



## An Uncertain Decision Making Model: An Application to AHP

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**Abstract:** This paper introduces uncertainty theory to deal with uncertain factors in decision making problems. The uncertain variable method (UVM) for generation weight vector, the definition of uncertain consistency and consistency theorem are proposed. An algorithm for checking whether or not an uncertainty comparison matrix is consistent is put forward. The final ranking is indicated by uncertainty weight vector if it is acceptable; otherwise we rely on the ranks of expected weight vector instead. Three numerical examples are examined to illustrate the validity and practicality of the proposed method.

**Keywords:** uncertainty theory; uncertain variable; uncertain measure; uncertainty distribution; uncertain variable method; AHP

### I. INTRODUCTION

The Analytic Hierarchy Process (AHP) is a decision making technique founded by Saaty (1977, 1980), that integrates pairwise comparison ratios into a ratio scale. The reciprocal ratio scale which is used by experts to provide their subjective opinion concerning each pair of stimuli considered. However, in practical application, non-deterministic factors appear in AHP, which leads to new situations. Such as what is the ratio criterion 1 over criterion 2, whether or not it is 2 or 3 cannot be provided exactly by expert. If the each element is non-deterministic, the consistency of comparison matrices cannot be checked and ranks of alternatives cannot be obtained, either.

This problem was first investigated by Vargas (1982). He considered such imprecise judgments as uncertainty in the stochastic or statistical context where the judgment ratio  $r_{ij}$  was viewed as a random variable. Subsequently, a number of techniques have been developed to use such a fuzzy or interval comparison matrix to generate weights. More details can be obtained in works [1-8].

Unfortunately, it is not suitable to regard every non-deterministic phenomenon as random phenomenon, especially when the uncertain phenomenon is caused by subjective judgment. In some works, fuzzy parameters are assumed to have known membership functions and credibility distributions. However, Atanu Sengupta and Tapan Kumar Pal [7] considered that in real-world to a decision maker (DM) it is difficult to specify the membership function or probability distribution in an ambiguous environment.

As is well known, the non-deterministic phenomenon above is caused by subjective judgment. Let us name it "uncertainty". In order to deal with these uncertainties, Liu [9] proposed uncertainty theory and refined it in 2010 [10]. This provides a motivation to introduce uncertainty theory into AHP.

In this paper, uncertain variable instead of precise ratio is used to represent human judgments. Then uncertain measure is used to indicate the belief degree of an uncertain event. Subsequently uncertainty distribution is used to describe uncertain variables in an incomplete but easy-to-use way. Based on uncertainty comparison matrix, weights are derived by using inverse uncertainty distribution and ranking with uncertain measure  $\alpha_0$  is obtained. Finally, consistency of matrices is tested and the inconsistent matrices are modified by expert. After that the reliable weights can be obtained. We call this decision problem model uncertain AHP when uncertain variable is brought in. This paper is focused only on the generating uncertainty weights and consistency check.

The remainder of this paper is organized as follows. Section 2 proposes uncertain variable method which includes constructing uncertainty comparison matrix and generating weight vector from both consistent and inconsistent uncertainty comparison matrices. Section 3 presents definition of uncertain consistency and theorem for identification of consistency, then an algorithm for checking consistency is discussed. In Section 4, three numerical examples are given to show the simplicity and practicality of the proposed the method. The paper is concluded in Section 5.

**II. UNCERTAIN VARIABLE METHOD (UVM)**

Uncertainty theory has become a branch of mathematics based on normality, self-duality, countable subadditivity, and product measure axioms. It is well developed in different fields. To explore the recent developments of uncertainty theory, readers may consult Liu [10]. In order to determine the uncertain AHP, we give the following fundamental concepts.

We consider treating elements in a comparison matrix as uncertain variables having uncertainty distributions. Then we get the definition of uncertainty comparison matrix.

**Definition 2.1** Let  $A$  be an uncertainty comparison matrix of order  $n$ , whose entries are uncertainty distributions  $\Phi_{ij} (i = 1, 2, \dots, n-1; j = i+1, \dots, n)$  of uncertain variables  $\xi_{ij} (i = 1, 2, \dots, n-1; j = i+1, \dots, n)$  representing the ratios criterion  $i$  over criterion  $j$ , then the uncertainty comparison matrix is denoted by

$$A = \begin{pmatrix} 1 & \Phi_{12} & & \Phi_{1n} \\ & 1 & \dots & \Phi_{2n} \\ & & \ddots & \vdots \\ & & & 1 \end{pmatrix}. \tag{1}$$

The lower triangular uncertainty ratios can be obtained by

$$\Phi_{ji}^{-1}(\alpha) = \frac{1}{\Phi_{ij}^{-1}(\alpha)} \quad (i = 1, 2, \dots, n-1; j = i+1, \dots, n), \tag{2}$$

where  $\alpha$  is the uncertain measure (confidence level) with  $\alpha \in [0, 1]$ .

