Volume 3, No. 4, July- August 2012

ISSN No. 0976-5697

International Journal of Advanced Research in Computer Science

RESEARCH PAPER

Available Online at www.ijarcs.info

Global Bacteria Optimization: A Metaheuristic Inspired on Bacteria Phototaxis to Solve Multi-objective Optimization Problems

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Abstract: A new metaheuristic, known as Global Bacteria Optimization (GBO), is known to solve multi-objective optimization problems and results from previous work has shown improvement over those solutions generated by other metaheuristics, such as genetic algorithms (GA), evolutionary algorithms (EA) and swarm algorithms (PSO). This metaheuristic is inspired on bacteria phototaxis behavior, where the solution space is reduced and gets far closer to the Pareto optima solutions. In this paper, the analytical aspect of the solution to multi-objective problems is approached, where it has been demonstrated how two mathematical functions, when minimized, produce Pareto Optima solutions. A review of the MCDM theory states the conditions that are required for two or more functions to reach their optimal solutions simultaneously. Metrics, such as extreme points and spacing, were compared to exact solutions obtained by MCDM techniques programmed in GAMS, proving that GBO not only produce Pareto Optima solutions, but robustness is also obtained.

Keywords**:** Metaheuristics; Multi-objective Optimization Problems; MCDM; Partial derivatives; GBO; Extreme Points; Spacing

I. INTRODUCTION

It has been widely known that today's decision making problems are based on several criteria or objectives, given that in a situation it is everyday more difficult to decide between two or several objectives. The reason for this is due to the fact that usually the objectives are conflicting; this means that if one is improved the other is affected. For example, if both time and cost are being minimized in order to operate any industrial process, it can be observed that the minimization of time requires a greater investment in cost and the minimization of the cost requires a greater time of execution; ending up both objectives in conflict. The existence of several objectives in an optimization problem is denominated multiobjective optimization and this gives rise to a theory designed by Vilfredo Pareto [1].

As for the optimization problems, it is important to understand the existence of the complexity of any optimization problem. This occurs when the problem can't be solved by exploring all possible scenarios and if this occurred the computer wouldn't finish in many years, depending on the size of the problem. This gives rise to heuristics and metaheuristics, which gives "good" solutions to large optimization problems in fairly small execution times.

Among the metaheuristic approaches, it can be observed that much work has been done with genetic algorithms (GA), evolutionary algorithms (EA), swarm algorithms (PSO), tabu search and ant colony optimization (ACO).

All of these metaheuristics were originated from problems observed in nature and were adapted to optimization problems, implementing new ways to obtain near to optimal or optimal solutions, by shortening the search space available, thus, improving algorithm execution times. For example, ACO algorithm is based on the behavior of ant colonies as they search for the best trail that leads to their source of food.

The metaheuristic named Global Bacteria Optimization (GBO) algorithm, which is introduced in this paper, is based on the process that takes place in phototropic bacteria, its mobility and life cycle in the participating colony. It has been proven that the, so called, "moving towards the light" process as a search space procedure produces excellent results, reaching objective values that are competitive in comparison to other metaheuristics [2].

In this paper, the analytical aspect of the solution to multi-objective problems is approached, where it has been demonstrated how two mathematical functions, when minimized, produce Pareto Optima solutions. Metrics such as extreme points and spacing, were compared to exact solutions obtained by MCDM techniques programmed in GAMS, proving that GBO not only produce Pareto Optima, but robustness in the solutions obtained.

II. MULTICRITERIA OPTIMIZATION THEORY

A. Multicriteria Decision Making (MCDM):

In the process of decision making there are a set of tools that permit a correct approach to an optimal solution of a problem. Many authors have presented significant contributions and, in general, the MCDM approach is more of a description where possible solutions are defined, including the attributes and evaluation of the criteria, but most importantly, there is a utility function where the criteria is incorporated. This utility function has to be maximized during this process and that is how optimal solutions are reached.

There are several axioms presented by Boysseu (1984) and Roy (1985) that are fundamental to MCDM [3]: 1) The decision maker always maximizes, implicitly or explicitly, a utility function; 2) An optimal solution exists for every situation; 3) No comparable solution exists, it will always need to have to choose or sort between a pair of decisions; 4) Decision maker's preferences can depend upon two binary relations: preference (P) and indifference (I).

Yet there are also some limitations to MCDM because problems are said to be unrealistic and this makes the theory less useful than what it should be. According to Zeleny (1992), MCDM is not useful when there is time pressure, when the problem is more completely defined, when using a strict hierarchical decision system, when there is changing environment, when there is limited or partial knowledge of the problem and when there is collective decision making in businesses; all this because it reduces the number of criteria being considered, leaving behind other possible alternatives [4].

Some authors, like Carlsson and Fuller (1995), agree that the traditional assumption used in MCDM, in which the criteria are taken as independent, is very limited and ideal to be applied to today's business decision making [5]. Reeves and Franz (1985) introduced a multicriteria linear programming problem, where they presume the decision maker has to determine his preferences in terms of the objectives but he must have more than an intuitive understanding of the trade-offs he is probably doing with the objectives [6]. For this reason, an assumption is made and that is, that a decision maker is taken to be a rational thinker and with a complete understanding of the whole situation in which his preferences have some basis with the use of a utility function.

