Volume 3, No. 3, May-June 2012

ISSN No. 0976-5697

International Journal of Advanced Research in Computer Science

RESEARCH PAPER

Available Online at www.ijarcs.info

New Aggregation Operator for Triangular Fuzzy Numbers based on the Geometric Means of L- and R- Apex Angles

Manju Pandey*
Research Scholar, Computer Science and Engineering
Maulana Azad National Institute of Technology
Bhopal, India
manjutiwa@gmail.com

Dr. Nilay Khare
Professor and Head, Computer Science and Engineering
Maulana Azad National Institute of Technology
Bhopal, India
nilay.khare@rediffmail.com

Dr. S.C. Shrivastava
Former Professor and Head, Electronics and Communication and Computer Science and Engineering
Maulana Azad National Institute of Technology Bhopal, India
scs_manit@yahoo.com

Abstract: In [1], authors have proposed a new aggregation operator for triangular fuzzy numbers (TFNs) in which the L- and R- apex angles of the piecewise continuous linear membership function of the composite or resultant or aggregate TFN are the arithmetic means of the corresponding L- and R- apex angles of the individual TFNs. Taking the basic idea of an aggregation operator for triangular fuzzy numbers based on the means of the corresponding L- and R- apex angles further, a new aggregation operator for triangular fuzzy numbers (TFNs) based on the geometric means of the corresponding L- and R- apex angles of the individual TFNs has been proposed in this paper. The L- and R- apex angles have been treated independently as has been done in the previous paper. Computation of the aggregate is demonstrated with a numerical example. Corresponding arithmetic and geometric aggregates have also been computed.

Keywords: LR Fuzzy Number, Triangular Fuzzy Number, Apex Angle, L-Apex Angle, R-Apex Angle, Aggregation Operator, Arithmetic and Geometric Mean

I. INTRODUCTION

Many different types of fuzzy numbers are defined in the literature dealing with fuzzy logic and applications. In this paper only one class of fuzzy numbers i.e., Triangular Fuzzy Numbers (TFNs) which are a special class of LR fuzzy numbers are treated.

A. Triangular Fuzzy Numbers :

TFNs are extensively used in fuzzy applications owing to their simplicity. TFNs are used in fuzzy applications where uncertainty exists on both sides of a value or parameter [2]. TFNs are characterized by an ordered triplet of real numbers <l,m,u>. Figure depicts a TFN (l,m,u) with values v on x-axis and membership or grade μ along y-axis. The L- and R- apex angles are shown with arcs having one and two dashes respectively. Vertices of the triangle are at (l, 0), (m, 1) & (u,0) moving clockwise. Line between (l, 0) and (m, 1) and between (m, 1) and (u,0) are the membership functions for the values in the intervals [l, m] and [m, u] respectively. Membership function of the TFN is the piecewise continuous linear function represented by the lines L and R respectively.

Intuitive meaning of such TFN is that the fuzzy number is approximately or around m [3]. The value of the fuzzy number varies in the range l and u. The possibility or membership grade of the number being a specified value v between l and u, $v \in [l, u]$ is represented by the ordinate of the projection of v on L or R accordingly as $l \le v \le m$ or $m \le v \le u$. [l,m] is the range or interval which depicts or represents the possibilities of the number being a specific value less than m and [m, u] is the interval which depicts the possibilities of the number being a specific value greater than m. Possibility takes on the maximum value of 1 at m and reduces with increasing distance on either side (left / right) of m, becoming zero beyond l at the left and m at the right respectively.

Mathematically, a TFN is defined as follows. Let $l, m, u \in \mathbf{R}$, $l < m < u$. The fuzzy number $t: \mathbf{R} \to [0, 1]$ denoted by

$$t = \begin{cases} 0, & if\ x < l \\ \dfrac{x-l}{m-l}, & if\ l \le x \le m \\ \dfrac{u-x}{u-m}, & if\ m \le x \le u \\ 0, & if\ x > u \end{cases}$$

is called a triangular fuzzy number [2][3][4][5].