As we know the pairwise comparison ratios are supplied by decision maker in the conventional-AHP. Here in this paper, uncertain variable (Liu [9])  $\xi_{ij}$  can be obtained by Delphi method and has a linear uncertainty distribution (Liu [9]). The consultation process is as follows:

Q1: What do you think is the minimum ratio criterion  $i$  over criterion  $j$ ?

A1:  $a$ . (an expert's experimental data  $(a, 0)$  is acquired)

Q2: What do you think is the maximum ratio criterion  $i$  over criterion  $j$ ?

A2:  $b$ . (an expert's experimental data  $(b, 1)$  is acquired)

Where  $(a, 0), (b, 1)$  represents  $\Phi(a) = M \{ \xi_{ij} \leq a \} = 0$ ,  $\Phi(b) = M \{ \xi_{ij} \leq b \} = 1$ , respectively. So the uncertainty ratio  $\xi_{ij}$  has an linear uncertainty distribution  $L_{ij}(a, b)$ .

Crawford and Williams [11] suggested for the Row Geometric Mean Method (RGMM), where the priorities (without the normalization factor) are given by

$$w_i = \left( \prod_{j=1}^n a_{ij} \right)^{1/n} \quad i = 1, 2, \dots, n, \tag{3}$$

where  $w_i$  is the priority of the  $i$ th attribute. Liu [12] proposed the concept of strictly increasing function of uncertain variables. A real-valued function  $f(x_1, x_2, \dots, x_n)$  is said to be strictly increasing if

$$f(x_1, x_2, \dots, x_n) \leq f(y_1, y_2, \dots, y_n)$$

whenever  $x_i \leq y_i$  for  $i = 1, 2, \dots, n$ , and

$$f(x_1, x_2, \dots, x_n) < f(y_1, y_2, \dots, y_n)$$

whenever  $x_i < y_i$  for  $i = 1, 2, \dots, n$ .

Eq. (3) can be expressed as

$$f(x_1, x_2, \dots, x_n) = \left( \prod_{j=1}^n x_j \right)^{1/n}, x_j > 0$$

then  $f$  is a strictly increasing function.

**Proof**

Apparently,

$$\begin{aligned} f(x_1, x_2, \dots, x_n) &= \left( \prod_{j=1}^n x_j \right)^{1/n} = \prod_{j=1}^n x_j^{1/n} \\ &= \prod_{j=1}^n y_j, (y_j = x_j^{1/n}) \end{aligned}$$

where  $g(y_j) = \prod_{j=1}^n y_j$  is a strictly increasing function and

$h(x_j) = x_j^{1/n}$  is a strictly increasing function, so the composite function  $f$  is a strictly increasing function of uncertain variables  $x_1, x_2, \dots, x_n$ . The proof is finished.

In order to derive the weights from the uncertainty comparison matrices, the inverse uncertainty distributions are used. The reason is that  $\Phi^{-1}(\alpha_0)$  is a precise number. Here we introduce a theorem proposed by Liu.

**Theorem 2.1** (Liu [10]) Let  $\xi_1, \xi_2, \dots, \xi_n$  be independent uncertain variables with regular uncertainty distribution  $\Phi_1, \Phi_2, \dots, \Phi_n$ , respectively. If  $f$  is a strictly increasing function, then

$$\xi = f(\xi_1, \xi_2, \dots, \xi_n)$$

is an uncertain variable with inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \dots, \Phi_n^{-1}(\alpha)).$$

