP to the time dependent traveling salesman problem, where distances between cities vary with time. ATSP with knapsack-like constraints on subpaths of the tour is a problem that arises in routing aircraft problem [12,22]. An  $m$ -period TSP solved by Paletta [17] is a traveling salesman problem where the salesman must visit each city a fixed number of times over a given  $m$ -day planning period.

Finally the Disk Scheduling Problem is related to the special case of the asymmetric traveling salesman problem with the triangle inequality (ATSP-n) in which all distances are either 0 or equal to some constant  $x$  [4,17].

The SOP problem occurs as a basic model in many industrial problems, such as in scheduling and routing decision [22,7,8]. It has found a wide range of applications: Timlin [17] applied it to helicopter routing, while Ascheuer [9] and Abdel- Hamid et al. [2] used it in stacker crane routing in an automatic storage system where the aim is to minimize the time needed for the unloading moves. Timlin [17] and Timlin and Pulleyblank [18] developed symmetric TSP models for minimizing the total distance on a daily

helicopter’s set of stops at oil-platforms, satisfying precedence constraints and helicopter capacity.

ATSPTW can also describe accurately many real life scheduling problems. Ascheuer et al. [6] simulated the control of a stacker crane in a warehouse with an ATSPTW. Applegate and Cook [5] related the ATSP-TW to the job-shop scheduling problem (JSSP) where just one of the machines is taken under consideration. Fagerholta and Christiansen [14] imposed on ATSP both precedence constrains and time windows (ATSP TW PC) and aimed at optimally sequencing a given set of port visits in a real bulk ship scheduling problem. Since the applications of ATSPs spread on such a wide range of problems, it is obvious that there is a great interest for developing efficient and extensive algorithms for solving them unobjectionably and unexceptionably. Although much work has been done for solving TSPs, disproportional work has been carried out for ATSPs despite their significant applicability on nonacademic practical problems.

**II. DESCRIPTION OF THE ALGORITHM**

**A. The threshold accepting method:**

Combined with gradually reinforced local search The basic idea of the threshold accepting algorithm is quite simple and similar to the one used in the simulated annealing algorithm. As mentioned in Section 2, in contrast to SA, TA does not require the generation of random numbers and exponential functions. Assuming that  $X^*$  is the set of all feasible solutions of the problem, TA starts with an element  $x_0 \in X^*$ , which may be randomly chosen. Then, the method proceeds in an iterative manner. In each iteration the algorithm decides if the current solution  $x_c$  will be replaced by a new one  $x_n$ . The new candidate is chosen (by use of local search moves) as a small perturbation of the current solution or—speaking in mathematical terms—in a given neighborhood of the current solution  $x_c$ . The value of the objective function is calculated for the new candidate and the results are compared:  $Df = f(x_n) - f(x_c)$ .

Up to this point, the procedure is similar to the one of a standard or trivial local search algorithm. The decision rule in a standard algorithm is to accept  $x_n$  as the new current solution if and only if  $Df \leq 0$  (assuming a minimization problem). If the number of iterations is large enough, the algorithm will end up in a local minimum with certainty. In general, the quality of the local minimum will be low, i.e., the difference from the global minimum will be large. Applications of the trivial local search algorithm to traveling salesman problems show differences in the order of magnitude of 10%. A further increase in the number of iterations cannot improve the quality of the results.

**B. Representation of the solution:**

To our knowledge, in any known heuristic algorithm that has been proposed for solving ATSPs or their variances, the current solution is an integrated schedule of the cities (jobs) to be visited (to be scheduled). In this work we consider as current solution a vector  $x \in X^*$ , where  $X^*$  is the set of all possible permutations of the cities. In a specific permutation  $x$ , the position of each city denotes the order in which it will be inserted in the schedule. Thus, for the rest of the paper, the name insertion order will be used for the sequence  $x$ , while the corresponding schedule will be denoted by  $S$ . This way the searching operations of the algorithm are imposed

on the unscheduled string  $x$ ,  $x$  in turn is translated into a schedule  $S$  and the solution  $x$  is surveyed right after the value of the respective cost has been calculated.

The proposed methods of insertion, guarantee that for a given insertion order the produced schedule is feasible and optimum for the two first groups of problems of problems, namely ATSPs.

**III. OPTIMIZATION STRATEGIES**

**A. Local optimization:**

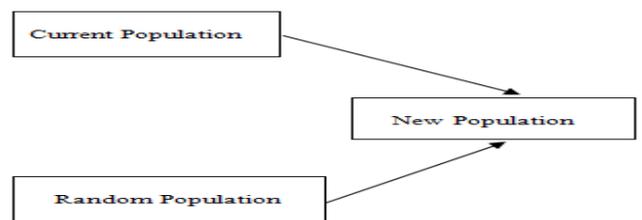
In this part, two local optimization approaches, the local determinate optimization (LDO) and the local stochastic optimization (LSO) are created to enhance the local optimization capability of our hybrid approach. The mechanism of these two regulations is shown in Figs. 6 and 7. The complexity of LDO and LSO is decreased by executing them only every several generations.

**B. Global optimization:**

In this part, the GO is established to improve the GO capability and convergence speed of our hybrid approach. The mechanism of the proposed GO is displayed as Fig. 8. The complexity of GO is decreased by executing it only every several generations.

