ISSN No. 0976-5697

Volume 1, No. 4, Nov-Dec 2010
International Journal of Advanced Research in Computer Science
RESEARCH PAPER
Available Online at www.ijarcs.info

Global Chaos Synchronization of Hyperchaotic Liu and Cai Systems by Active Nonlinear Control

Dr. V. Sundarapandian*
Professor, R & D Centre
Vel Tech Dr. RR & Dr. SR Technical University
Avadi, Chennai-600 062, INDIA
sundarvtu@gmail.com

R. Suresh
Lecturer, Department of Mathematics
Vel Tech Dr. RR & Dr. SR Technical University
Avadi, Chennai-600 062, INDIA
mrpsuresh83@gmail.com

Abstract: This paper investigates the global exponential synchronization of hyperchaotic systems, *viz.* identical hyperchaotic Liu systems (Wang and Liu, 2006), identical hyperchaotic Cai systems (Wang, Cai, Miao and Tian, 2010), and synchronization of hyperchaotic Liu and Cai systems. Active nonlinear feedback control is the method used to achieve the synchronization of the hyperchaotic systems addressed in this paper. Our theorems on global exponential synchronization for hyperchaotic Liu and Cai systems are established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the nonlinear feedback control method is effective and convenient to synchronize identical and different hyperchaotic Liu and Cai systems. Numerical simulations are also given to illustrate and validate the synchronization results for hyperchaotic Liu and Cai systems.

Keywords: Chaos Synchronization, Nonlinear Control, Hyperchaotic Liu System, Hyperchaotic Cai System, Active Control.

I. Introduction

Chaotic systems are dynamical systems that are highly sensitive to initial conditions. This sensitivity is popularly referred to as the *butterfly effect* [1].

Chaos synchronization problem was first described by Fujisaka and Yemada [2] in 1983. This problem did not receive great attention until Pecora and Carroll ([3]-[4]) published their results on chaos synchronization in early 1990s. From then on, chaos synchronization has been extensively and intensively studied in the last three decades ([3]-[22]). Chaos theory has been explored in a variety of fields including physical [5], chemical [6], ecological [7] systems, secure communications ([8]-[10]) etc.

Synchronization of chaotic systems is a phenomenon that may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem.

In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

Since the seminal work by Carroll and Pecora ([3]-[4]), a variety of impressive approaches have been proposed for the synchronization for the chaotic systems such as PC method ([3]-[4]), the sampled-data feedback synchronization method ([10]-11]), OGY method [12], time-delay feedback approach [13], backstepping design method [14], adaptive design method ([15]-[19]), sliding mode control method [20], Lyapunov stability theory method [21], hyperchaos [22], etc.

Hyperchaotic system is usually defined as a chaotic system with at least two positive Lyapunov exponents, implying that its dynamics are expanded in several different directions simultaneously. For a continuous dynamical system to exhibit hyperchaotic behaviour, the system must be at least four-dimensional. Hyperchaotic systems have more complex dynamical behavious, which can be used to improve the security of a chaotic communication system.

This paper has been organized as follows. In Section II, we give the problem statement and our methodology. In Section III, we discuss the chaos synchronization of two identical hyperchaotic Liu systems ([23], 2006). In Section IV, we discuss the chaos synchronization of two identical hyperchaotic Cai systems ([24], 2010). In Section V, we discuss the heterogeneous synchronization of hyperchaotic Liu and Cai systems. In Section VI, we present the conclusions of this paper.

II. Problem Statement and Our Methodology

Consider the chaotic system described by the dynamics

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in \mathbf{R}^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : \mathbf{R}^n \to \mathbf{R}^n$ is the nonlinear part of the system. We consider the system (1) as the *master* or *drive* system.

As the *slave* or *response* system, we consider the following chaotic system described by the dynamics

$$\dot{y} = By + g(y) + u \tag{2}$$

where $y \in \mathbf{R}^n$ is the state vector of the response system, B is the $n \times n$ matrix of the system parameters, $g : \mathbf{R}^n \to \mathbf{R}^n$ is the nonlinear part of the response system and $u \in \mathbf{R}^n$ is the controller of the response system.

If $A = B$ and $f = g$, then x and y are the states of two *identical* chaotic systems. If $A \neq B$ and $f \neq g$, then x and y are the states of two *different* chaotic systems.

In the nonlinear feedback control approach, we design a feedback controller u, which synchronizes the states of the master system (1) and the slave system (2) for all initial conditions $x(0), z(0) \in \mathbf{R}^n$.