II. FUZZY AGGREGATION

Aggregation operations on fuzzy numbers are operations by which several fuzzy numbers are combined to produce a single fuzzy number [6]. An excellent account of Mathematical Aggregation Operators is given in [7]

A. Arithmetic Mean:

The arithmetic mean aggregation operator [6][8] defined on n TFNs $<l_1,m_1,u_1>$, $<l_2,m_2,u_2>$, …, $<l_i,m_i,u_i>$, …, $<l_n,m_n,u_n>$ produces the result $<\bar{l}, \bar{m}, \bar{u}>$ where

$$\bar{l} = \frac{1}{n}\sum_{1}^{n} l_i\ ,\ \bar{m} = \frac{1}{n}\sum_{1}^{n} m_i\ , \text{and}\ \bar{u} = \frac{1}{n}\sum_{1}^{n} u_i$$

B. Geometric Mean:

The geometric mean aggregation operator [6][9] defined on n TFNs $<l_1,m_1,u_1>$, $<l_2,m_2,u_2>$, …, $<l_i,m_i,u_i>$, …, $<l_n,m_n,u_n>$ produces the result $<\bar{l}, \bar{m}, \bar{u}>$ where

$$\bar{l} = \left(\prod_1^n l_i\right)^{\frac{1}{n}},\ \bar{m} = \left(\prod_1^n \mathrm{m}_i\right)^{\frac{1}{n}},\ \text{and}\ \bar{u} = \left(\prod_1^n u_i\right)^{\frac{1}{n}}$$

Other aggregation operators have also been defined in literature. For examples see [9][10][11][12].

C. Applications of Aggregation:

The combination/aggregation/fusion of information from different sources is at the core of knowledge based systems. Applications include decision making, subjective quality evaluation, information integration, multi-sensor data fusion, image processing, pattern recognition, computational intelligence etc. An application of aggregation operators in fuzzy multicriteria decision making is discussed in [8][9]. Another application in sensor data fusion is discussed in [11].

Other aggregation operators have also been defined in literature. For examples see [9][10][11][12].

D. Organization of the Paper:

A new aggregation operator for TFNs in which the L- and R- apex angles of the composite or resultant or aggregate TFN are the geometric means of the corresponding L- and R- apex angles of the individual TFNs has been proposed in this paper. The L- and R- apex angles have been treated independently as has been done in the previous paper. The operator is described in the next section and a numerical example is given. Results from the proposed operator are presented alongside those obtained from arithmetic and geometric mean aggregate operators.

III. PROPOSED AGGREGATION OPERATOR

Consider the TFN shown in Fig. 1. The most likely value of this TFN is m where the possibility μ= 1. The apex angle of this TFN is L lpu

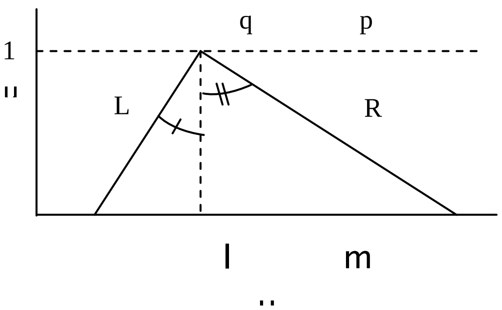

Figure 1: Triangular Fuzzy Number

However, L lpu = L lpm + L upm
But, L lpm = π/2 - L mlp

L lpm is the contribution of the left-side line L to the apex angle referred to here as the L-apex angle.

Computing the geometric mean over n TFNs we have

$$\left(\prod_1^n (\mathcal{L}\, lpm)_i\right)^{\frac{1}{n}} = \left(\prod_1^n (\mathcal{L}\,(\pi/2 - mlp_i))\right)^{\frac{1}{n}}$$

The left side of the above equation represents the contributions of the left lines i.e., L's of all TFNs to the aggregate apex angle. It can be seen that

$$\tan\left(\prod_1^n (\mathcal{L}\, lpm)_i\right)^{\frac{1}{n}} = \frac{1}{\tan\left(\prod_1^n (\mathcal{L}\, mlp)_i\right)^{\frac{1}{n}}}$$

It can be shown that

$$\tan\left(\prod_1^n (\mathcal{L}\, mlp)_i\right)^{\frac{1}{n}} = \tan\left(\prod_1^n \tan^{-1}(\mathrm{m}_i - \mathrm{l}_i)\right)^{\frac{1}{n}}$$

This represents the slope of the resultant aggregate left line $\bar{\mathrm{L}}$. Similarly, it can be shown that the slope of the resultant fuzzy aggregate right line $\bar{\mathrm{R}}$ is

$$-\left(\tan\left(\prod_1^n \tan^{-1}(\mathrm{u}_i - \mathrm{m}_i)\right)^{\frac{1}{n}}\right)^{-1}$$

Under identical mathematical treatment, it can be shown that $\bar{m} = \left(\prod_1^n \mathrm{m}_i\right)^{\frac{1}{n}}$. Subsequently it is possible to show that

$$\bar{l} = \left(\prod_1^n \mathrm{m}_i\right)^{\frac{1}{n}} - \tan\left(\prod_1^n \tan^{-1}(\mathrm{m}_i - \mathrm{l}_i)\right)^{\frac{1}{n}},\ \text{and}$$

$$\bar{u} = \left(\prod_1^n \mathrm{m}_i\right)^{\frac{1}{n}} + \tan\left(\prod_1^n \tan^{-1}(\mathrm{u}_i - \mathrm{m}_i)\right)^{\frac{1}{n}}$$

