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# Nonlinear Observer Design for the Undamped Oscillator

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*Abstract:* This paper investigates the nonlinear observer design for the undamped oscillator. Explicitly, Sundarapandian's theorem (2002) for observer design for nonlinear systems is used to solve the problem of local exponential observer design for the undamped oscillator. In this paper, we derive results for exponential observer design for the undamped oscillator. Numerical examples and simulations of nonlinear observer design for undamped oscillator are shown to illustrate the results and validate the proposed observer design for the undamped oscillator.

*Keywords:* Undamped Oscillator, Observer Design, Nonlinear Observers, Exponential Observers, Stability, Nonlinear Systems.

## I. INTRODUCTION

In the control systems design, it is often necessary to construct estimates of state variables, which are not available for direct measurement. In such cases, the state vector of the control system can be approximately reconstructed by building an observer which is driven by the available outputs and inputs of the original control system. Local observer design for nonlinear control systems is one of the central problems in the control systems literature.

The problem of designing observers for linear control systems was first introduced by Luenberger ([1], 1966) and that for nonlinear control systems was proposed by Thau ([2], 1973). Over the past three decades, significant attention has been paid in the control systems literature to the construction of observers for nonlinear control systems.

A necessary condition for the existence of an exponential observer for nonlinear control systems was obtained by Xia and Gao ([3], 1988). Explicitly, in [3], Xia and Gao showed that an exponential observer exists for the nonlinear system only if the linearization of the nonlinear system is detectable.

On the other hand, sufficient conditions for nonlinear observers have been obtained in the control systems literature from an impressive variety of points of view. Kou, Elliott and Tarn ([4], 1975) obtained conditions for the existence of exponential observers using Lyapunov-like method. In ([5]-[10]), suitable coordinate transformations were found under which a nonlinear control system is transferred into a canonical form, where the observer design is carried out. In [11], Kazantzis and Kravaris obtained results on nonlinear observer design using Lyapunov auxiliary theorem. In ([12]-[13]), Tsinias derived sufficient Lyapunov-like conditions for the existence of asymptotic observers for nonlinear systems. A harmonic analysis approach was proposed by Celle *et al.* ([14], 1989) for the synthesis of nonlinear observers.

Necessary and sufficient conditions for the existence of local exponential observers for nonlinear control systems were obtained using differential geometric techniques by Sundarapandian ([15], 2002). Krener and Kang ([16], 2003) introduced a new method for the design of observers for nonlinear systems using backstepping.

In this paper, we shall use Sundarapandian's theorem (2002) for observer design for nonlinear systems to solve the problem of designing observers for the undamped oscillator, which is an important model of stable systems in mechanical engineering.

This paper is organized as follows. Section II reviews the definition of nonlinear observers and the results of observability and observers. Section III details the stability result and examples for the undamped oscillator. Section IV details the design of nonlinear observers for the undamped oscillator. Numerical examples and simulations of nonlinear observer design for the undamped oscillator are also contained in this section. Finally, Section V provides the conclusions of this paper.

## II. REVIEW OF OBSERVERS FOR NONLINEAR SYSTEMS

By the concept of a *state observer* or *state estimator* for a nonlinear system, it is meant that from the observation of certain states of the system considered as outputs or indicators, it is desired to estimate the state of the whole system as a function of time. Mathematically, observers for nonlinear systems are defined as follows.

Consider the nonlinear system described by

$$\dot{x} = f(x) \qquad (1a)$$

$$y = h(x) \qquad (1b)$$

where $x \in R^n$ is the state and $y \in R^p$ the output. It is assumed that $f: R^n \to R^n$, $h: R^n \to R^p$ are $C^1$ maps and for some $x^* \in R^n$, the following hold:

$$f(x^*) = 0, \; h(x^*) = 0.$$

Note that the solutions $x^*$ of the equation $f(x) = 0$ are called the *equilibrium points* of (1a).