If we define the *synchronization error* as

$$e = y - x, \tag{3}$$

then the synchronization error dynamics is obtained as

$$\dot{e} = By - Ax + g(y) - f(x) + u \tag{4}$$

Thus, the global synchronization problem is essentially to find a feedback controller u so as to stabilize the error dynamics (4) for all initial conditions $e(0) \in \mathbf{R}^n$, i.e.

$$\lim_{t \to \infty} \|e(t)\| = 0 \tag{5}$$

for all initial conditions $e(0) \in \mathbf{R}^n$.

We use Lyapunov function technique as our methodology. We take as a candidate Lyapunov function

$$V(e) = e^T P e, \tag{6}$$

where P is a positive definite matrix. Note that $V : \mathbf{R}^n \to \mathbf{R}^n$ is a positive definite function by construction. We assume that the parameters of the master and slave systems are known and that the states of both systems (1) and (2) are measurable.

If we we find a feedback controller u so that

$$\dot{V}(e) = -e^T Q e, \tag{7}$$

where Q is a positive definite matrix, then $\dot{V} : \mathbf{R}^n \to \mathbf{R}^n$ is a negative definite function.

Thus, by Lyapunov stability theory [26], the error dynamics (4) is globally exponentially stable and hence the condition (5) will be satisfied for all initial conditions $e(0) \in R^n$. Then the states of the master system (1) and slave system (2) are globally exponentially synchronized.

III. Synchronization of Identical Hyperchaotic Liu Systems

In this section, we apply the nonlinear control technique for the synchronization of two identical hyperchaotic Liu systems ([23], 2006) described by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= bx_1 - kx_1x_3 + x_4 \\ \dot{x}_3 &= -cx_3 + hx_1^2 \\ \dot{x}_4 &= -dx_1 \end{aligned} \tag{8}$$

which is the *master* or *drive* system and

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= by_1 - ky_1y_3 + y_4 + u_2 \\ \dot{y}_3 &= -cy_3 + hy_1^2 + u_3 \\ \dot{y}_4 &= -dy_1 + u_4 \end{aligned} \tag{9}$$

which is the *slave* or *response* system, where all the parameters a, b, c, d, h, k are positive real constants and

$$u = [u_1 \quad u_2 \quad u_3 \quad u_4]^T$$

is the nonlinear controller to be designed.

The hyperchaotic Liu system (8) is a new 4-D hyperchaotic system derived from the Liu system by Wang and Liu ([23], 2006).

The four-dimensional system (8) is hyperchaotic when $a = 10, b = 40, k = 1, c = 2.5, d = 10.6$ and $h = 4$.

Figure 1 depicts the portrait of the hyperchaotic Liu system (8).

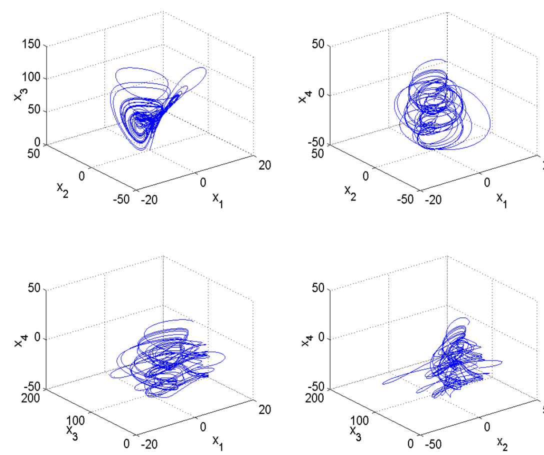

Figure 1. Portrait of the Hyperchaotic Liu System (8)

The synchronization error e is defined by

$$e_i = y_i - x_i, \qquad (i = 1, 2, 3, 4) \tag{10}$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + u_1 \\ \dot{e}_2 &= be_1 + e_4 - k(y_1y_3 - x_1x_3) + u_2 \\ \dot{e}_3 &= -ce_3 + h(y_1^2 - x_1^2) + u_3 \\ \dot{e}_4 &= -de_1 + u_4 \end{aligned} \tag{11}$$

In order to find the synchronizing controller, we first let

$$\begin{aligned} u_2 &= u_{2a} + u_{2b} \\ u_3 &= u_{3a} + u_{3b} \end{aligned} \tag{12}$$

where

$$\begin{aligned} u_{2b} &= k\left(y_1y_3 - x_1x_3\right) \\ u_{3b} &= -h\left(y_1^2 - x_1^2\right) \end{aligned} \tag{13}$$