It has been universally recognized that there is no such thing as an optimal solution valid for any multiobjective problem. In literature, much has been found in terms of different approaches to solving MCDM problems. Delgado, et. al. (1999) used, for example, fuzzy sets and possibility theory not only to involve MCDM but also, multiobjective programming [7]. Also, Felix (1992) worked with fuzzy relations between criteria by presenting a novel theory for multiple attribute decision making [8]. Carlsson, on the other hand, "used fuzzy Pareto optimal set of non-dominated alternatives to find the best compromise solution to MCDM problems with interdependent criteria". In order to understand more about the interdependencies between criteria, it is important to notice the problem defined by Carlsson and Fúller in terms of multiple objectives [5]:

$$\max_{x \in X} \{ f_1(x), \ldots, f_k(x) \}$$

$$\textit{where } f_1 : \Re^n \to \Re \textit{ are objrective functions}$$

$$x \in \Re^n \textit{ is a decision variable and } x \textit{ is a subset of } \Re^n.$$

Definition: Let f_i and f_j be the two objective functions of the problem defined above.

i. f_i supports f_j on X (denoted $f_i \uparrow f_j$) if $f_i(x') \geq f_j(x)$ *entails* $f_j(x') \geq f_j(x)$, *for all* $x', x \in X$;

ii. f_i is in conflict with f_j on X (denoted $f_i \downarrow f_j$) if $f_i(x') \geq f_j(x)$ *entails* $f_j(x') \leq f_j(x)$, *for all* $x', x \in X$;

iii. Otherwise, f_i and f_j are independent on X.

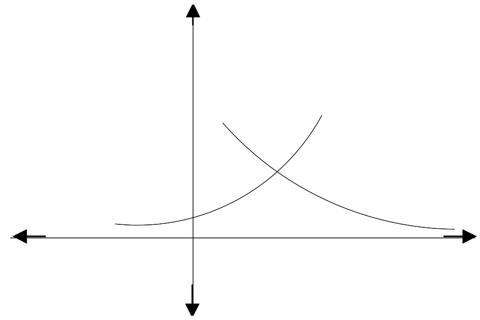

Figure: 1 Conflictive Objective Functions.

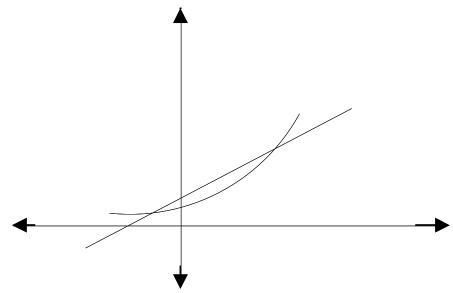

Figure: 2 Supportive Objective Functions.

In traditional MCDM it has been found that the criteria should be independent, yet there are some methods that deal with conflictive objectives but do not recognize other interdependencies that can be present, which makes the problem more unrealistic. Zeleny (1992) recognized that there are objectives that might support each other when he shows the fallacy with using weights independent from criterion performance [4].

B. Multiobjective Optimization Problems:

When problems have more than one objective, they are said to be multicriteria-based or multiobjective. It is important to understand the theory that they have considered to solve these types of problems. The multicriteria optimization theory takes basically a set of priorities established by the decision maker and provides the best solution under their preferences. T'Kindt and Billaut (2006) show a mathematical definition of the multicriteria optimization problems expressing them as a special case of vector optimization problems where the solution space is S and the criteria space, Z(S), are vectorial euclidian spaces of finite dimension, Q and K respectively [3].

$$\begin{aligned}
&\textit{Min } Z(x) \textit{ with } Z(x) = [Z_1(x); \ldots; Z_K(x)]^T \\
&\textit{Subject to} \\
&\quad x \in S \\
&\quad S = \{ x / [g_1(x); \ldots; g_M(x)]^T \leq 0 \} \\
&\textit{i.e. } S \subset \Re^Q \textit{ and } Z(S) \subset \Re^K \textit{ with } 1 \leq Q, K < \infty.
\end{aligned}$$

Definition of Optimality [3]: Let $S \subset \Re^Q$ be a set of solutions and $Z(S) \subset \Re^K$ the image in the criteria space of *S* by *K* criteria Z_i. $\forall x, y \in \Re^K$:

$$x \le y \Leftrightarrow x_i \le y_i,\ \forall i = 1,...,K$$

$$x = y \Leftrightarrow x_i = y_i,\ \forall i = 1,...,K$$

This is valid for K > 2, because for single criterion problems (K=1), there is no way to compare between two solutions, for which the optimal solution is given right away. In the case of multiple objectives, this is no longer the case because there will be various solutions that minimize several criteria and they need to be compared. To approach it, Pareto Optima, a general definition of optimality, is used. Figure 3 shows the graphical representation of Pareto Optima.

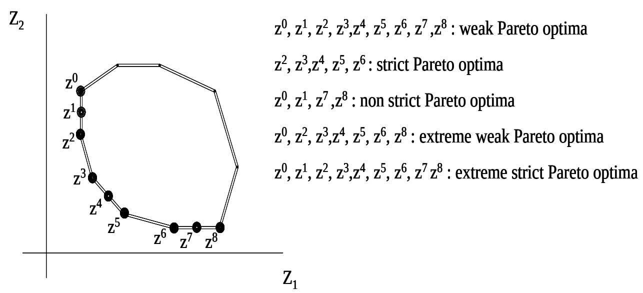

Figure: 3 Graphical representation of Pareto Optima.