**Definition 1.** The nonlinear system (1) is called **locally observable** at the equilibrium $x^*$ over a given time interval $[0, T]$, if there exists $\varepsilon > 0$ such that for any two different solutions $x(t)$ and $\overline{x}(t)$ of the system (1a) with

$$| x(t) - x^* | < \varepsilon \text{ and } | \bar{x}(t) - x^* | < \varepsilon \text{ for } t \in [0, T],$$

the observed functions $h \circ x$ and $h \circ \bar{x}$ are different, *i.e.* there exists some $\tau \in [0, T]$ such that

$$(h \circ x)(\tau) \neq (h \circ \bar{x})(\tau).$$

For the formulation of a sufficient condition for local observability of the nonlinear system (1), consider the linearization of (1) at the equilibrium $x^*$ given by

$$\dot{x} = Ax \tag{2a}$$

$$y = Cx \tag{2b}$$

where

$$A = \left[ \frac{\partial f}{\partial x} \right]_{x=x^*} \quad \text{and} \quad C = \left[ \frac{\partial h}{\partial x} \right]_{x=x^*}.$$

**Theorem 1.** (Lee and Markus, [17], 1971)

If the observability matrix for the linear system (2) given by

$$O(C, A) = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

has rank $n$, then the nonlinear system (1) is locally observable at $x^*$.

**Definition 2.** (**Hurwitz Matrices**)

An $n \times n$ matrix $A$ is called **Hurwitz** if all eigenvalues of $A$ have negative real parts.

Next, the definition of nonlinear observers for the given nonlinear system (1) is given. Basically, an observer for a nonlinear system is a state estimator.

**Definition 3.** (Sundarapandian, [15], 2002)

A $C^1$ dynamical system described by

$$\dot{z} = g(z, y), \qquad (z \in R^n) \tag{3}$$

is a local asymptotic (respectively, exponential) observer for the nonlinear system (1) if the composite system (1) and (3) satisfies the following two requirements:

(i) If $z(0) = x(0)$, then $z(t) = x(t), \quad \forall t \geq 0.$

(ii) There exists a neighbourhood $V$ of the equilibrium $x^*$ of $R^n$ such that for all $z(0), x(0) \in V$, the error $e(t) = z(t) - x(t)$ decays asymptotically (resp. , exponentially) to zero.

**Theorem 2.** (Sundarapandian, [15], 2002)

Suppose that the nonlinear system (1) is Lyapunov stable at the equilibrium $x^*$ and that there exists a matrix $K$ such that $A - KC$ is Hurwitz. Then the dynamical system defined by

$$\dot{z} = f(z) + K[y - h(z)] \tag{4}$$

is a local exponential observer for the nonlinear system (1).

**Remark 1.** If the estimation error $e$ is defined as

$$e = z - x,$$

then the estimation error is governed by the dynamics

$$\dot{e} = f(x+e) - f(x) - K[h(x+e) - h(x)] \tag{5}$$

Linearizing the error dynamics (5) at $x^*$, we obtain the linear system

$$\dot{e} = Ee, \qquad \text{where } E = A - KC. \tag{6}$$

If $(C, A)$ is observable, ie. if the observability matrix $O(C, A)$ has full rank, then the eigenvalues of $E = A - KC$ can be arbitrarily assigned in the complex plane. Since the linearization of the error dynamics (5) is governed by the system matrix $E = A - KC$, it follows that when $(C, A)$ is observable, then a local exponential observer of the form (4) can be always found so that the transient response of the error decays quickly with any desired speed of convergence.

## III. Stability Result and Examples for the Undamped Oscillator

In this section, we discuss the model and stability result for the undamped oscillator [18], which is a classical example of a stable system in Mechanical Engineering.

