



## A Comparative Analysis for Wavelets and Threshold Estimation Selection for Denoising of Audio Signals of Some Indian Musical Instruments

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**Abstract:** It is known that noise is present in all communication channels, therefore, the generated signal, when transmitted through these channels, get corrupted. Denoising of such noisy signals without losing its features is a challenging task. The wavelet based methods has proved to be one of the best tool for denoising purposes. The proper selection of wavelet function and noise estimation algorithm is a complex task. As not all wavelet function and all noise estimation methods work well for all types of signals. In this paper an effort has been made to find a suitable wavelet function and noise estimation method to give good denoising results of audio signals from some Indian musical instruments such as Tabla, Pakhawaj, Flute, Harmonium and Taanpura. For this purpose Haar, Db10, Coif5 and Bior6.8 wavelets are considered and some well known threshold estimation methods i.e. Sqrtwolog, SURE (Rigrsure and Heursure) and Minimaxi are considered for comparative analysis. The quality of denoised musical signal is expressed in terms of PSNR as compared to original signals.

**Keywords:** Wavelets; Denoising; Haar; Db10; Coif5; Bior6.8; Minimaxi; Rigrsure; Heursure; Square-Root-Log.

### I. INTRODUCTION

Signal distortion created by noise is the major problem occurring in generation and transmission of signals and therefore, affects the performance of associated systems. Many techniques have been developed for removing noise from signals while retaining details. To achieve the goal of noise reduction without losing much detail, wavelet transform based techniques have shown promising and encouraging results in these areas because of some of their excellent properties like time frequency localization and multi resolution analysis.

Denoising of audio signal of musical instruments has become an important research field. To achieve good audio quality noise reduction of musical signal is desirable. A noise is an unwanted signal which deteriorates the characteristics of original signal. There are various types of noises present in environment such as colored noise, burst noise, White noise etc. This noise signal may occupy either some specific frequency band or entire frequency band. When noise also share the frequency band of signal, then it becomes very difficult to remove this noise without losing some signal information. Therefore, noise removal without losing original features of signal is a challenging task and has become an active area of research. In wavelet based techniques denoising is done by soft and hard thresholding of wavelet coefficients.

In wavelet analysis low frequency coefficients mainly represent signal and high frequency coefficients with randomness represent noise. Denoising is achieved by selecting a threshold for such high frequency coefficients. A

sizable amount of work has been done in area of wavelet based denoising and reported in literature. An effective method based on wavelet transform of signal denoising

utilizing soft thresholding is given in [1]. It suggests that non-linear denoising known as wavelet shrinkage, of high frequency components performs well over conventional frequency selective filter approach. The threshold values for correlated noise signals are calculated by a method proposed by Johnston and Silverman [2]. A modified threshold selection method is reported in [3]. It suggests that high threshold values for audio signals cut part of the original signal too. Another method which improves SNR of a signal in presence of transients and harmonics utilizing block thresholding is proposed in [4]. A multi-wavelet transform method proposed in [5] with appropriate initialization represent signal in a better way than the conventional wavelet thresholding methods. It clearly identifies noise and only the shrinkage function is modified to multivariate shrinkage.

A noise reduction filter is proposed in [6]. It passes the noisy signal to an adaptive prediction filter and first and second signal component is obtained. The first component represents predictable part of noisy signal and second to a prediction error. Both the components are then attenuated according to the signal and noise content level which are further recombined to form enhanced output signal with low delay, low computation and reduced colored and white noise.

A shift invariance DWT based thresholding is suggested in [7]. It improves performance of audio signals but increases the computational load and storage by a logarithmic multiplied factor due to its non-orthogonal transformation matrix. An average improvement in segmental SNR, speech quality and log-spectral distortion is proposed by using non-causal estimation than decision directed approach in [8]. A novel approach in audio denoising is presented in [9] by grouping signal blocks together. These blocks are then filtered and replaced in their original positions. The blocks overlap each other and therefore, detail estimation of every element is obtained. In transformation procedure noise is removed and

signal is further reconstructed with the application of Haar wavelet transform. It gives good denoising performance both in terms of PSNR and audible quality of the audio signal. A performance comparison of various denoising methods is presented in [10]. It suggests Coif5 wavelet and lowest decomposition level to be most appropriate for denoising of speech signals. Also Heursure method is suggested in [11] to give better denoising performance of audio signals from Indian musical instruments. The rest of the paper is organized as follows, section II gives brief introduction to wavelet transform analysis, section III describes denoising schemes, section IV displays the experimental results, section V represents the conclusion and references are given in section VI.