C. MCDM Theory to solve multi-objective problems:

When reaching for Pareto optima, the decision maker has to look for the "best trade-off" solutions between conflicting criteria, and it is assumed to be done by optimizing a utility function. When searching for the solution, the decision maker must choose for an algorithm or heuristic that can determine the whole Pareto optima set. The decision maker provides weights to the different criteria being analyzed in order to determine the priorities. In literature many ways have been used to determine Pareto optima, it is just a matter of choosing the correct one depending on the quality of the calculable solutions and the ease of the application [3].

MCDM presents various methods to generate Pareto Optima solutions, such as Convex Combination of Criteria, Parametric Analysis, Means of the -constraint approach, Tchebycheff Metric, Goal-Attainment Approach and Use of Lexicographical Order. This paper references the method that uses Convex Combination of Criteria and solutions generated were compared with the metaheuristic GBO proposed.

Convex Combination of Criteria [9]: Let *S* be the convex set of solutions and *K* criteria Z_i convex on *S*. x^0 is a proper Pareto optimum if and only if,

$$\exists \alpha \in \Re^K,\ \text{with}\ \alpha_i \in \left]0;1\right[\ and\ \sum_{i=1}^{K} \alpha_i = 1$$

such that x^0 is an optimal solution of the problem (Pα):

$$Min\ g(Z(x))\ with\ g(Z(x)) = \sum_{i=1}^{K} \alpha_i Z_i(x)$$

Subject to

$$x \in S$$

The above theorem, Geoffrion's Theorem, the parameters α_i cannot be equal to zero because, otherwise, not all the results found will correspond to proper Pareto optima. So another condition is needed to determine a weak Pareto optima:

Let S be the convex set of solutions and K criteria Z_i convex on S. x^0 is a set of weak Pareto optimum if and only

$$\text{IF}\ \exists \alpha \in \Re^K,\ \text{WITH}\ \alpha_i \in [0;1]\ and\ \sum_{i=1}^{K} \alpha_i = 1$$

Such that x^0 is an optimal solution of the problem (Pα):

T'Kindt and Billaut introduce how graphical representations of the different optimization problems can be done by using level curves. For minimizing the convex combination of criteria, problem (Pα) can be represented by defining first the set of level curves in the decision space, using the conditions for this specific approach:

$$Let\ X\_(a) = \left\{ x \in S / \sum_{i=1}^{K} \alpha_i Z_i(x) = a\ with\ \alpha_i \left]0;1\right[\ and \sum_{i=1}^{K} \alpha_i = 1 \right\}$$

By writing $L_=(a) = Z(X\_(a))$ in order to construct the curves in the graphs, the curve of minimal value g* is found, where the line $L_=(g^*)$ is tangent to Z in the criteria space. See figure 4 for a geometric representation of the problem described above.

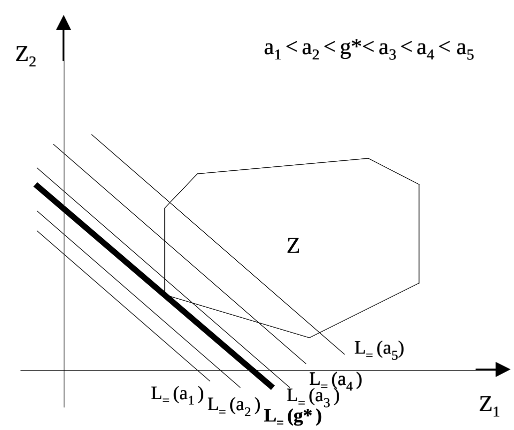

Figure: 4 Geometric Interpretation of the Convex Combination of Criteria.

III. THE USE OF METAHEURSITICS TO SOLVE MULTI-OBJECTIVE PROBLEMS

A. A Review on Metaheuristics [10]:

Metaheuristic algorithms have been developed over the last three decades and most of them have been nature-inspired. Today they have become a very powerful tool in solving global optimization problems. In metaheuristic algorithms, meta- means 'beyond' or 'higher level', and they generally perform better than simple heuristics. All metaheuristic algorithms use certain tradeoff of local search and global exploration. Most solutions are often realized via randomization because it provides a good way to move away from local search to the search on the global scale. Among the most applied metaheuristics are the genetic algorithms (GA), simulated annealing (SA), tabu search and ant colony optimization (ACO).

Metaheuristics can be an efficient way to obtain acceptable solutions through experimentation to complex problems in reasonable practical time. Depending on the type of problem and its complexity, will it be feasible to generate all possible solutions and from those select the best one. Yet, for many problems there is no significant improvement in a solution that deserves an exhaustive search, which at the end will be penalized by large execution times and significant computational effort in obtaining them. When using metaheuristics, there is no guarantee that the best solutions will be found, more if for that type of problem the optimal has not been found yet.

The main components of any metaheuristic algorithms are: intensification and diversification (Blum and Roli, 2003). Diversification is defined as generating diverse solutions so as to explore the search space on the global scale. Intensification focuses on the search on a local region and exploits it, given that a good solution was obtained there. The selection of the best solution ensures the solutions will converge to optimality. Yet, the use of randomization to increase the diversity of the solution avoids solutions being trapped at local optima. A combination of both components will usually ensure that global optimality is achievable.