The undamped oscillator is described by the second-order differential equation

$$\ddot{u} + \varphi(u) = 0 \tag{7}$$

where $u$ is the displacement of a moving object. Throughout this paper, we shall assume that the function $\varphi$ is continuously differentiable on $-\infty < u < \infty$ and that $\varphi$ satisfies

$$u\ \varphi(u) > 0 \text{ for } u \neq 0. \tag{8}$$

For our analysis, it is convenient to express the second-order differential equation (7) as a system of two differential equations. This is carried out by defining the phase variables

$$\begin{aligned} x_1 &= u \\ x_2 &= \dot{u} \end{aligned} \tag{9}$$

Note that (7) is equivalent to the system of differential equations given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\varphi(x_1) \end{aligned} \tag{10}$$

Next, we shall prove the following result, which is well-known in Lyapunov stability theory [18].

**Theorem 3.** [18] The system (10) has a Lyapunov stable equilibrium at $x = 0$. Moreover, the solution curves of (10) are simple closed orbits in the $(x_1, x_2)$ plane.

**Proof.** The total energy of the moving object is given by

$$V(x_1, x_2) = \int_0^{x_1} \varphi(\tau) d\tau + \frac{1}{2} x_2^2 \tag{11}$$

We shall establish the Lyapunov stability of the equilibrium $x = 0$ by showing that $V$ is a Lyapunov function for the system (10).

First, we note that $V$ is a positive definite function on $R^2$.

Next, differentiating $V$ along the trajectories of (10), we obtain

$$\dot{V} = \varphi(x_1)\dot{x}_1 + x_2\dot{x}_2 = \varphi(x_1)x_2 - x_2\varphi(x_1) \equiv 0 \quad (12)$$

which shows that $\dot{V}$ is a negative semi-definite function on $R^2$. Thus, by Lyapunov stability theory [18], it follows that $x = 0$ is a Lyapunov stable equilibrium of the system (10).

Integrating Eq. (12), it is immediate that

$$V(x_1(t), x_2(t)) = V(x_1^0, x_2^0) \equiv \text{constant} \qquad (13)$$

which shows that the solution curves of (10) are simple closed orbits in the $(x_1, x_2)$ plane.

**Example 1. (Undamped Pendulum)**

If we take

$$\varphi(u) = \frac{g}{L}\sin u, \qquad (14)$$

then we note that

$$u\ \varphi(u) = \frac{g}{L} u \sin u > 0 \ \text{ for } \ u \neq 0, u \in (-\pi, \pi). \ (15)$$

Thus, the undamped pendulum is an undamped oscillator and is described by the nonlinear dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{g}{L}\sin x_1 \end{aligned} \qquad (16)$$

By Theorem 3, it is immediate that $x = 0$ is a Lyapunov stable equilibrium of the system (16).

Note also that the energy function for the undamped pendulum is given by

$$V(x_1, x_2) = \int_0^{x_1} \varphi(\tau) d\tau + \frac{1}{2} x_2^2$$

*i.e.*

$$V(x_1, x_2) = \int_0^{x_1} \frac{g}{L} \sin\tau \ d\tau + \frac{1}{2} x_2^2$$

*i.e.*

$$V(x_1, x_2) = \frac{g}{L}(1 - \cos x_1) + \frac{1}{2} x_2^2 \qquad (17)$$

Note that $V$ is a positive definite function on $\boldsymbol{R}^2$.

Thus, the integral curves of (16) are simple closed orbits in the $(x_1, x_2)$ plane described by

$$V(x_1, x_2) \equiv c$$

*i.e.*

$$\frac{g}{L}(1 - \cos x_1) + \frac{1}{2} x_2^2 = c$$

For numerical simulation, we take $L = 2g$..

The state orbits of the undamped pendulum are depicted in Figure 1.