## II. DISCRETE WAVELET TRANSFORM

The Wavelet Transform (WT) is a mathematical tool useful in the analysis of signals. Its representation involves the decomposition of the signals in wavelet basis functions  $\psi(t)$  given by,

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \quad a, b \in \mathbb{R}(1)$$

Here  $a, b$  are called scale and position parameters respectively. If scales and positions are chosen based on powers of two, so called dyadic scales and positions, then analysis becomes much more efficient and just as accurate. It was developed in 1988 by S. Mallat. In this case, wavelet function becomes,

$$\psi_{m,n}(k) = 2^{-\frac{m}{2}} \psi(2^{-m}k - n) \quad m, n \in \mathbb{Z}(2)$$

In orthonormal basis for  $\mathbb{L}^2(\mathbb{R})$ . For a given function  $S(k)$ , the inner product  $\langle S, \psi_{m,n} \rangle$  then gives the discrete wavelet transform as, [12]

$$DWT(m, n) = \langle S, \psi_{m,n} \rangle = 2^{-\frac{m}{2}} \sum_{k=-\infty}^{\infty} S(k) \cdot \psi^*(2^{-m}k - n) \quad (3)$$

The multi resolution theory given by S. Mallat and Meyer proves that any conjugate mirror filter characterizes a wavelet  $\psi$  that generates an orthonormal basis of  $\mathbb{L}^2(\mathbb{R})$ , and that a fast discrete wavelet transform is implemented by cascading these conjugate mirror filters. The wavelet decomposition of a signal  $S(k)$  based on the multi resolution theory can be obtained using digital FIR filters [13]. The FIR filter based wavelet decomposition scheme is shown as shown in figure 1.

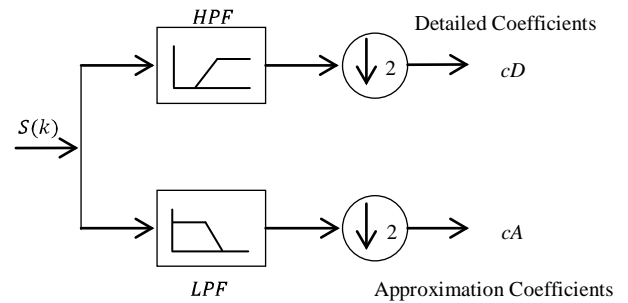


Figure 1. One level wavelet decomposition (Analysis)

The arrangement shown above has used two wavelet decomposition (Analysis) filters which are High Pass and Low Pass respectively followed by down sampling by 2 producing half of input data point of High and Low frequency. The High frequency coefficients are called Detailed Coefficients (cD) and Low frequency coefficients are called Approximation Coefficients (cA). After decomposition, the signal can be reconstructed back by Inverse Wavelet Transform. The corresponding Filter Bank structure for reconstruction is shown in figure 2.

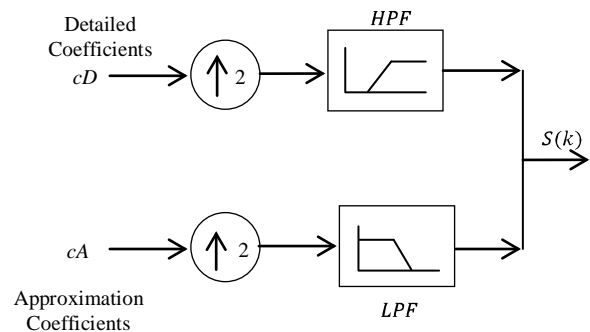


Figure 2. One level wavelet reconstruction (Synthesis)

The signal  $S(k)$  can be decomposed in several levels. A three level wavelet decomposition tree is shown in figure 3 [14].