Metaheuristics have been classified as population-based and trajectory-based. GA, for example, are population-based, whereas SA is trajectory-based.

Throughout history, the main approach to problem solving has been through a heuristic or metaheuristic. In the 1960's and 1970's, evolutionary algorithms were introduced, started by a study made by John Holland in 1962. A GA is a search method based on the abstraction of Darwinian evolution and natural selection of biological systems and representing them in the mathematical operators: crossover or recombination, mutation, fitness and selection of the fittest. In the early 1960's, Lawerence J. Fogel intended to use simulated evolution as a learning process, as a tool to study artificial intelligence. Then, in 1966, L.J. Fogel, A.J. Owen and M.J. Walsh developed an evolutionary programming technique by representing solutions as finite-state machines and randomly mutating one of these machines (Fogel, et al, 1966). In 1983, the optimization technique known today as simulated annealing (SA), was introduced by S. Kirkpatrick, C.D. Gellat and M.P. Vecchi, inspired by the annealing process of metals. It is a trayectory-based search algorithm starting with an initial guess solution at a high temperature and gradually cooling down the system (Kirkpatrick, et.al, 1983).

Yet, the first step to the use of memory in modern metaheuristics was due to Fred Glover's tabu search in 1968 (Glover and Luguna, 1997). Then, in 1989, through the work of P. Moscato, the memetic algorithm was developed. This was, more than a metaheuristic, a a hyper-heuristic algorithm (Moscato, 1989).

At the beginning of the 1990's, ACO algorithm was developed by Marco Dorigo (Dorigo, 1992), a search technique inspired on the swarm intelligence of social ants using pheromone as a chemical messenger. Then, in 1992, John Koza published work on genetic programming, which laid the foundation to a whole new concept on machine learning (Koza, 1992) and in 1986 another interesting concept known as the artificial immune system was developed (Farmer et.al., 1986) based on the use of memory and learning as a novel solution approach.

In 1995, the PSO algorithm was developed by James Kennedy and Russell Eberbart. This metaheuristic was inspired on swarm intelligence of fish and birds and even by human behavior, where multiple agents, called particles, swarm around the search space, starting from some initial random solution and communicating the current best found through the swarm. Since the development of PSO, many variants have been developed and applied to many areas of tough optimization problems. Around 1996, Storn and Price developed the vector-based evolutionary algorithms known as differential evolution (DE) and this algorithm proves to be better than GA in numerous applications.

At the turn of the 21st century, many other metaheuristics were developed: the harmony search (HS) algorithm (Geem, et.al, 2001); bacterial foraging optimization (Passino, 2002), which was inspired by social foraging behavior of certain bacteria such as the *Escherichia coli*; the novel bee algorithm (Pham, et.al, 2005); the artificial bee colony (ABC) (Karaboga, 2005); the glowworm algorithm (Krishnan and Ghose, 2005); honey-bees mating optimization algorithm (2006); monkey search algorithm (Mucherino and Seraf, 2008); firefly algorithm (FA) (Yang, 2008; Yang, 2009; Yang, 2010); efficient cuckoo search (CS) algorithm; bat-inspired for continuous optimization (2010). In 2011, a unified view of metaheuristics was implemented in a generalized evolutionary walk algorithm (GEWA) [10].

B. Global Bacteria Optimization (GBO) Algorithm:

GBO is a population-based metaheuristic that combines both diversification and intensification concepts that characterize today's metaheuristics. It was developed as a result of a graduate thesis [11] which was directly applied to a scheduling problem whose results improve MOEA algorithm [12]. After observing the behavior of Bacteria phototaxis and the different processes that bacteria incur naturally, this metaheuristic was developed based on this processes, which was converted into a mathematical function that works as a process within the algorithm. The algorithm is described as follows:

```
START
Generate Valid Bacteria Colony: C
Assign Size of population: Tam
Assign Bacterial Loop Size: A
DO WHILE (A >= 1)
  {NewTam = 0;
  REPEAT integer i=1:Tam times
   { IF C[i].energy > rotation-and-race-wear THEN
        {DO Bacteria Rotation for C[i]
         Save directions sets in D[i]
         Race to Light C[i] Random Select of the Best
         Light Direction in D[i]
C[i].energy=C[i].energy – rotation-and-race-wear}
     }
END REPEAT
```

```
REPEAT integer i=1:Tam times
   { IF C[i].energy > binary-fission-wear THEN
{ Create CT set with the Bacteria separated;
      CT[NewTam] = C[i]
      C[i].energy = C[i].energy – binary-fission-wear
      NewTam = NewTam + 1;}
}
END REPEAT
REPEAT i=1:Tam times
   { IF C[i].energy > Spontaneous-Mutation-wear THEN {
      IF random > Spontaneous-Mutation-Probability
THEN
      {Save C*[i] = C[i]
       Mutate C[i] to feasible solution.
       C[i].energy = C[i].energy – Spontaneous-
                    Mutation-wear}
 }
       IF random > Reverse-Mutation-Probability THEN
         {IF C[i].energy > Reverse-Mutation-wear THEN
        {IF C[i] was not better than C*[i] THEN
           { Apply reverse mutation C[i] = C*[i]
C[i].energy = C[i].energy – Reverse-Mutation-wear }
        }
}
IF C[i].energy > 0 THEN
{Add energy to C[i] with GLS
 CT[NewTam] = C[i];
 NewTam = NewTam + 1;}
 }
END REPEAT
A = A - 1
}
END WHILE
Select NO-Dominated
END
```