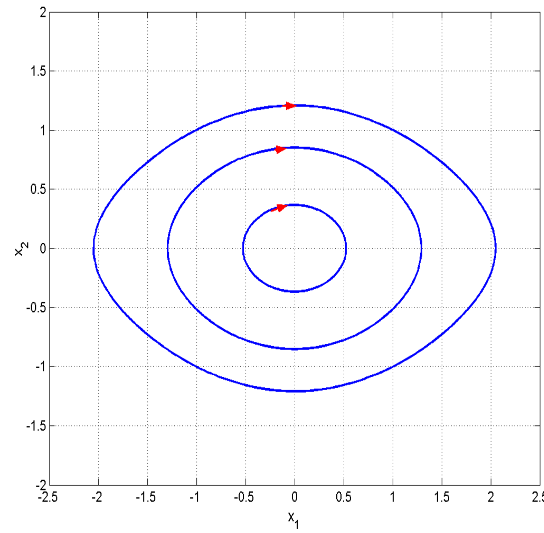


Figure 1. State Orbits of the Undamped Pendulum (16)

**Example 2 (Simple Harmonic Oscillator)**

If we take

$$\varphi(u) = ku, \quad (k > 0) \qquad (18)$$

then we note that

$$u\ \varphi(u) = k\ u^2 > 0 \ \text{ for } \ u \neq 0\ . \qquad (19)$$

To simplify the notation, we take $k = \omega^2$.

Thus, the simple harmonic motion is an undamped oscillator and is described by the linear dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\omega^2 x_1 \end{aligned} \qquad (20)$$

By Theorem 3, it is immediate that $x = 0$ is a Lyapunov stable equilibrium of the simple harmonic oscillator (20).

Note also that the energy function for the simple harmonic oscillator is given by

$$V(x_1, x_2) = \int_0^{x_1} \varphi(\tau) d\tau + \frac{1}{2} x_2^2$$

*i.e.*

$$V(x_1, x_2) = \int_0^{x_1} \omega^2 \tau d\tau + \frac{1}{2} x_2^2 = \frac{1}{2}\omega^2 x_1^2 + \frac{1}{2} x_2^2 \quad (21)$$

Thus, the integral curves of (20) are simple closed orbits in the $(x_1, x_2)$ plane described by

$$V(x_1, x_2) \equiv c$$

*i.e.*

$$\omega^2 x_1^2 + x_2^2 = \alpha^2$$

which are ellipses in the $(x_1, x_2)$ plane.

The state orbits of the simple harmonic oscillator are depicted in Figure 2. (For simulation, we take $\omega = 1/2$).

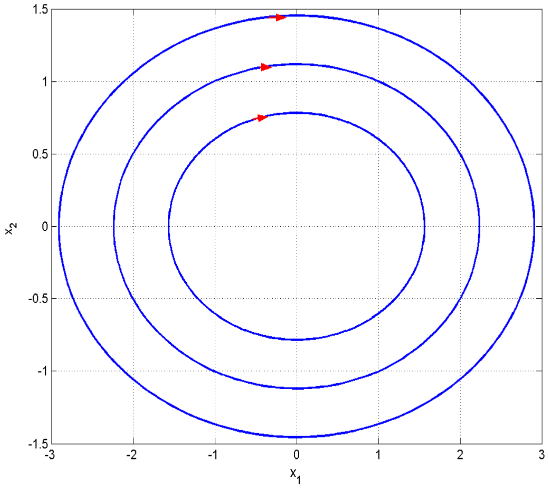


Figure 2. State Orbits of the Harmonic Oscillator (20)

**Example 3**

If we take

$$\varphi(u) = k\ u\ \left(a^2 - u^2\right),\ \ (k, a > 0) \qquad (22)$$

then we note that

$$u\varphi(u) = k\ u^2\ \left(a^2 - u^2\right) > 0 \ \text{ for } \ |u| < a, u \neq 0.$$

Thus, we have an undamped oscillator described by the nonlinear dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -kx_1\left(a^2 - x_1^2\right) \end{aligned} \qquad (23)$$

By Theorem 3, it is immediate that $x = 0$ is a Lyapunov stable equilibrium of the undamped oscillator (23).