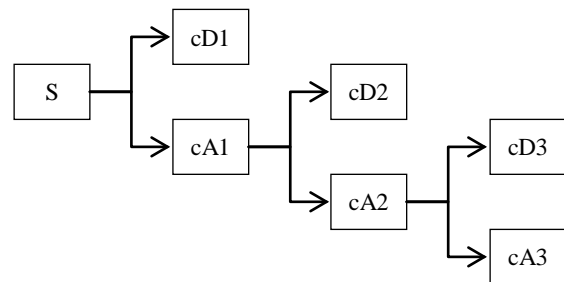


Figure 3. Three level wavelet decomposition tree

## III. WAVELET DENOISING SCHEMES

To prevent deterioration in quality of instrumental signals, denoising of the signal is required. Let assume an instrumental signal  $S(k)$  is corrupted by the noise  $w(k)$  as  $S'(k) = S(k) + w(k)$ , where  $w(k)$  is White Gaussian Noise. White Gaussian noise is difficult to remove as it is located at all frequencies.

The wavelet based overall denoising scheme is shown in figure 4.

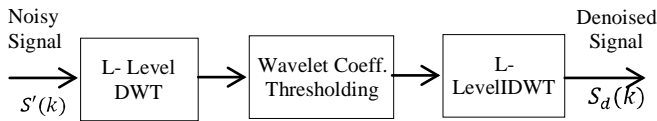


Figure 4. Overall Denoising Scheme

As seen from figure 4, the denoising scheme involves three main steps,

- a. L-level Wavelet Decomposition of input noisy signal.
- b. Threshold estimation and thresholding of wavelet coefficients.
- c. L-level Inverse Wavelet Transform for reconstruction of denoised signal.

Wavelet denoising involves thresholding in which coefficients below a specific threshold value ( $\lambda$ ) are set to zero. It helps in eliminating noise but main characteristics of the original signal are preserved. This is called Hard Thresholding while Soft-Thresholding set the wavelet coefficients to zero which are below threshold as well as it simply shrinks or scales other coefficients which are above the threshold value [1]. Threshold selection is an important process which directly affects the quality of output denoised signal. There are several well-known threshold estimation methods available in literature. Some of them are discussed here briefly. In this paper, performances of four well-known standard threshold estimation methods are investigated for audio signals of Indian musical instruments corrupted by white Gaussian Noise. The effect of wavelet decomposition level (L) is also investigated. These four methods are briefly described as follows:

**A. Minimaxi Criterion:**

This method finds threshold ( $\lambda$ ) using minimaxi principle. It uses a fixed threshold to yield minimaxi performance for mean square error against an ideal procedure. The Minimaxi principle is used in statistics to design estimators. Since the de-noised signal can be assimilated to the estimator of the unknown regression function, the minimaxi estimator is the option that realizes the minimum, over a given set of functions of the maximum Mean Square Error (MSE). This procedure finds optimal thresholds [15]. The threshold is given by:

$$\lambda = \begin{cases} \sigma(0.3936 + 0.1829 \log_2 N)N > 32 \\ 0 & N < 32 \end{cases} \quad (4)$$

Where  $\sigma = \text{median}\left(\frac{|\omega|}{0.6745}\right)$  and  $\omega$  is the detailed wavelet coefficient vector at unit scale and  $N$  is the length of signal vector.

**B. Sqtwolog Criterion:**

The threshold values ( $\lambda$ ) are calculated by universal threshold (square root log) method given by,

$$\lambda_j = \sigma_j \sqrt{2 \log(N_j)} \quad (5)$$

Where,  $N_j$  is the length of the noisy signal at  $j^{\text{th}}$  scale and  $\sigma_j$  is Median Absolute Deviation (MAD) at  $j^{\text{th}}$  scale given by,

$$\sigma_j = \frac{MAD_j}{0.6745} = \frac{\text{median}(|\omega|)}{0.6745} \quad (6)$$

Where,  $\omega$  represent wavelet coefficients at scale  $j$ .