By the Theorem 2.1, it is clear that  $w_i = f(\xi_{i1}, \xi_{i2}, \dots, \xi_{in})$  is an uncertain variable with inverse uncertainty distribution

$$\Phi_i^{-1}(\alpha) = \left( \prod_{j=1}^n \Phi_{ij}^{-1}(\alpha) \right)^{1/n} = \left( \prod_{j=1}^n \xi_{ij} \right)^{1/n} = w_i \tag{4}$$

where  $\alpha \in [0, 1]$  is the uncertain measure (Liu [9]).  $w_i$  is defined as uncertainty weight of the  $i$ th attribute. For all  $i = 1, 2, \dots, n$ , the uncertainty weight vector  $w = (w_1, w_2, \dots, w_n)$  is obtained. The uncertainty weight vector is acceptable when the ranking order does not alter if  $\alpha \geq 0.5$ . Otherwise experts will be required to supply new judgments. Without any modification or new information, we rely on the ranks of expected values of uncertainty weights instead. Liu and Ha [12] give the expected value  $E[\xi]$  of monotone function of uncertain variables

$$E[\xi] = \int_0^1 \left( \prod_{j=1}^n \Phi_{ij}^{-1}(\alpha) \right)^{1/n} d\alpha = \bar{w}_i, \quad (5)$$

which is a precise number showing expected value of uncertain variable  $w_i$  (priority of  $i$ th attribute) in formula (4). However, only the numerical solution of the expected value could be calculated by computer. Finally the expected weight vector  $\bar{w} = (\bar{w}_1, \bar{w}_2, \dots, \bar{w}_n)$  is obtained.

The uncertainty weight  $w_i$  is changed along with different uncertain measures. A certain group of weights will be used to check consistency of uncertainty comparison matrix and indicate the ranking if uncertainty weight vector is unacceptable. The expected value of weight plays an important role here.

### III. CONSISTENCY TEST AND RANKS OF WEIGHTS

As is well known, only comparison matrices passing the test of satisfactory consistency can be used to derive reliable weights. We must check the consistency of comparison matrices in the process of generation weights. Since the complexity and uncertainty of real-world decision analysis problems and the subjectivity of expert judgments, it is inevitable to generate inconsistent comparisons. In previous works, the definition of fuzzy consistency [4] and consistency of interval comparison matrix [13] were proposed. In this paper, the definition of uncertain consistency and a theorem for identification of consistency are given.

**Definition 3.1** Let  $A$  is an uncertainty comparison matrix defined by (1) with  $\Phi_{ij}^{-1}(0) \leq \xi_{ij} \leq \Phi_{ij}^{-1}(1)$  and  $\xi_{ii} = \Phi_{ii}^{-1}(0) = \Phi_{ii}^{-1}(1) = 1$  for  $i, j = 1, 2, \dots, n$ . If the convex feasible region

$$S_w = \{w = (w_1, w_2, \dots, w_n) \mid \Phi_{ij}^{-1}(0) \leq w_i/w_j \leq \Phi_{ij}^{-1}(1), \sum_{i=1}^n w_i = 1, w_j > 0, j = 1, 2, \dots, n\}$$

is nonempty, then  $A$  is considered to be a consistent uncertainty comparison matrix.

**Theorem 3.1**  $A$  is a consistent uncertainty comparison matrix if and only if it satisfies the following inequality constraints:

$$\max(\Phi_{ik}^{-1}(0) \cdot \Phi_{kj}^{-1}(0)) \leq \min(\Phi_{ik}^{-1}(1) \cdot \Phi_{kj}^{-1}(1)), \quad \text{for all } k = 1, 2, \dots, n. \quad (6)$$

#### Proof

If  $A$  is a consistent uncertainty comparison matrix, it means the convex feasible region  $S_w$  is nonempty and there is no contradiction among the following inequality constraints:

$$\Phi_{ik}^{-1}(0) \leq w_i/w_k \leq \Phi_{ik}^{-1}(1) \quad i, k = 1, 2, \dots, n, \quad (7)$$

$$\Phi_{kj}^{-1}(0) \leq w_k/w_j \leq \Phi_{kj}^{-1}(1) \quad k, j = 1, 2, \dots, n. \quad (8)$$

Multiplying (7) by (8) leads to the following inequalities

$$\Phi_{ik}^{-1}(0) \cdot \Phi_{kj}^{-1}(0) \leq \Phi_{ik}^{-1}(1) \cdot \Phi_{kj}^{-1}(1), \quad i, j, k = 1, 2, \dots, n. \quad (9)$$

Since (9) holds for any  $k = 1, 2, \dots, n$ , it follows that  $\max(\Phi_{ik}^{-1}(0) \cdot \Phi_{kj}^{-1}(0)) \leq \min(\Phi_{ik}^{-1}(1) \cdot \Phi_{kj}^{-1}(1))$  holds for all  $i, j, k = 1, 2, \dots, n$ .