Note also that the energy function for the simple harmonic oscillator is given by

$$V(x_1, x_2) = \int_0^{x_1} \varphi(\tau)d\tau + \frac{1}{2}x_2^2$$

*i.e.*

$$V(x_1, x_2) = \frac{k}{4}x_1^2\left(2a^2 - x_1^2\right) + \frac{1}{2}x_2^2 \qquad (24)$$

Thus, the integral curves of (20) are simple closed orbits in the $(x_1, x_2)$ plane described by

$$V(x_1, x_2) \equiv c$$

The state orbits of the undamped oscillator are depicted in Figure 3. (For simulation, we take $k = 1, a = 3.$ ).

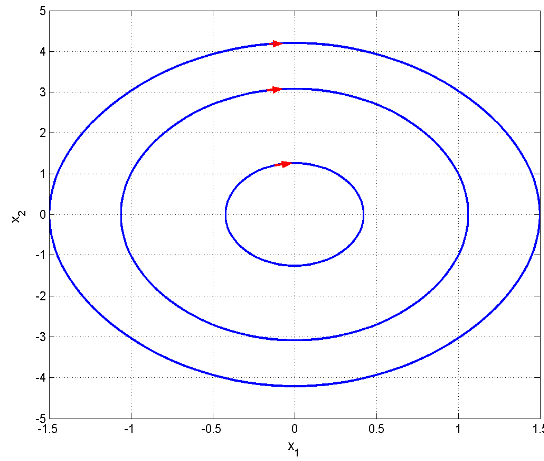


Figure 3. State Orbits of the Undamped Oscillator (23)

## III. NONLINEAR OBSERVER DESIGN FOR THE UNDAMPED OSCILLATOR

In this section, we discuss the nonlinear observer design for the undamped oscillator [18], which is a classical example of a stable system in Mechanical Engineering.

The undamped oscillator is described by the dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\varphi(x_1) \end{aligned} \qquad (25)$$

where the function $\varphi : \boldsymbol{R} \to \boldsymbol{R}$ is continuously differentiable and satisfies the assumption that

$$u\ \varphi(u) > 0 \text{ for } u \neq 0. \qquad (26)$$

Suppose that the displacement $u$ is available for measurement, i.e. the output function for the undamped oscillator (25) is given by

$$y = x_1 \qquad (27)$$

By Theorem 1, the undamped oscillator (25) is Lyapunov stable about the equilibrium $x = 0.$

Thus, we can apply Sundarapandian's theorem (2002) to construct nonlinear observers for the undamped oscillator given by (25).

Linearizing the undamped oscillator (25) and the output function (27) at $x = 0,$ we obtain the system matrices

$$A = \begin{bmatrix} 0 & 1 \\ -\alpha & 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

where $\alpha = \dot{\varphi}(0).$

Thus, the observability matrix is obtained as

$$\boldsymbol{O}(C, A) = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

which has full rank. Thus, by Kalman's rank condition [19], the pair $(C, A)$ is observable.

Thus, we can always find a gain matrix $K$ such that the eigenvalues of the error matrix $E = A - KC$ is Hurwitz.

Hence, by Theorem 2 (Sundarapandian, 2000), we obtain the following result.

**Theorem 4.** A local exponential observer for the undamped oscillator (25) is described by the dynamics

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_2 \\ -\varphi(z_1) \end{bmatrix} + K\left[y - z_1\right] \qquad (28)$$

where $K$ is a gain matrix chosen so that $A - KC$ is Hurwitz. Since $(C, A)$ is observable, a gain matrix $K$ can be found so that the error matrix $E = A - KC$ has arbitrarily assigned set of eigenvalues with negative real parts.

**Example 4**. Here, we describe the construction of local exponential observer for the undamped pendulum described in Example 1 with $L = 2g$.