The main functions that are described in this algorithm are the following:

a. Generation of Initial Bacteria Colonies: Initial bacteria colonies are generated as feasible solutions to the problem.

b. Bacteria Rotation: refers to a function (GLS) that is created to measure the amount of energy the bacteria is able to release in a rotation, in which the search directions are based on by the program. The following equation refers to this function.

$$GLS_i = \frac{\prod_{r=1}^{\left\lfloor \frac{nobj}{2} \right\rfloor} \exp\left(\frac{-f_{2*r-1}(B_i)}{\max\left(f_{2*r-1}(C)\right)}\right)}{\prod_{r=1}^{\left\lfloor \frac{nobj}{2} \right\rfloor} \exp\left(\frac{-f_{2*r}(B_i)}{\max\left(f_{2*r}(C)\right)}\right)}; B_i \in C$$

c. Race to light: a function that is known for each bacteria colony in order to move from one location to another where the light is more intense. The intensity of the light is determined randomly among the four mayor intensities in order to get a more diverse search space.

d. Binary Fission: a process that undergoes bacteria when it is duplicated, generating a new bacteria with the initial intensity and position as the mother bacteria.

e. Spontaneous mutation: a function where bacteria change its structure and its position with respect to the light, which can improve a solution or make it worse. This mutation is only done to some bacteria chosen randomly, but subject to the energy they have to divide.

f. Mutation by reversion: this process is done on a percentage of mutated bacteria, given that some bacteria were made worse during mutation process.

g. Bacteria selected for death: Some bacteria do not have sufficient energy to rotate, so they will no longer continue in the colony. These bacteria are selected for death, while the rest undergo binary fission.

h. Photosynthesis: bacteria are fed by energy, according to the natural process of photosynthesis, expressed in ATP. This function assigns an ATP to each bacteria according to the following function:

$$ATP_i = energia\_por\_fotosi\ ntesis * \sum_{r=1}^{nobj} \exp\left(\frac{-f_r(B_i)}{\max\left(f_r(C)\right)}\right); B_i \in C$$

ATPi represents the energy for each bacteria I, which is given in terms of the objective functions to be optimized.

IV. MATHEMATICAL FORMULATION AND ANALYSIS OF PARETO OPTIMA OF THE MULTI-OBJECTIVE PROBLEMS

A. *Example 1: Two supportive functions:*

In this example, two supportive functions were considered:

$$f_1 = x^2 + y^2$$
$$f_2 = (x-4)^2 + (y-4)^2$$
$$x, y \in [-10, 10]$$

As shown in figure 5, the functions are graphed in a coordinate graph where, f_1 is traced in blue, f_2 is traced in pink and the sum of both is traced in green.

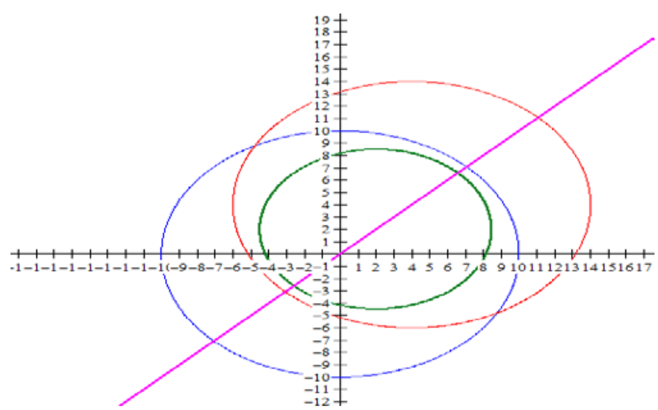

Figure: 5 Graphical representation of Example 1.

For each function an analysis of derivatives was formulated in order to obtain the equation that optimizes the function and generate a Pareto Front:

Finding the derivative of the first function with respect to x:

$$\frac{df}{dx}(f1(x,y)) = \frac{df}{dx}[x^2 + y^2 - 100]$$

$$\frac{df}{dx}\big(f1(x,y)\big) = \frac{df}{dx}(x^2) + \frac{df}{dx}(y^2) + \frac{df}{dx}(100)$$

$$0 = \frac{df}{dx}\big(f1(x,y)\big) = 2x + 0 + 0$$

$$x = 0$$

Finding the derivative of the 1st with respect to y:

$$\frac{df}{dy}(f1(x,y)) = \frac{df}{dy}[x^2 + y^2 - 100]$$

$$\frac{df}{dy}\big(f1(x,y)\big) = \frac{df}{dy}(x^2) + \frac{df}{dy}(y^2) + \frac{df}{dy}(100)$$

$$0 = \frac{df}{dy}\big(f1(x,y)\big) = 0 + 2y + 0$$

$$y = 0$$

So the optimal point for the first function is [0.0]

Now, finding derivative of the second function with respect to x:

$$\frac{df}{dx}(f2(x,y)) = \frac{df}{dx}[(x-4)^2 + (y-4)^2 - 100]$$

$$\frac{df}{dx}\big(f2(x,y)\big) = \frac{df}{dx}(x-4)^2 + \frac{df}{dx}(y-4)^2 + \frac{df}{dx}(100)$$

$$0 = \frac{df}{dx}\big(f2(x,y)\big) = 2(x-4) + 0 + 0$$

$$x = \frac{4}{2} = 2$$

Finding derivative of the second function with respect to y:

$$\frac{df}{dy}(f2(x,y)) = \frac{df}{dy}[(x-4)^2 + (y-4)^2 - 100]$$

$$\frac{df}{dy}\big(f2(x,y)\big) = \frac{df}{dy}(x-4)^2 + \frac{df}{dy}(y-4)^2 + \frac{df}{dy}(100)$$

$$0 = \frac{df}{dy}\big(f2(x,y)\big) = 0 + 2(y-4) + 0$$

$$y = \frac{4}{2} = 2$$

So the optimal point for the first function is [2.2].