Conversely, if (6) holds for all  $i, j, k$ , then  $\Phi_{ij}^{-1}(0) \leq w_i/w_j \leq \Phi_{ij}^{-1}(1)$  holds for any  $i, j = 1, 2, \dots, n$ . So,  $S_w$  is nonempty and  $A$  is a consistent uncertainty comparison matrix in the sense of Definition 3.1.

Certainly, the definition of consistency test for uncertainty comparison matrices can be used to check the consistency. It is clear that it computes inefficiently when the matrix order is large. Here we introduce an algorithm enlightened by R. Islam et al. [14],

$$a = \sum_{i=1}^{n-1} \sum_{j=i+1}^n (p_{ij} + p'_{ij}) \quad (10)$$

subject to

$$-\bar{w}_i + \Phi_{ij}^{-1}(0)\bar{w}_j + n_{ij} - p_{ij} = 0,$$

$$\bar{w}_i - \Phi_{ij}^{-1}(1)\bar{w}_j + n'_{ij} - p'_{ij} = 0,$$

$$i = 1, 2, \dots, n-1; j = i+1, \dots, n,$$

where  $n_{ij} p_{ij} = 0, n'_{ij} p'_{ij} = 0, p_{ij} p'_{ij} = 0$ , and  $\bar{w}_i$  ( $i = 1, 2, \dots, n$ ) is the normalized expected weights.

There must be a group of weights  $w_i$  ( $i = 1, 2, \dots, n$ ) taken into formula (10). Using the UVM, we consider the expected values of uncertainty weights as the estimations of weights  $w_i$  ( $i = 1, 2, \dots, n$ ). Note that  $w_i$  ( $i = 1, 2, \dots, n$ ) are normalized expected values. After that, the ratios  $w_i/w_j$  for all  $i$  and  $j$  may or may not belong to the interval  $[\Phi_{ij}^{-1}(0), \Phi_{ij}^{-1}(1)]$ . The  $p_{ij}$  or  $p'_{ij}$  reflects the deviation  $w_i/w_j$  exceeding the set  $[\Phi_{ij}^{-1}(0), \Phi_{ij}^{-1}(1)]$ .

It is clear that the matrix is consistent if and only if  $a = 0$ . The expert will be asked to modify the uncertainty comparison matrix if  $a > 0$ .

Because the expected weight vector or uncertainty weight vector (with uncertain measure  $\alpha_0$ ) is an exact priority vector, the ranking is obvious. The whole process introduced for generating uncertainty weights and expected weights from uncertainty comparison matrices is summarized in Figure 1.

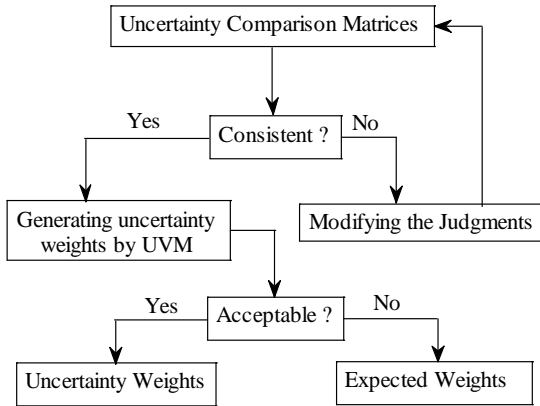


Figure 1 Process for generating priorities from uncertainty comparison matrices.

#### IV. NUMERICAL EXAMPLES

In this section, we offer three numerical examples that are solved using the proposed UVM. The linear uncertainty distributions are obtained by Delphi method.

Table 1 Uncertainty weights in Example 1

Criterion weights	$\alpha$										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Criterion 1	0.3047	0.3738	0.4032	0.4294	0.4530	0.4742	0.4936	0.5112	0.5274	0.5422	0.5560
Criterion 2	0.2026	0.2073	0.2103	0.2120	0.2129	0.2132	0.2129	0.2123	0.2114	0.2103	0.2091
Criterion 3	0.1703	0.1609	0.1525	0.1449	0.1380	0.1317	0.1260	0.1208	0.1159	0.1115	0.1073
Criterion 4	0.2865	0.2580	0.2341	0.2137	0.1961	0.1809	0.1675	0.1558	0.1453	0.1360	0.1276