Thus, we consider the nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{1}{2}\sin x_1 \\ y &= x_1 \end{aligned} \qquad (29)$$

The nonlinear system (29) has the linearization pair

$$A = \begin{bmatrix} 0 & 1 \\ -1/2 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Clearly, the pair $(C, A)$ is observable.

Using the Ackermann formula for the observer gain matrix ([20], p.822), we can choose the gain matrix $K$ so that the error matrix $E = A - KC$ has the eigenvalues $\{-2, -2\}$.

A simple calculation using MATLAB yields

$$K = \begin{bmatrix} 4.0 \\ 3.5 \end{bmatrix}.$$

By Theorem 4, a local exponential observer for the undamped pendulum (29) near the equilibrium $x = 0$ is given by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_2 \\ -\frac{1}{2}\sin z_1 \end{bmatrix} + \begin{bmatrix} 4.0 \\ 3.5 \end{bmatrix}\left[y - z_1\right] \qquad (30)$$

If we define the estimation error as

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} z_1 - x_1 \\ z_2 - x_2 \end{bmatrix},$$

then $e_1(t) \to 0$ and $e_2(t) \to 0$ exponentially as $t \to \infty$.

For simulation, we take the initial conditions as

$$x(0) = \begin{bmatrix} 1.8 \\ 1.5 \end{bmatrix} \quad \text{and} \quad z(0) = \begin{bmatrix} 2.5 \\ 0.5 \end{bmatrix}.$$

Figure 4 depicts the exponential convergence of the error trajectories for the observer design of the pendulum (29).

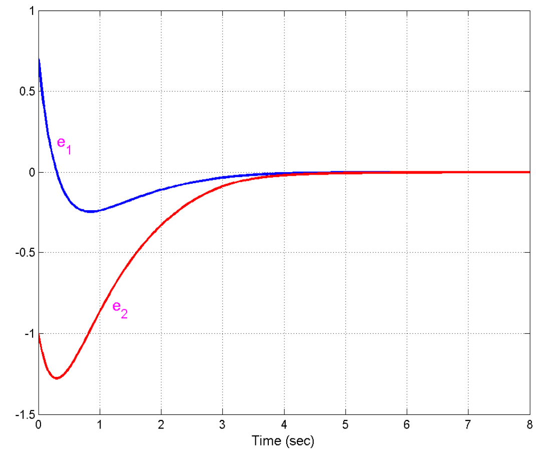


Figure 4. Observer for the Undamped Pendulum (29)

**Example 5**. Here, we describe the construction of local exponential observer for the simple harmonic motion described in Example 2 with $\omega = 1/2$.

Thus, we consider the linear harmonic system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -0.25 x_1 \\ y &= x_1 \end{aligned} \qquad (31)$$

The linear system (31) has the system matrices given by

$$A = \begin{bmatrix} 0 & 1 \\ -0.25 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Clearly, the pair $(C, A)$ is observable.

Using the Ackermann formula for the observer gain matrix ([20], p.822), we can choose the gain matrix $K$ so that the error matrix $E = A - KC$ has the eigenvalues $\{-2, -2\}$.

A simple calculation using MATLAB yields

$$K = \begin{bmatrix} 4.00 \\ 3.75 \end{bmatrix}.$$

By Theorem 4, a local exponential observer for the undamped pendulum (31) near the equilibrium $x = 0$ is given by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_2 \\ -0.25 z_1 \end{bmatrix} + \begin{bmatrix} 4.00 \\ 3.75 \end{bmatrix}\left[y - z_1\right] \qquad (32)$$

If we define the estimation error as

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} z_1 - x_1 \\ z_2 - x_2 \end{bmatrix},$$

then $e_1(t) \to 0$ and $e_2(t) \to 0$ exponentially as $t \to \infty$.

For simulation, we take the initial conditions as

$$x(0) = \begin{bmatrix} 2.3 \\ 1.9 \end{bmatrix} \quad \text{and} \quad z(0) = \begin{bmatrix} 0.2 \\ 3.5 \end{bmatrix}.$$

Figure 5 depicts the exponential convergence of the error trajectories for the observer design of the system (31).