By unifying both points, the line obtained, which minimizes each of the functions, is $x = y$. Through this equation and by permitting a spacing of 0.25 between each point, the Pareto Optima is obtained, which is shown in figure 6.

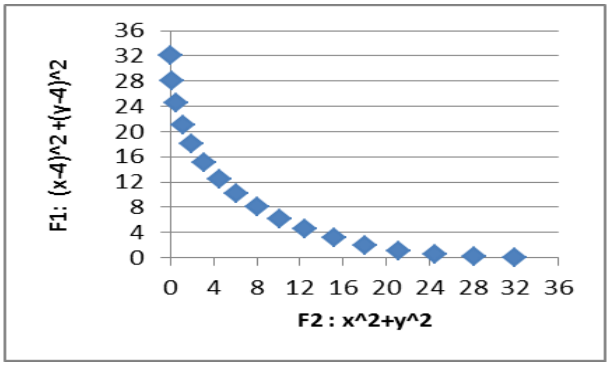

Figure: 6 Pareto Front generated by partial derivatives for example 1.

B. *Example 2: Two conflictive functions:*

This example considered two conflictive functions:

$$f_1 = x^2 + 2xy + y^2$$

$$f_2 = (3x-1)^2 + (y-3)^2$$

$$x, y \in [-10, 10]$$

As shown in figure 7, the functions are graphed in a coordinate graph where, f_1 is traced in blue and f_2 is traced in red.

Figure 7. Graphical representation of example 2

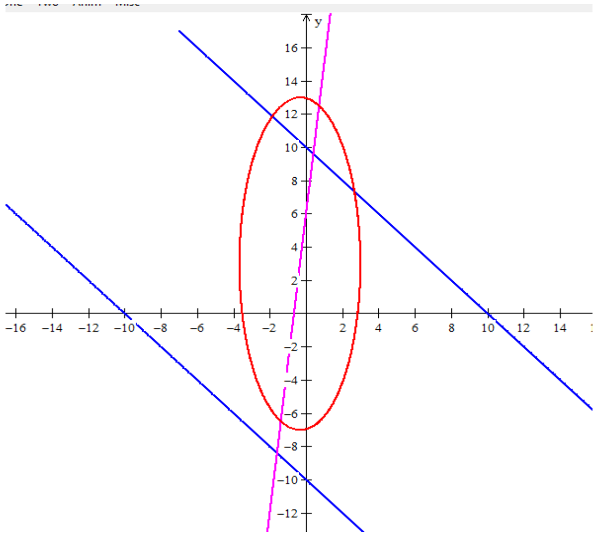

Figure: 7 Graphical representation of Example 2.

For each function an analysis of derivatives was formulated in order to obtain the equation that optimizes the function and generate a Pareto Front. After doing the same process as in example 1, the equation obtained, which minimizes each of the functions, is $y = 9x + 6$. Through this equation and by permitting a spacing of 0.25 between each point, the Pareto Optima is obtained, which is shown in figure 8.

Figure viii. Pareto Front generated by equation generated through partial derivatives in example 2

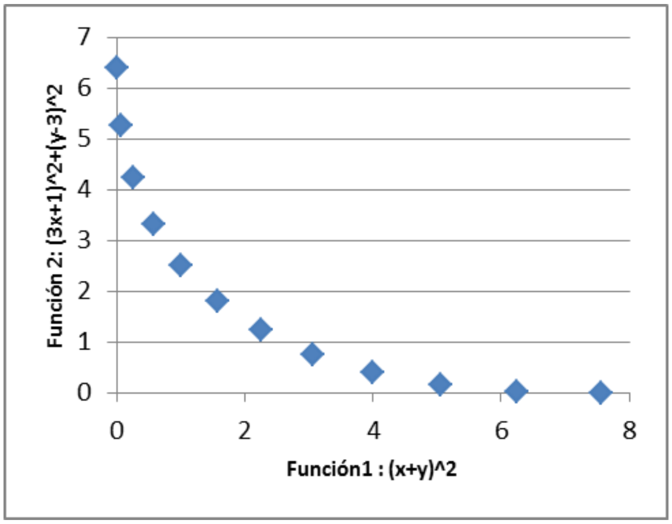

Figure: 8 Pareto Front generated by partial derivatives for example 2.

V. GENERATION OF RESULTS THROUGH MCDM AND GBO

In order to demonstrate the efficiency of GBO, these same problems were approached with GBO and with the MCDM technique known as Convex Combination of Criteria.