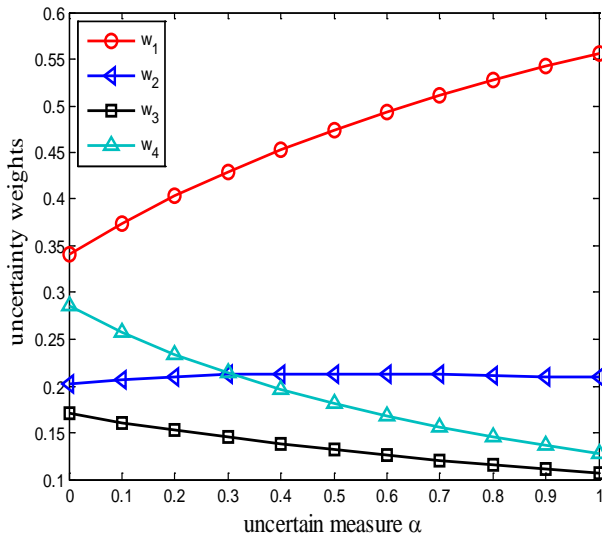


Figure 2 Preference relations in Example 1

**Example 1** Consider the following uncertainty comparison matrix

$$A = \begin{pmatrix} 1 & L_{12}(2,5) & L_{13}(2,4) & L_{14}(1,3) \\ & 1 & L_{23}(1,3) & L_{24}(1,2) \\ & & 1 & L_{34}(1/2,1) \\ & & & 1 \end{pmatrix}$$

As introduced in section 2, the uncertainty weights and expected weights are obtained by formulas (2), (4) and (5). The normalized expected weight vector is  $(0.4514, 0.2134, 0.1388, 0.1963)$ . Using the formula (10),  $a = 0$  is obtained and  $A$  is consistent uncertainty comparison matrix. Table 1 expresses the weights with different uncertain measures, from which it is clear that criterion 1 is the most important because its weight is greater than weights of all the other criteria holds for  $\alpha \in [0, 1]$ . To provide a complete ranking order for the four uncertainty weights, a directed diagram is depicted in Figure 2, from which it is quite clear that the ranking order is generated to be  $w_1 > w_2 > w_4 > w_3$  (if  $\alpha > 0.4$ ), and the result is acceptable. Our ranking order provides the information about uncertain measure of preference, which reflects uncertain nature of the ranking.

**Example 2** Consider the following decision analysis given by uncertainty comparison matrix,

$$A = \begin{pmatrix} 1 & L_{12}(1,2) & L_{13}(1,2) & L_{14}(2,3) \\ & 1 & L_{23}(3,5) & L_{24}(4,5) \\ & & 1 & L_{34}(6,8) \\ & & & 1 \end{pmatrix}$$

The problem of weight determination from  $A$  is calculated expected weight vector is  $(0.3150, 0.3922, 0.2226, 0.0701)$ , respectively, with  $a = 0.6973$ . Then the comparison matrix is inconsistent. Only comparison matrices passing the test of satisfactory consistency can be used to derive reliable weights. Unreliable weights and ranking orders for alternatives may be caused by high inconsistency. So the uncertainty comparison matrix  $A$  should be modified by expert until it becomes consistent.

**Example 3** The problem is about a government agency’s goal (G) to rank chemicals  $A_1, A_2, A_3$  in terms of their level of harm to the environment. The goal is affected by three criteria with criterion  $C_1$ : Air,  $C_2$ : Water, and  $C_3$ : Soil, see Figure 3.

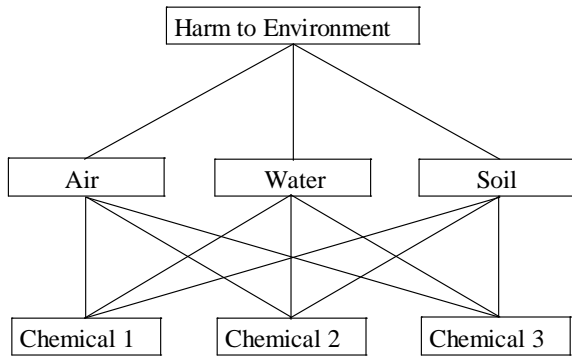


Figure 3 Hierarchy Structure in Example 3.