**IV. EVALUATION AND COMPARATIVE STUDIES**

In this section, the proposed approach is compared with other published algorithms on sixteen ATSP instances for the effectiveness evaluation. The termination criteria for our approach can be listed as follows: (1) the known optimal solution is achieved, (2) maximal iterative generation (MG) is exhausted and (3) the global best solution from the beginning trials is not improved in the successive (SG) generations. When one of the termination criteria is satisfied, our approach stops and provides the coordinates of the located point, and the objective



The distance matrix of an ATSP instance is listed as follows.

	A	B	C	D
A	0	3	4	5
B	2	0	3	6
C	8	4	0	5
D	9	1	3	0

(1) Select an individual from current population randomly, such as,  $C \rightarrow B \rightarrow A \rightarrow D$ .

(2) Arrange the first two cities  $C$  and  $B$  according to distances, for example, the distance of  $B \rightarrow C$  is 3, and the distance of  $C \rightarrow B$  is 4, so the new partial solution is  $B \rightarrow C$ .

Potential partial solution	Total Distance
$A \rightarrow B \rightarrow C$	7
$B \rightarrow A \rightarrow C$	6
$B \rightarrow C \rightarrow A$	11

so the new partial solution is selected as,  $B \rightarrow A \rightarrow C$ .

(4) Insert fourth city  $D$  into the obtained partial solution according to distances, such as,

Potential partial solution	Total Distance
$D \rightarrow B \rightarrow A \rightarrow C$	7
$B \rightarrow D \rightarrow A \rightarrow C$	19
$B \rightarrow A \rightarrow D \rightarrow C$	10
$B \rightarrow A \rightarrow C \rightarrow D$	11

so the new partial solution is selected as,  $D \rightarrow B \rightarrow A \rightarrow C$ .

(5) The obtained tour is improved after the LDO. The existing tour is  $C \rightarrow B \rightarrow A \rightarrow D$ , and its length is 15. The improved tour is  $D \rightarrow B \rightarrow A \rightarrow C$ , and its length is 12.

Figure. 6. The mechanism of the local determinate optimization.

The distance matrix of an ATSP instance is listed as follows.

	A	B	C	D
A	0	3	4	5
B	2	0	3	6
C	8	4	0	5
D	9	1	3	0

(1) Select an individual from current population randomly, such as,  
 $C \rightarrow B \rightarrow A \rightarrow D$ .

(2) Arrange the first two cities  $C$  and  $B$  according to distances, for example, the distance of  $B \rightarrow C$  is 3, and the distance of  $C \rightarrow B$  is 4, so the new partial solution is

$$B \rightarrow C.$$

(3) Insert third city  $A$  into the obtained partial solution according to distances, such as,

Potential partial solution	Total Distance
$A \rightarrow B \rightarrow C$	7
$B \rightarrow A \rightarrow C$	6
$B \rightarrow C \rightarrow A$	11

so the new partial solution is selected as,

$$B \rightarrow A \rightarrow C.$$

(4) Insert fourth city  $D$  into the obtained partial solution according to distances, such as,

Potential partial solution	Total Distance
$D \rightarrow B \rightarrow A \rightarrow C$	7
$B \rightarrow D \rightarrow A \rightarrow C$	19
$B \rightarrow A \rightarrow D \rightarrow C$	10
$B \rightarrow A \rightarrow C \rightarrow D$	11

so the new partial solution is selected as,

$$D \rightarrow B \rightarrow A \rightarrow C.$$

(5) The obtained tour is improved after the LDO.

The existing tour is  $C \rightarrow B \rightarrow A \rightarrow D$ , and its length is 15.

The improved tour is  $D \rightarrow B \rightarrow A \rightarrow C$ , and its length is 12.

Figure 7. The mechanism of the local stochastic optimization.

Value at this point. In this paper, the global optimal value of all ATSP instances is pre-obtained by Buriioletal.

The distance matrix of an ATSP instance is listed as follows.

	A	B	C	D	E
A	0	3	4	5	6
B	2	0	3	6	4
C	8	4	0	5	2
D	9	1	3	0	8
E	2	8	7	3	0

(1) Select an individual from current population randomly, i.e.,

$$C \rightarrow A \rightarrow E \rightarrow B \rightarrow D.$$

(2) Generate two operational points randomly, such as, operational point 1 is position 1 and operational point 2 is position 5.

(3) Find a near-optimal solution of the sub-problem which consists of cities between two operational points by using IGA. For example, its near-optimal solution is,

$$A \rightarrow B \rightarrow E.$$

(4) Apply the solution obtained from step (3) to replace the corresponding positions of the existing solution. After this step, the new improved solution is,

$$C \rightarrow A \rightarrow B \rightarrow E \rightarrow D.$$

(5) The obtained tour is improved after the GO.

The existing tour is  $C \rightarrow A \rightarrow E \rightarrow B \rightarrow D$ , and its length is 31.

The improved tour is  $C \rightarrow A \rightarrow B \rightarrow E \rightarrow D$ , and its length is 21.

Figure 8. The mechanism of global optimization.

## V. CONCLUSIONS

The contributions of this paper can be summarized as follows. A hybrid approach that combines an IGA and optimization strategies was presented for solving the ATSP. Both the crossover operation and the mutation operation in this IGA were enhanced by selecting the optimum from a set of solutions. At the same time, three regulations (immigration, local optimization and GO) were established based on several empirical optimization strategies to enhance the evolution of the IGA. The comparative study shows that the proposed approach outperforms several other published algorithms.

In the work presented in this paper, the design of the abovementioned operators and regulations to some extent minimizes the influence of some parameters on optimization performance. But, the design of an appropriate parameter combination for finding the optimal solution quickly should be studied in the future. That is, 'parameter adaptation techniques' have to be considered for future development to make the proposed approach self-adaptive.

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