A. Example 1: Two supportive functions:

In this example, Figure 9 shows the Pareto Front generated by the MCDM approach, Figure 10 shows the Pareto Front generated by GBO and Figure 11 shows the combined Pareto Front with the different approaches, including the derivative.

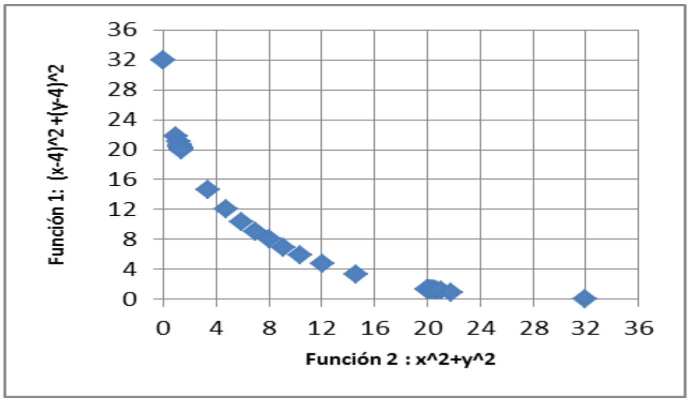

Figure: 9 Pareto Front generated by MCDM approach for example 1.

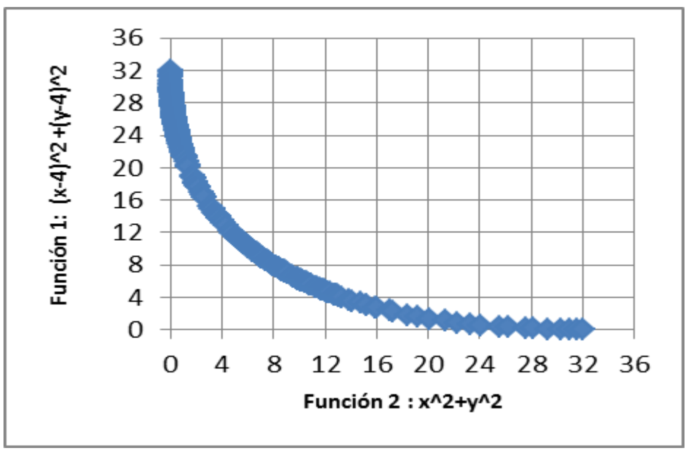

Figure: 10 Pareto Front generated by GBO approach for example 1.

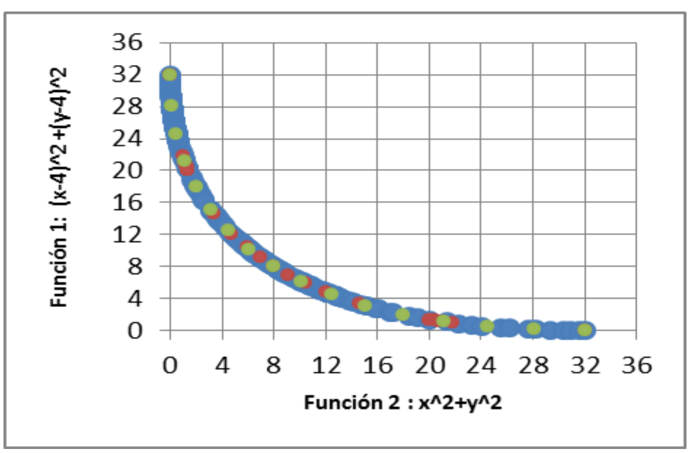

Figure: 11 Pareto Fronts compared in example 1.

In the previous figure, the green dots represent the Pareto Front generated by partial derivatives, the red dots represent the MCDM approach and the blue dots represent GBO approach.

B. Example 2: Two conflictive functions:

In this example, Figure 12 shows the Pareto Front generated by the MCDM approach, Figure 13 shows the Pareto Front generated by GBO and Figure 14 shows the combined Pareto Front with the different approaches, including the derivative.

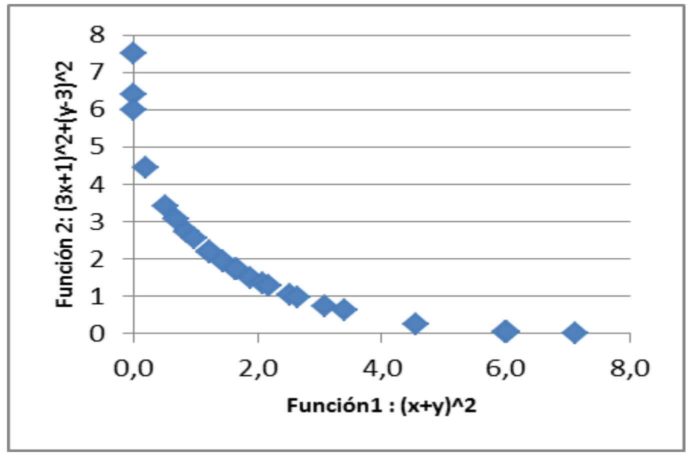

Figure: 12 Pareto Front generated by MCDM approach for example 2.