Substituting (12) and (13) into (11), we obtain the error dynamics as

$$\begin{aligned}\dot{e}_1 &= a(e_2 - e_1) + u_1\\ \dot{e}_2 &= be_1 + e_4 + u_{2a}\\ \dot{e}_3 &= -ce_3 + u_{3a}\\ \dot{e}_4 &= -de_1 + u_4\end{aligned} \quad (14)$$

Next, we consider the candidate Lyapunov function

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right) \quad (15)$$

A simple calculation gives

$$\begin{aligned}\dot{V}(e) &= -ae_1^2 - ce_3^2 + (a+b)e_1e_2 + e_2e_4 - de_1e_4\\ &\quad + e_1u_1 + e_2u_{2a} + e_3u_{3a} + e_4u_4\end{aligned} \quad (16)$$

Therefore, we choose

$$\begin{aligned}u_1 &= -(a+b)e_2\\ u_{2a} &= -e_2\\ u_{3a} &= 0\\ u_4 &= -e_4 + de_1\end{aligned} \quad (17)$$

Substituting (17) into (14), the error dynamics (14) simplifies to

$$\begin{aligned}\dot{e}_1 &= -ae_1 - be_2\\ \dot{e}_2 &= be_1 - e_2\\ \dot{e}_3 &= -ce_3\\ \dot{e}_4 &= -e_4\end{aligned} \quad (18)$$

Substituting (17) into (16), we also obtain

$$\dot{V}(e) = -ae_1^2 - e_2^2 - ce_3^2 - e_4^2 \quad (19)$$

which is a negative definite function on $\boldsymbol{R}^4$ since a and c are positive constants.

Hence, by Lyapunov stability theory [25], the error dynamics (18) is globally exponentially stable.

Combining (12), (13) and (17), the synchronizing nonlinear controller u is obtained as

$$\begin{aligned}u_1 &= -(a+b)e_2\\ u_2 &= -e_2 + k(y_1y_3 - x_1x_3)\\ u_3 &= -h\left(y_1^2 - x_1^2\right)\\ u_4 &= -e_4 + de_1\end{aligned} \quad (20)$$

Thus, we have proved the following result.

Theorem 1. The identical hyperchaotic Liu systems (8) and (9) are exponentially and globally synchronized for any initial conditions with the nonlinear controller u defined by (19).

Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems using MATLAB with time-step equal to 10^{-6}.

For the hyperchaotic Liu system (8), the parameter values are taken as those which result in the hyperchaotic behaviour of the system, *viz.* $a = 10, b = 40, k = 1, c = 2.5, d = 10.6$ and $h = 4.$ [23].

The initial values of the master system (8) are taken as

$$x_1(0) = 5,\ x_2(0) = 7,\ x_3(0) = 8,\ x_4(0) = 3$$

while the initial values of the slave system (9) are taken as

$$y_1(0) = 9,\ y_2(0) = 2,\ y_3(0) = 4,\ y_3(0) = 8.$$

Figure 2 shows that synchronization between the states of the master system (8) and the slave system (9) occur in 8 seconds.

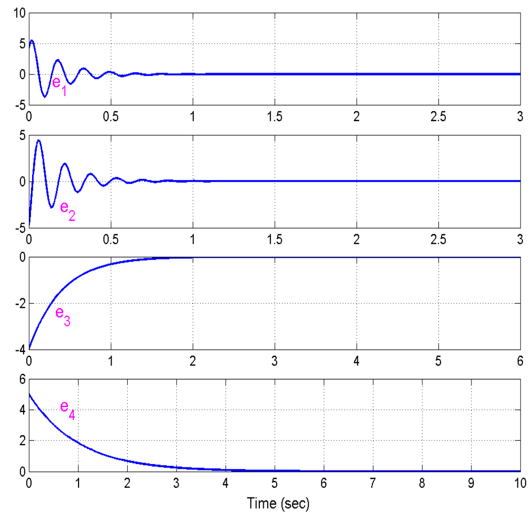

Figure 2. Synchronization of the States of (8) and (9)

IV. SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC CAI SYSTEMS

In this section, we apply the nonlinear control technique for the synchronization of two identical hyperchaotic Cai systems ([24], 2010) described by

$$\begin{aligned}\dot{x}_1 &= \alpha(x_2 - x_1)\\ \dot{x}_2 &= \beta x_1 + \gamma x_2 - x_1x_3 + x_4\\ \dot{x}_3 &= x_2^2 - rx_3\\ \dot{x}_4 &= -kx_1\end{aligned} \quad (21)$$

which is the *master* or *drive* system and

$$\begin{aligned}\dot{y}_1 &= \alpha(y_2 - y_1) + u_1\\ \dot{y}_2 &= \beta y_1 + \gamma y_2 - y_1y_3 + y_4 + u_2\\ \dot{y}_3 &= y_2^2 - ry_3 + u_3\\ \dot{y}_4 &= -ky_1 + u_4\end{aligned} \quad (22)$$

which is the *slave or response* system, where all the parameters $\alpha, \beta, \gamma, r, k$ are positive real constants and

$$u = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T$$

is the nonlinear controller to be designed.