**C. Rigrsure:**

It is a soft threshold evaluator of unbiased risk. Suppose  $W = [\omega_1, \omega_2, \dots, \omega_N]$  is a vector consists of the square of wavelet coefficients from small to large. Select the minimum value  $r_b(b^{\text{th}}r)$  from risk vector, which is given as,

$$R = \{r_i\}_{i=1,2,\dots,N} = \frac{[N-2i+(N-i)\omega_i + \sum_{k=1}^i \omega_k]}{N} \quad (7)$$

as the risk value. The selected threshold is  $\lambda = \sigma \sqrt{\omega_b}$  where,  $\omega_b$  is the  $b^{\text{th}}$  squared wavelet coefficient (coefficient at minimum risk) chosen from the vector  $W$  and  $\sigma$  is the standard deviation of the noisy signal.

**D. Heursure:**

Threshold is selected using a combination of Sqtwolog and Rigrsure methods. If the signal to noise ratio is very small, the SURE method's estimation is poor. In such case, fixed form threshold of Sqtwolog method gives better threshold estimation [15]. Let threshold obtained from Sqtwolog method is  $\lambda_1$  and threshold obtained from Rigrsure is  $\lambda_2$  then Heuristic SURE gives the threshold given by,

$$\lambda = \begin{cases} \lambda_1 A > B \\ \min(\lambda_1, \lambda_2) A \geq B \end{cases} \quad (8)$$

Where,  $A = \frac{s-N}{N}$  and  $B = (\log_2 N)^{3/2} \sqrt{N}$ . The  $N$  is length of wavelet coefficient vector and  $s$  is the sum of squared wavelet coefficients given as  $s = \sum_{i=1}^N \omega_i^2$ . Threshold determination is an important problem. A small threshold may yield a result which may be noisy and large threshold can cut significant part of signal thus losing the important details of the signal.

**IV. EXPERIMENTAL RESULTS**

As noisy musical test samples, five audio signals each of 10 seconds duration sampled at 8000 samples per second are analyzed for the experiment. These audio signals are taken from Indian musical instruments viz. Tabla, Pakhawaj, Flute, Harmonium and Taanpura. For performance comparison of various methods of denoising, with decomposition level  $L = 2$  is selected. The effect of various denoising methods on audio musical samples is investigated. For comparison and measurement of the quality of denoising, the Peak Signal to Noise Ratio (PSNR) is calculated between original musical signal  $S(k)$  and denoised musical signal  $S_d(k)$  given by,

$$PSNR = 10 \log_{10} \left( \frac{S_{max}^2}{MSE} \right) \quad (9)$$

Where,  $S_{max}$  is maximum value of signal and is given by,

$$S_{max} = \max(\max(S(k)), \max(S_d(k))) \quad (10)$$

and MSE is mean Square Error given by,

$$MSE = \frac{1}{N} \sum_{k=1}^N [S_d(k) - S(k)]^2 \quad (11)$$

PSNR values for various musical signals are shown comparatively in table1 to 5.

Table 1: Comparison of PSNR (db) for Tabla at L=2

Method	Haar	Db10	Coif5	Bior6.8
Sqtwolog	30.605	36.550	36.564	36.486
Minimaxi	30.973	36.609	36.619	36.554
Heursure	33.112	36.746	36.574	36.661
Rigrsure	33.349	36.746	36.752	36.659

Table 2: Comparison of PSNR (db) for Pakhawaj at L=2

Method	Haar	Db10	Coif5	Bior6.8
Sqtwolog	37.076	42.172	42.224	42.117
Minimaxi	37.542	42.232	42.294	42.185
Heursure	39.359	42.319	42.554	42.291
Rigrsure	39.359	42.274	42.385	42.248

Table 3: Comparison of PSNR (db) for Flute at L=2

Method	Haar	Db10	Coif5	Bior6.8
Sqtwolog	31.646	38.299	38.240	38.361
Minimaxi	31.874	38.299	38.240	38.361
Heursure	34.238	38.299	38.241	38.361
Rigrsure	34.271	38.299	38.238	38.343

Table 4: Comparison of PSNR (db) for Harmonium at L=2

Method	Haar	Db10	Coif5	Bior6.8
Sqtwolog	23.756	32.349	32.506	31.959
Minimaxi	24.560	33.339	33.529	33.058
Heursure	31.384	38.519	38.672	38.505
Rigrsure	31.384	38.519	38.672	38.505