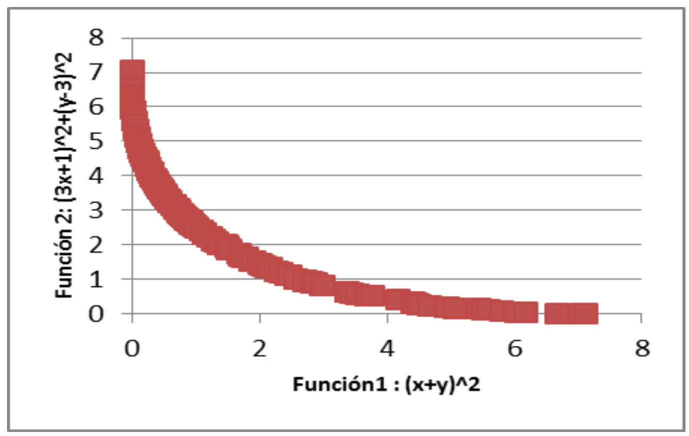

Figure: 13 Pareto Front generated by GBO approach for example 2.

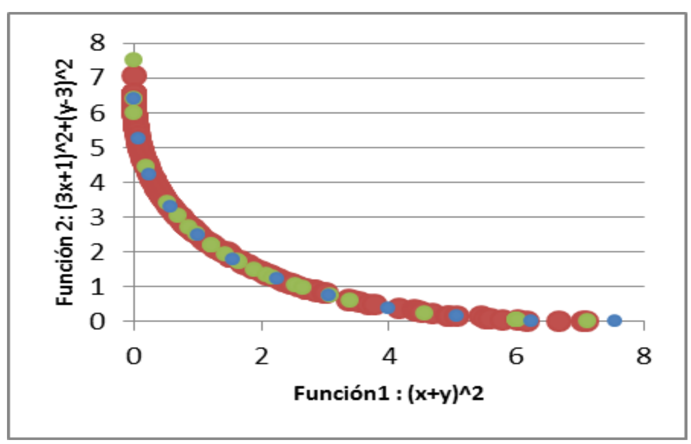

Figure: 14 Pareto Fronts compared in example 2.

In the previous figure, the blue dots represent the Pareto Front generated by partial derivatives, the green dots represent the MCDM approach and the red dots represent GBO approach.

VI. COMPARISON OF RESULTS

As mentioned at the beginning, the efficiency of the GBO approach was demonstrated by just observing how the Pareto Front generated by GBO cover all the points generated by the exact methodologies, reaching Pareto Optima. Yet the metrics calculated below and shown in tables 1 and 2, which are spacing and extreme points, detail on how efficient and robust is the Pareto Front generated by the GBO algorithm.

Table I. Comparison of results for extreme points metric

Examples	***Results for Extreme Points***		
	Derivatives	***MCDM***	***GBO***
Example 1			
Min F1 [f1,f2]	[0,32]	[0,31.99]	[0,32]
Min F2 [f1,f2]	[32,0]	[32,0]	[32,0]
Example 2			
Min F1 [f1,f2]	[0,6.4]	[0,7.489]	[0,7.05]
Min F2 [f1,f2]	[7.5625,0.006]	[7.11, 0]	[7.11,0]

Table II. Comparison of results for spacing metric

Examples	***Results for Spacing***		
	Derivatives	***MCDM***	***GBO***
Example 1	0,83721	2,66612	0,25767
Example 2	0,20404	0,45734	0,06899

When comparing the extreme points metric, it can be observed that GBO reaches the Pareto Optima solutions for the three out of four cases, the fourth case (which was the extreme point that minimizes the first function in the second example) was 10% higher, but the MCDM approach was 17% higher. The latter is due to the fact that the second example was composed by conflictive functions and the definition for Pareto Optima in the MCDM technique didn't cover this type of functions.

On the other hand, the comparison for spacing shows that robustness that is achieved in the solutions generated by the GBO approach, given that this one outperforms the other approaches. For the first example spacing was reduced with respect to the MCDM in 69% and with respect to the derivative approach in 90%. For the second example, it reduced with respect to the MCDM in 66% and with respect to the derivative approach in 85%.

VII. CONCLUSIONS AND FURTHER RESEARCH

Through this research paper, Global Bacteria Optimization (GBO) was introduced as a population-based metaheuristic that combines both diversification and intensification concepts that characterize today's metaheuristics.

Work performed on this metaheuristic [2,11], including results generated in this paper, demonstrate how GBO is a solution approach to multi-objective optimization problems, obtaining Pareto Optima solutions that are both efficient and robust. In this paper, different types of mathematical functions were compared, all of them, searching for their minimum, simultaneously. Pareto Fronts generated by GBO clearly outperform results generated by MCDM approach and have shown to reach Pareto Optima, as compared to the exact solutions generated by the equation derived from both functions. Both extreme points and spacing metrics demonstrate the efficiency of this methodology as a multi-objective solution approach.

Future work on this area is needed, especially by approaching NP-Hard problems, since this are to be considered and compared to other metaheuristics as already done by Gomez-Vizcaino [2]. This research group is already working on an application to the Resource Constrained Project Scheduling problem (RCPSP) [13], clearly an NP-Hard problem. Other work in combinatorial optimization problems is greatly encouraged. GBO is a new metaheuristic that has shown to produce excellent results and it can be easily implemented to any type of problems.

VIII. ACKNOWLEDGMENT

This paper was supported by the Colombian Institute for Development in Science and Technology "Francisco José de Caldas" (COLCIENCIAS) No. 2333-502-27695 and Fundación Centro de Investigación en Modelación Empresarial del Caribe (FCIMEC).

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