The hyperchaotic Cai system (21) is a new 4-D hyperchaotic system derived by Wang, Cai, Miao and Tan ([24], 2010).

The Cai system (21) is hyperchaotic when

$\alpha = 27.5,\ \beta = 3,\ \gamma = 19.3,\ r = 2.9$ and $k = 3.3.$

Figure 3 illustrates the portrait of the hyperchaotic Cai system (21).

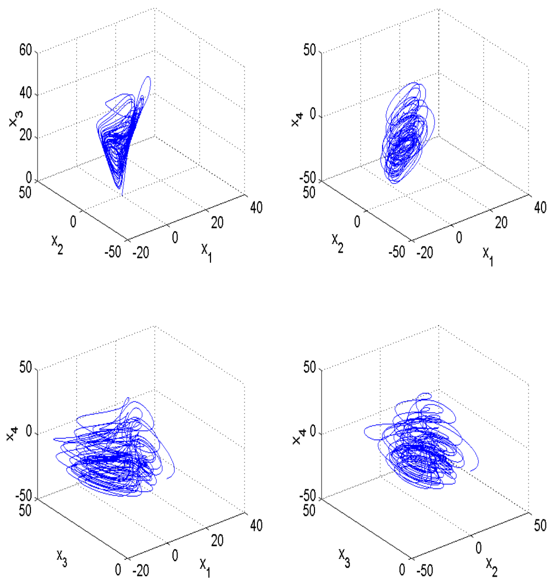

Figure 3. Portrait of the Hyperchaotic Cai System (21)

The synchronization error e is defined by

$$e_i = y_i - x_i, \qquad (i = 1,2,3,4) \tag{23}$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= \alpha(e_2 - e_1) + u_1 \\ \dot{e}_2 &= \beta e_1 + \gamma e_2 + e_4 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -re_3 + y_2^2 - x_2^2 + u_3 \\ \dot{e}_4 &= -ke_1 + u_4 \end{aligned} \tag{24}$$

In order to find the synchronizing controller, we first let

$$\begin{aligned} u_2 &= u_{2a} + u_{2b} \\ u_3 &= u_{3a} + u_{3b} \end{aligned} \tag{25}$$

where

$$\begin{aligned} u_{2b} &= y_1 y_3 - x_1 x_3 \\ u_{3b} &= -y_2^2 + x_2^2 \end{aligned} \tag{26}$$

Substituting (25) and (26) into (24), we obtain the error dynamics as

$$\begin{aligned} \dot{e}_1 &= \alpha(e_2 - e_1) + u_1 \\ \dot{e}_2 &= \beta e_1 + \gamma e_2 + e_4 + u_{2a} \\ \dot{e}_3 &= -re_3 + u_{3a} \\ \dot{e}_4 &= -ke_1 + u_4 \end{aligned} \tag{27}$$

Next, we consider the candidate Lyapunov function

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right) \tag{28}$$

A simple calculation gives

$$\begin{aligned} \dot{V}(e) &= -\alpha e_1^2 + \gamma e_2^2 - re_3^2 + (\alpha+\beta)e_1 e_2 - ke_1 e_4 \\ &\quad + e_1 u_1 + e_2 u_{2a} + e_3 u_{3a} + e_4 u_4 \end{aligned} \tag{29}$$

Therefore, we choose

$$\begin{aligned} u_1 &= -(\alpha+\beta)e_2 \\ u_{2a} &= -(\gamma+1)e_2 \\ u_{3a} &= 0 \\ u_4 &= ke_1 - e_4 \end{aligned} \tag{30}$$

Substituting (30) into (27), the error dynamics (27) simplifies to

$$\begin{aligned} \dot{e}_1 &= -\alpha e_1 - \beta e_2 \\ \dot{e}_2 &= \beta e_1 - e_2 \\ \dot{e}_3 &= -re_3 \\ \dot{e}_4 &= -e_4 \end{aligned} \tag{31}$$

Substituting (30) into (29), we also obtain

$$\dot{V}(e) = -\alpha e_1^2 - e_2^2 - re_3^2 - e_4^2 \tag{32}$$

which is a negative definite function on $\boldsymbol{R}^4$ since α and r are positive constants.