Table 5: Comparison of PSNR (db) for Taanpura at L=2

Method	Haar	Db10	Coif5	Bior6.8
Sqtwolog	25.393	37.202	37.502	36.333

Minimaxi	26.130	37.213	37.569	36.609
Heursure	31.597	37.645	37.850	37.359
Rigrsure	31.597	37.644	37.848	37.357

The part of original audiosignal (noise free) of musical instrument Pakhawaj, its noisy version (corrupted by white Gaussian noise) and denoised waveform using Heursure method at level 2 and Coif5 wavelet, are shown in figure 5.

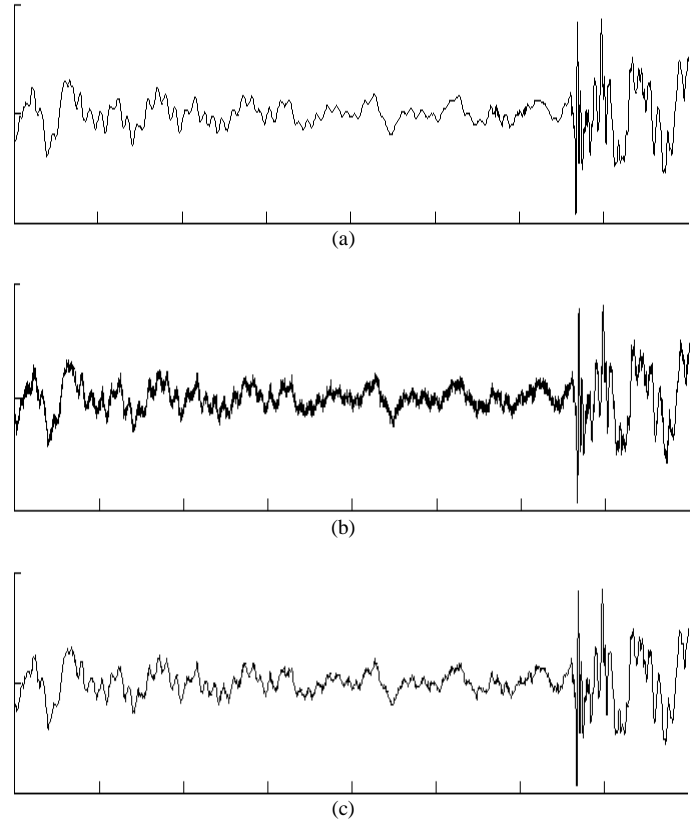


Figure 5. (a) Original noise free signal of Pakhawaj, (b) Signal corrupted by white Gaussian noise, (c) Denoised signal using Heursure at L=2

As it can be seen from the figure 5, the denoised version of sound is much similar to the original noise free sound of pakhawaj and hearing perception is also very good.

## V. CONCLUSIONS

In this paper, performance of various wavelet based thresholding methods for denoising of audio signals of some Indian musical instruments viz. Tabla, Pakhawaj, Flute, Harmonium and Taanpura, corrupted by white Gaussian Noise is presented along with comparative analysis of suggesting the suitable wavelet function. For denoising wavelet decomposition at level 2 is considered. The methods considered for threshold estimation is Sqtwolog, Minimaxi, Heursure, Rigrsure and various wavelets such as Haar, Db10, Coif5 and Bior6.8 are considered for comparison. The results show that coif5 wavelet gives highest PSNR value for audio signals from Tabla, Pakhawaj, Harmonium and Taanpura, while for Flute, the performance of Bior6.8 is better

as compared to other wavelets. The Haar wavelet is performing poorer as compared to others also it gives unwanted distortion in reconstructed voice when heard, as it is not a smooth wavelet. In general, there is slight difference in the performance of Db10, Coif5 and Bior6.8 as all are higher order and smooth wavelets and can be selected for denoising of such audio signals. The results show that the best suitable thresholding method for these specific audio musical signals is SURE method. Both Rigrsure and Heursure are almost equivalent in all cases. Finally it is concluded that the combination of Coif5 and SURE can be efficiently used for better denoising of audio signals of these Indian musical instruments.

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