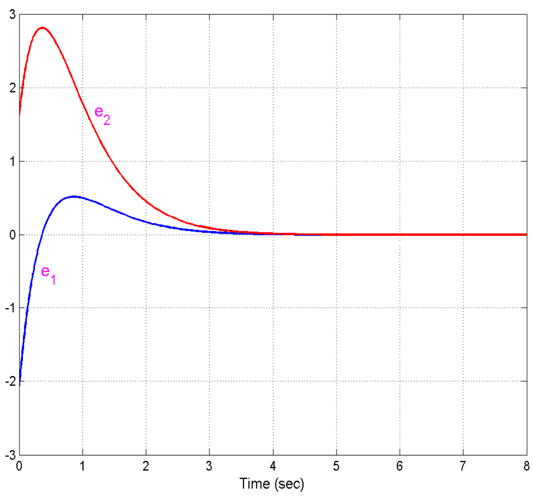


Figure 5. Observer for the Undamped System (31)

**Example 6**. Here, we describe the construction of local exponential observer for the undamped oscillator described in Example 3 with $k = 1, a = 3.$

Thus, we consider the nonlinear undamped oscillator

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1(9 - x_1^2) \\ y &= x_1 \end{aligned} \tag{33}$$

The nonlinear system (33) has the linearization matrices

$$A = \begin{bmatrix} 0 & 1 \\ -9 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Clearly, the pair $(C, A)$ is observable.

Using the Ackermann formula for the observer gain matrix ([20], p.822), we can choose the gain matrix $K$ so that the error matrix $E = A - KC$ has the eigenvalues $\{-2, -2\}$.

A simple calculation using MATLAB yields

$$K = \begin{bmatrix} 4 \\ -5 \end{bmatrix}.$$

By Theorem 4, a local exponential observer for the undamped oscillator (33) near the equilibrium $x = 0$ is given by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} z_2 \\ -z_1(9 - z_1^2) \end{bmatrix} + \begin{bmatrix} 4 \\ -5 \end{bmatrix} [y - z_1]. \tag{34}$$

If we define the estimation error as

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} z_1 - x_1 \\ z_2 - x_2 \end{bmatrix},$$

then $e_1(t) \to 0$ and $e_2(t) \to 0$ exponentially as $t \to \infty$.

For simulation, we take the initial conditions as

$$x(0) = \begin{bmatrix} 0.4 \\ 1.1 \end{bmatrix} \quad \text{and} \quad z(0) = \begin{bmatrix} 1.2 \\ 0.6 \end{bmatrix}.$$

Figure 5 depicts the exponential convergence of the error trajectories for the observer design of the system (33).

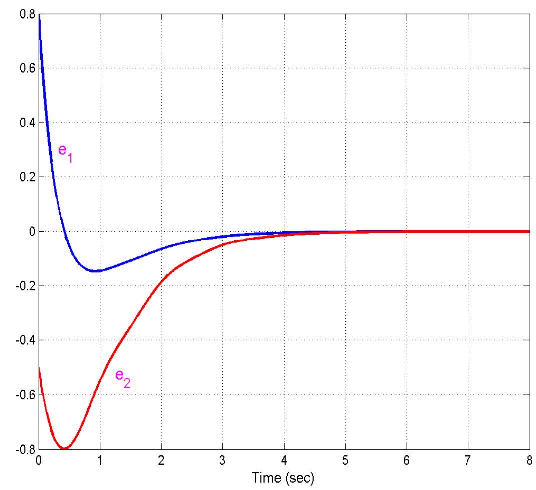


Figure 6. Observer for the Undamped Oscillator (33)