Hence, by Lyapunov stability theory [25], the error dynamics (32) is globally exponentially stable.

Combining (25), (26) and (30), the synchronizing nonlinear controller u is obtained as

$$\begin{aligned} u_1 &= -(\alpha+\beta)e_2 \\ u_2 &= -(\gamma+1)e_2 + y_1 y_3 - x_1 x_3 \\ u_3 &= -y_2^2 + x_2^2 \\ u_4 &= -e_4 + ke_1 \end{aligned} \tag{33}$$

Thus, we have proved the following result.

Theorem 2. The identical hyperchaotic Cai systems (21) and (22) are exponentially and globally synchronized for any initial conditions with the nonlinear controller u defined by (33).

Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems using MATLAB with time-step equal to 10^{-6}.

For the hyperchaotic Cai system (21), the parameter values are taken as those which result in the hyperchaotic behaviour of the system, *viz.*

$$\alpha = 27.5,\ \beta = 3,\ \gamma = 19.3,\ r = 2.9 \text{ and } k = 3.3.$$

The initial values of the master system (21) are taken as

$$x_1(0) = 10,\ x_2(0) = 6,\ x_3(0) = 4,\ x_4(0) = 9$$

while the initial values of the slave system (22) are taken as

$$y_1(0)=4,\ y_2(0)=12,\ y_3(0)=15,\ y_3(0)=16.$$

Figure 4 shows that synchronization between the states of the master system (21) and the slave system (22) occur in 8 seconds.

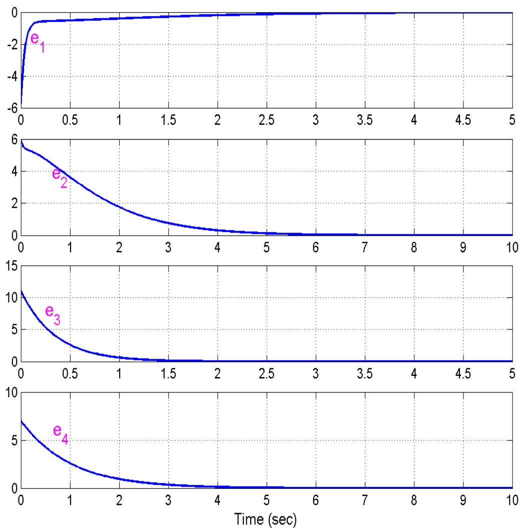

Figure 4. Synchronization of the States of (21) and (22)

V. Synchronization of Hyperchaotic Liu and Cai Systems

In this section, we apply the nonlinear control technique for the synchronization of non-identical hyperchaotic Liu and Cai systems. As the master system, we consider the hyperchaotic Liu system ([23], 2006) described by

$$\begin{aligned}\dot{x}_1 &= a(x_2-x_1)\\ \dot{x}_2 &= bx_1-kx_1x_3+x_4\\ \dot{x}_3 &= -cx_3+hx_1^2\\ \dot{x}_4 &= -dx_1\end{aligned}\tag{34}$$

As the slave system, we consider the hyperchaotic Cai system ([24], 2010) described by

$$\begin{aligned}\dot{y}_1 &= \alpha(y_2-y_1)+u_1\\ \dot{y}_2 &= \beta y_1+\gamma y_2-y_1y_3+y_4+u_2\\ \dot{y}_3 &= y_2^2-ry_3+u_3\\ \dot{y}_4 &= -ky_1+u_4\end{aligned}\tag{35}$$

where all the parameters are positive real constants and

$$u=\begin{bmatrix}u_1 & u_2 & u_3 & u_4\end{bmatrix}^T$$

is the nonlinear controller to be designed.