IV. NUMERICAL EXAMPLE

Consider the two triangular fuzzy numbers **<1,1.5,3>** and **<5,6.5,9>** aggregate TFN is computed as $\bar{\mathrm{m}} = \sqrt{(1.5)\times(6.5)}$ =3.1224;

$\bar{l} = 3.1224 - \tan\left(\sqrt{\tan^{-1}(0.5)\times\tan^{-1}(1.5)}\right)$ =2.3219;

Similarly, $\bar{u}$ can be computed as 5.00

Thus we have the aggregate as **<2.3219,3.1224,5.00>.** The arithmetic mean aggregate is **<3,4,6>** and the geometric mean aggregate is **<2.24,3.12,5.19>** respectively.

V. CONCLUSIONS

In this paper we have defined a new aggregate of triangular fuzzy numbers based on the geometric mean of the apex angle. The L- and R-apex angles have been treated independently. A numerical example has been worked out. The aggregate is the resultant piecewise continuous membership function line whose L- and R- apex angles are the geometric means of the L- and R- apex angles of the individual TFNs. The suitability of the aggregation operator proposed in this paper in different fuzzy logic applications involving fuzzy number aggregation remains to be explored.

VI. ACKNOWLEDGEMENTS

Manju Pandey gratefully acknowledges Shri Harendra Bikrol, Associate Professor, and Shri Mithilesh Atulkar, Associate Professor and Head of Computer Applications Department at National Institute of Technology (NIT), Raipur, Chhattisgarh, India, for their constant help and support. She is also thankful to Dr. Shrish Verma, Head of Electronics and Information Technology Departments and Central Computer Centre at NIT Raipur for his encouragement and support.

VII. REFERENCES

[1]. Manju Pandey, Nilay Khare, and S.C. Shrivastava, “A New Aggregation Operator for Triangular Fuzzy Numbers based on the Arithmetic Means of the L- and R- Apex Angles”, Communicated

[2]. A.K. Verma, A.Srividya and R. S Prabhu Gaonkar, “Fuzzy-Reliability Engineering Concepts and Applications”, Narosa Publishing House Pvt. Limited, 2007`

[3]. Hung T, Nguyen, Elbert A. Walker, “A First Course in Fuzzy Logic”, Third Edition, Chapman & Hall/CRC, Taylor and Francis Group, 2006

[4]. Michael Haans, “Applied Fuzzy Arithmetic – An Introduction with Engineering Applications”**,** Springer-Verlag, Berlin Heidelberg, 2005

[5]. Kauffman, A. and Gupta, M.M., “Introduction to Fuzzy Arithmetic – Theory and Applications”, Van Nostrand Reinhold Company, New York, 1985

[6]. George J. Klir and Tina A. Folger, “Fuzzy Sets, Uncertainty, and Information”, Prentice-Hall of India Pvt. Ltd., New Delhi, 1988

[7]. Marcin Detyniecki, “Mathematical Aggregation Operators and their Application to Video Querying”, PhD Thesis in Artificial Intelligence Specialty, University of Paris, © 2000 Detyniecki

[8]. Mehdi Fasangarhi, Farzad Habibipour Roudsari, “The Fuzzy Evaluation of E-Commerce Customer Satisfaction”, World Applied Sciences Journal 4 (2): 164-168, IDOSI Publications 2008

[9]. Oliver Meixner, “Fuzzy AHP Group Decision Analysis and its Application for the Evaluation of Energy Sources”, Proceedings of the 10th International Symposium on the Analytic Hierarchy/Network Process Multi-criteria Decision Making, July 29 - August 1 2009, University of Pittsburgh , Pittsburgh, Pennsylvania, USA

[10]. Bing Yi Kang, Ya Juan Zhang, Xin Yang Deng, Ji Yi Wu, Xiao Hong Sun and Yong Deng “A New Method of Aggregation of Fuzzy Number Based on the Dempster/Shafer Theory”, Chinese Control and Decision Conference, ©IEEE 2011

[11]. Genevieve Cron, Bernard Dubuisson, “A Weighted Fuzzy Aggregation Method”, ©IEEE 1998

[12]. Ondrej Pavlacka, Jana Talasova, “Application of the Fuzzy Weighted Average of Fuzzy Numbers in Decision Making Models”, Proceedings of EUSFLAT Conference 2007