## IV. Conclusions

For many real problems of science and engineering, undamped oscillator is a classical mechanical system which is stable in the sense of Lyapunov. It has important applications in several stability problems arising in Mechanic. In this paper, we first established a stability result for the undamped oscillator using the concept of energy function and Lyapunov stability theory. Explicitly, we showed that the undamped oscillator has a Lyapunov stable equilibrium at the origin and that all its integral curves are simple closed orbits in the plane. Next, we applied Sundarapandian's theorem (2002) on nonlinear observer design to construct local exponential observers for the undamped oscillator. We had explained the stability result and observer construction using lucid examples of (i) simple pendulum, (ii) simple harmonic motion and (iii) a simple undamped oscillator. Numerical examples have been worked out in detail for all the three special cases of undamped oscillator system.

## V. References

[1] D.G. Luenberger, "Observrers for multivariable linear systems", IEEE Transactions on Automatic Control, AC-2, pp. 190-197, 1966.

[2] F.E. Thau, "Observing the states of nonlinear dynamical systems", International J. Control, vol. 18, pp. 471-479, 1973.

[3] X.H. Xia and W.B. Gao, "On exponential observers for nonlinear systems", Systems and Control Letters, vol. 11, pp. 319-325, 1988.

[4] S.R. Kou, D.L. Elliott and T.J. Tarn, "Exponential observers for nonlinear dynamical systems", Inform. Control, vol. 29, pp. 204-216, 1975.

[5] D. Bestle and M. Zeitz, "Canonical form observer design for nonlinear time-varying systems", International J. Control, vol. 38, pp. 419-431, 1983.

[6] A.J. Krener and A. Isidori, "Linearization by output injection and nonlinear observers", Systems and Control Letters, vol. 3, pp. 47-52, 1983.

[7] A.J. Krener and W. Respondek, "Nonlinear observers with linearizable error dynamics", SIAM J. Control and Optimization, vol. 23, 197-216, 1985.

[8] X.H. Xia and W.B. Gao, "Nonlinear observer design by canonical form", International J. Control, vol. 47, pp. 1081-1100, 1988.

[9] J.P. Gauthier, H. Hammouri and I. Kupka, "Observers for nonlinear systems", Proceedings of the 30th IEEE Conference on Decision and Control, vol. 2, pp. 1483-1489, 1991.

[10] J.P. Gauthier, H. Hammouri and S. Othman, "A simple observer for nonlinear systems – Applications to bioreactors", IEEE Transactions on Automatic Control, vol. 37, pp. 875-880, 1992.

[11] N. Kazantzis and C. Kravaris, "Nonlinear observer design using Lyapunov auxiliary theorem", Systems and Control Letters, vo. 34, pp. 241-247, 1998.

[12] J. Tsinias, "Observer design for nonlinear systems", Systems and Control Letters, vol. 13, 135-142, 1989.

[13] J. Tsinias, "Further results on the observer design problem", Systems and Control Letters, vol. 14, pp. 411-418, 1990.

[14] F. Celle, J.P. Gauthier, D. Kazakos and G. Salle, "Synthesis of nonlinear observers: A harmonic analysis approach", Math. Systems Theory, vol. 22, pp. 291-322, 1989.

[15] V. Sundarapandian, "Local observer design for nonlinear systems", Mathematical and Systems Theory, vol. 35, pp. 25-36, 2002.

[16] A.J. Krener and W. Kang, "Locally convergent nonlinear observers", SIAM J. Control Optimization, vol. 42, pp. 155-177, 2003.

[17] E.B. Lee and L. Markus, Foundations of Optimal Control Theory, New York: Wiley, 1971.

[18] F. Brauer and J.A. Nohel, Qualitative Theory of Differential Equations, New York: Dover Publications, 1969.

[19] I.J. Nagrath and M. Gopal, Control Systems Engineering, 3rd edition, New Delhi: New Age International Ltd., 2002.

[20] K. Ogata, Modern Control Engineering, 3rd edition, New Jersey: Prentice Hall, 1997.


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