The synchronization error e is defined by

$$e_i=y_i-x_i,\qquad (i=1,2,3,4)\tag{36}$$

The error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= \alpha(e_2-e_1)-(a-\alpha)(x_2-x_1)+u_1\\ \dot{e}_2 &= \beta e_1+\gamma e_2+e_4-(b-\beta)x_1\\ &\quad +\gamma x_2-y_1y_3+kx_1x_3+u_2\\ \dot{e}_3 &= -re_3+(c-r)x_3+y_2^2-hx_1^2+u_3\\ \dot{e}_4 &= -ke_1+(d-k)x_1+u_4\end{aligned}\tag{37}$$

In order to find the synchronizing controller, we first let

$$\begin{aligned}u_1 &= u_{1a}+u_{1b}\\ u_2 &= u_{2a}+u_{2b}\\ u_3 &= u_{3a}+u_{3b}\\ u_4 &= u_{4a}+u_{4b}\end{aligned}\tag{38}$$

where

$$\begin{aligned}u_{1b} &= (a-\alpha)(x_2-x_1)\\ u_{2b} &= (b-\beta)x_1-\gamma x_2+y_1y_3-kx_1x_3\\ u_{3b} &= (r-c)x_3-y_2^2+hx_1^2\\ u_{4b} &= (k-d)x_1\end{aligned}\tag{39}$$

Substituting (38) and (39) into (37), the error dynamics simplifies to

$$\begin{aligned}\dot{e}_1 &= \alpha(e_2-e_1)+u_{1a}\\ \dot{e}_2 &= \beta e_1+\gamma e_2+e_4+u_{2a}\\ \dot{e}_3 &= -re_3+u_{3a}\\ \dot{e}_4 &= -ke_1+u_{4a}\end{aligned}\tag{40}$$

Next, we consider the candidate Lyapunov function

$$V(e)=\frac{1}{2}e^Te=\frac{1}{2}\left(e_1^2+e_2^2+e_3^2+e_4^2\right)\tag{41}$$

A simple calculation gives

$$\begin{aligned}\dot{V}(e) &= -\alpha e_1^2+\gamma e_2^2-re_3^2+(\alpha+\beta)e_1e_2-ke_1e_4\\ &\quad +e_1u_{1a}+e_2u_{2a}+e_3u_{3a}+e_4u_{4a}\end{aligned}\tag{42}$$

Therefore, we choose

$$\begin{aligned}u_{1a} &= -(\alpha+\beta)e_2\\ u_{2a} &= -(\gamma+1)e_2\\ u_{3a} &= 0\\ u_{4a} &= ke_1-e_4\end{aligned}\tag{43}$$

Substituting (43) into (40), the error dynamics reduces to

$$\begin{aligned}\dot{e}_1 &= -\alpha e_1-\beta e_2\\ \dot{e}_2 &= \beta e_1-e_2\\ \dot{e}_3 &= -re_3\\ \dot{e}_4 &= -e_4\end{aligned}\tag{44}$$

Substituting (43) into (42), we also obtain

$$\dot{V}(e)=-\alpha e_1^2-e_2^2-re_3^2-e_4^2\tag{45}$$

which is a negative definite function on $\boldsymbol{R}^4$ since α and r are positive constants.

Hence, by Lyapunov stability theory [25], the error dynamics (43) is globally exponentially stable.

Combining (38), (39) and (43), the synchronizing nonlinear controller u is obtained as follows:

$$\begin{aligned} u_1 &= -(\alpha+\beta)e_2 + (a-\alpha)(x_2 - x_1) \\ u_2 &= -(\gamma+1)e_2 + (b-\beta)x_1 - \gamma x_2 + y_1 y_3 - k x_1 x_3 \\ u_3 &= (r-c)x_3 - y_2^2 + h x_1^2 \\ u_4 &= k e_1 - e_4 + (k-d)x_1 \end{aligned} \tag{46}$$

Thus, we have proved the following result.

Theorem 4. The non-identical hyperchaotic Liu system (34) and hyperchaotic Cai system (35) are exponentially and globally synchronized for any initial conditions with the nonlinear controller u defined by (46).

Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method is used to solve the system using MATLAB with time-step equal to 10^{-6}.

For the hyperchaotic Liu system (34), the parameter values are taken as those which result in the hyperchaotic behaviour of the system, *viz.*

$$a = 10, b = 40, k = 1, c = 2.5, d = 10.6 \text{ and } h = 4.$$

For the hyperchaotic Cai system (35), the parameter values are taken as those which result in the hyperchaotic behaviour of the system, *viz.*

$$\alpha = 27.5,\ \beta = 3,\ \gamma = 19.3,\ r = 2.9 \text{ and } k = 3.3.$$

The initial values of the hyperchaotic Liu system (34) are taken as

$$x_1(0) = 10,\ x_2(0) = 6,\ x_3(0) = 12,\ x_4(0) = 5$$

while the initial values of the Cai system (33) are taken as

$$y_1(0) = 2,\ y_2(0) = 16,\ y_3(0) = 8,\ y_4(0) = 18.$$

Figure 5 shows that synchronization between the states of the hyperchaotic Liu system (34) and hyperchaotic Cai system (35) occur in 5 seconds.