The uncertainty comparison matrices for the three criteria as well and for the three alternatives are summarized in Table 2. Both the local and global expected weights are showed in Table 3, from which our uncertainty comparison matrices all turn out to be consistent. It is can be seen that the expected weights indicate the ranking  $A_1 > A_3 > A_2$ . Based on the consistent matrices, the UVM are used to generate the global uncertainty weights and plotted in Figure 4. It can be seen from Figure 4 that the ranking given by uncertainty weights is unacceptable, because the ranking alters with  $\alpha > 0.5$ . It is clear that  $A_1$  is preferred over  $A_2$  and  $A_3$ , but whether  $A_3$  is preferred over  $A_2$  is ambiguous. Perhaps the reason is that the consistent matrices include contradictory information. M. Kwiesielewicz and E. Van Uden [15] expressed that the consistency test is performed to ensure that judgments are neither random nor illogical. They proposed that even if a matrix will pass a consistency test successfully, it can be contradictory. Maybe the judgments for  $A_3$  have been given carelessly. Without any new information, we take the ranking  $A_1 > A_3 > A_2$  given by expected weights as the final ranks of alternatives.

Table 2 Uncertainty comparison matrices in Example 3.

$$G : \begin{pmatrix} 1 & L_{12}(1,3) & L_{13}(1,3) \\ & 1 & L_{23}(1/2,2) \\ & & 1 \end{pmatrix}$$

$$C_1 : \begin{pmatrix} 1 & L_{12}(1/4,1/2) & L_{13}(1/6,1/3) \\ & 1 & L_{23}(1/3,1) \\ & & 1 \end{pmatrix}$$

$$C_2 : \begin{pmatrix} 1 & L_{12}(2,5) & L_{13}(2,5) \\ & 1 & L_{23}(1/2,2) \\ & & 1 \end{pmatrix}$$

$$C_3 : \begin{pmatrix} 1 & L_{12}(1,2) & L_{13}(2,6) \\ & 1 & L_{23}(2,4) \\ & & 1 \end{pmatrix}$$

Table 3 Local expected weights and global expected weights in Example 3.

Chemicals	$C_1$	$C_2$	$C_3$	Global weights
	0.4260	0.2870	0.2870	
$A_1$	0.1597	0.5361	0.4619	0.3545
$A_2$	0.3421	0.2320	0.3627	0.3164
$A_3$	0.4982	0.2320	0.1725	0.3283

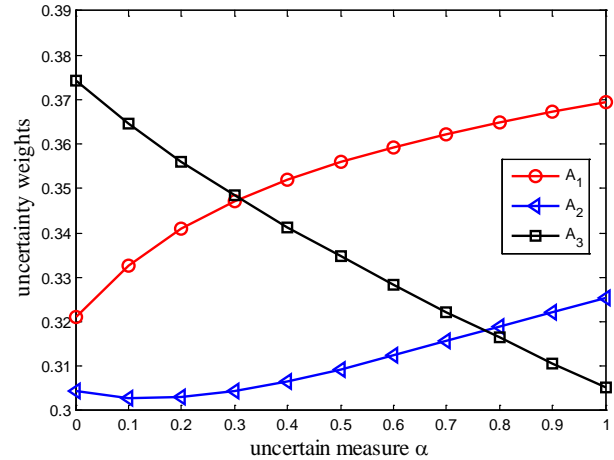


Figure 4 Preference relations in Example 3.

### V. CONCLUSION

In order to deal with subjective uncertainty in decision making, this paper introduced uncertainty theory into analytic hierarchy process (AHP). Uncertain variable method (UVM), a new approach to consistency and inconsistency of uncertainty comparison matrices was proposed to generate uncertainty weights. The definition of uncertain consistency and a theorem for identification of consistency were provided and a simple yet pragmatic approach for testing whether or not an uncertainty comparison matrix is consistent was put forward without solving any optimization model. Both the uncertainty weight vector and expected weight vector indicated the ranks of alternatives. We considered the uncertainty weight vector as the final ranking if it was acceptable. Otherwise the ranks of expected weight vector would be adopted. Three numerical examples illustrated the simplicity and wide applicability of the proposed methods. Because interval comparison matrices can be transformed into uncertainty comparison matrices, the proposed method is applicable to interval comparison matrices. We can find that uncertainty comparison matrices with zigzag uncertainty distributions can be applied to the fuzzy comparison matrices with triangular membership functions. Therefore they can be widely used to deal with decision analysis problems.

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