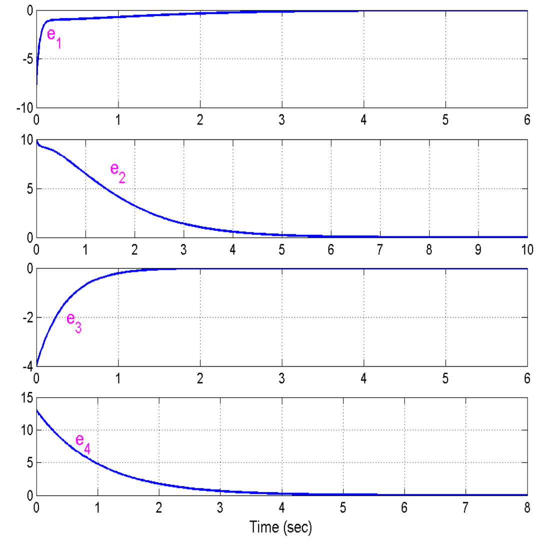

Figure 5. Synchronization of the States of (34) and (35)

VI. CONCLUSIONS

Since the hyperchaotic systems have more complex dynamical behaviours, they can be used to improve the security of a chaotic communication system. In this paper, we have used nonlinear control method based on Lyapunov stability theory to achieve global chaos synchronization for the following three cases of hyperchaotic systems:

(A) Identical Hyperchaotic Liu systems (2006).

(B) Identical Hyperchaotic Cai systems (2010).

(C) Non-Identical Hyperchaotic Liu and Cai Systems.

Numerical simulations are also given to validate the proposed synchronization approach for the global chaos synchronization of the hyperchaotic systems. Since the Lyapunov exponents are not required for these calculations, the nonlinear control method is very effective and convenient to achieve global chaos synchronization for the three cases of hyperchaotic systems discussed in this paper.

VII. REFERENCES

[1] K.T. Alligood, T. Sauer and J.A. Yorke, Chaos: An Introduction to Dynamical Systems, Springer, New York, 1997.

[2] H. Fujisaka and T. Yamada, "Stability theory of synchronized motion in coupled-oscillator systems", Progress of Theoretical Physics, vol. 69, no. 1, pp. 32-47, 1983.

[3] T.L. Carroll and L.M. Pecora, "Synchronization in chaotic systems", Phys. Rev. Lett., vol. 64, pp. 821-824, 1990.

[4] T.L. Carroll and L.M. Pecora, "Synchronizing chaotic circuits", IEEE Trans. Circ. Sys., vol. 38, pp. 453-456, 1991.

[5] M. Lakshmanan and K. Murali, Chaos in Nonlinear Oscillators: Controlling and Synchronization, World Scientific, Singapore, 1996.

[6] S.K. han, C. Kerrer and Y. Kuramoto, "D-phasing and bursting in coupled neural oscillators", Phys. Rev. Lett., vol. 75, pp. 3190-3193, 1995.

[7] B. Blasius, A. Huppert and L. Stone, "Complex dynamics and phase synchronization in spatially extended ecological system", Nature, vol. 399, pp. 354-359, 1999.

[8] J. Lu, X. Wu, X. Han and J. Lu, "Adaptive feedback synchronization of a unified chaotic sytem", Phys. Lett. A, vol. 329, pp. 327-333, 2004.

[9] L. Kocarev and U. Parlitz, "General approach for chaotic synchronization with applications to communications", Phys. Rev. Lett., vol. 74, pp. 5028-5030, 1995.

[10] K. Murali and M. Lakshmanan, "Secure communication using a compound signal using sampled-data feedback", Applied Mathematics and Mechanics, vol. 11, pp. 1309-1315, 2003.

[11] T. Yang and L.O. Chua, "Generalized synchronization of chaos via linear transformations", International Journal of Bifurcation and Chaos, vol. 9, pp. 215-219, 1999.

[12] E. Ott, C. Grebogi and J.A. Yorke, "Controlling chaos", Phys. Rev. Lett., vol. 64, pp. 1196-1199, 1990.

[13] J.H. Park and O.M. Kwon, "A novel criterion for delayed feedback control of time-delay chaotic systems", Chaos, Solitons and Fractals, vol. 17, pp. 709-716, 2003.

[14] X. Wu and J. Lü, "Parameter identification and backstepping control of uncertain Lu system", Chaos, Solitons and Fractals, vol. 18, pp. 721-729, 2003.

[15] J. Lu, X. Wu, X. Han and J. Lu, "Adaptive feedback synchronization of a unified chaotic system", Phys. Lett. A, vol. 329, pp. 327-333, 2004.

[16] Y.G. Yu and S.C. Zhang, "Adaptive backstepping synchronization of uncertain chaotic systems", Chaos, Solitons and Fractals, vol. 27, pp. 1369-1375, 2006.

[17] J.H. Park, S.M. Lee and O.M. Kwon, "Adaptive synchronization of Genesio-Tesi chaotic system via a novel feedback control", Physcs Letters A, vol. 371, no. 4, pp. 263-270, 2007.

[18] J.H. Park, "Adaptive control for modified projective synchronization of a four-dimensional chaotic system with uncertain parameters", J. Computational and Applied Math., vol. 213, no. 1, pp. 288-293, 2008.

[19] J.H. Park, "Chaos synchronization of nonlinear Bloch equations", Chaos, Solitons and Fractals, vol. 27, no. 2, pp. 357-361, 2006.

[20] H.T. Yau, "Design of adaptive sliding mode controller for chaos synchronization with uncertainties", Chaos, Solitons and Fractals, vol. 22, pp. 341-347, 2004

[21] R. Suresh and V. Sundarapandian, " Synchronization of an optical hyper-chaotic system", International J. Comp. Applied Math., vol. 5, no. 2, pp. 199-207, 2010.

[22] R. Vicente, J. Dauden, P. Colet and R. Toral, "Analysis and characterization of the hyperchaos generated by a semiconductor laser object," IEEE J. Quantum Electronics, vol. 41, pp. 541-548, 2005.

[23] F.Q. Wang and C.X. Liu, Hyperchaos evolved from the Liu chaotic system," Chinese Physics, vol. 15, pp. 963-968, 2006.

[24] H.X. Wang, G.L. Cai, S. Miao and L.X. Tian, "Nonlinear feedback control of a novel hyperchaotic system and its circuit implementation," Chinese Physics B, vol. 19, no.3, 030509, 2010.

[25] W. Hahn, The Stability of Motion, Springer, New York, 1967.

AUTHORS

Dr. V. Sundarapandian is a Research Professor (Systems and Control Engineering) in the Research and Development Centre, Vel Tech Dr. RR & Dr. SR Technical University, Avadi, Chennai since September 2009. Previously, he was a Professor and Academic Convenor at the Indian Institute of Information Technology and Management-Kerala, Trivandrum. He earned his Doctor of Science Degree from the Department of Electrical and Systems Engineering, Washington University, St. Louis, Missouri, USA in May 1996. He has published over 70 research papers in refereed International Journals and presented over 80 research papers in National Conferences in India and International Conferences in India and abroad. He is an Associate Editor of the *International Journal on Control Theory* and is in the Editorial Boards of the Journals – *Scientific Research and Essays*, *International Journal of Engineering, Science and Technology*, *ISST Journal of Mathematics and Computing System*, *International Journal of Soft Computing and biofinformatics*, etc. He is a regular reviewer for the reputed journals – *International Journal of Control*, *Systems and Control Letters*, etc. His research areas are: Linear and Nonlinear Control Systems, Optimal Control and Operations Research, Soft Computing, Mathematical Modelling and Scientific Computing, Dynamical Systems and Chaos, Stability Theory, Nonlinear Analysis etc. He has authored two text-books for PHI Learning Private Ltd., namely *Numerical Linear Algebra* (2008) and *Probability, Statistics and Queueing Theory* (2009). He has given several key-note lectures on Modern Control Systems, Mathematical Modelling, Scientific Computing with SCILAB, etc.

Mr. R. Suresh is a Lecturer in the Department of Mathematics at Vel Tech Dr. RR & Dr. SR Technical University, since June 2010. He previously held positions at Vel Tech Multi Tech Dr. Rangarajan Dr. Sakunthala Engineering College (Chennai), Maharaja Institute of Technology (Coimbatore) and Nanda Arts and Science College (Erode). He earned his M.Sc. degree in Mathematics in 2005 from Bharathiar University, Coimbatore, Tamil Nadu, India and M.Phil in Mathematics in 2008 from Bharathiar University, Coimbatore, Tamil Nadu India. He is currently pursuing his Ph.D degree in Mathematics from Vel Tech Dr.

RR & Dr. SR Technical University under the guidance of Prof. V. Sundarapandian. He has published five International journal papers, two contributed papesr in Springer-Verlag Lecture Notes and over ten papers in National and International Conferences. His research areas are Chaos, Soft Computing and Control.