DOI: http://dx.doi.org/10.26483/ijarcs.v13i2.6811

ISSN No. 0976-5697

Volume 13, No. 2, March-April 2022

International Journal of Advanced Research in Computer Science

RESEARCH PAPER

Available Online at www.ijarcs.info

A STUDY ON OPTIMAL STRATEGY N-POLICY FUZZY VACATION QUEUEING SYSTEM WITH SERVER START UP AND TIME OUT USING L-R METHOD

P Sudhakara Babu
Research Scholar Dept.of Statistics
Acharya Nagarjuna University,
Guntur, Andhra Pradesh, India

K Satish Kumar
Assistant Professor, Dept. Of Statistics,
KRU,Dr, MARAP College P.G Studies
Nuzvid, Krishna Dist. Andhra Pradesh, India

K Chandan
Professor (Rtd.),
Dept. Of Statistics,
Acharya Nagarjuna University,
Guntur, Andhra Pradesh,India

Abstract: In this study, the process of finding performance measures of N* is the optimal threshold , L is the expected system length and T(N*) is the minimum expected cost for an optimal strategy analysis of N-policy FM / FM / 1 Vacation Queueing system with server Start-up and Time-out, in which the arrival rate and service rate triangular and trapezoidal fuzzy numbers are proposed. By using L-R method the L-R method is smaller and more convenient differentiate to alpha-cuts method. The cogency of the model is study by numerical example.

Keywords: N-policy ,fuzzy vacation queue, performance measures, L-R method, triangular and trapezoidal fuzzy numbers

I. INTRODUCTION

Queueing theory is the analysis of the mark of persons, focus or information across a line. Considering congestion and its causes in a exercise can be used to help design well organized and cost efficient services and systems. Examples ration shops, post offices, medical shops and vaccination centers, etc.

Vacation models are described by their scheduling sections, according to which when the service stops, the holiday begins. These estimates help us to assess the condition of the system and take appropriate action to reduce the queue. In most of the models in the queue, the service starts as soon as the customers arrive. But in some physical systems idle servers leave the system for some other undisturbed work mention as vacation.

In this paper, we will examine the n-policy FM / FM / 1 queuing system with server start up and time out by L-R Method. The customer will come according to the fuzzy arrival rate . The waiting customer is personally served fuzzy service. Prior to the holiday, the server ended up waiting for the scheduled time. If a unit arrives at this scheduled time and the queue is empty he will serve that unit. Otherwise, he will take leave after the allotted time. The server turns off every time the system is empty. When the queue length reaches N units the server turns on. Prior to service, the system needs a random start time for pre-service. Enables the service to all users in the server queue immediately after the start period.

Zadeh [1] initiated fuzzy logic in 1965. fuzzy queues has been researched by many researchers like Li and Lee [2], R. Srinivasan [3], etc. G. kannadasan and D. devi [7], Discussed for analysis of single server variant vacation queuing system with ambiguous parameters. S.Aarthi , M.Shanmugasundari , Saranya.V [6], studied the characteristics of N-policy fuzzy queue under vogue data. Many of these studies are committed to finding system performance measurements using the alpha-cuts method.
Here we calculate the optimal threshold N*,the expected system length L and the minimum expected cost T(N*) in the fuzzy queue by the L-R method, especially based on the L-R fuzzy arithmetic.

.

II. PRELIMINARIES

A. fuzzy set

A fuzzy set is a set (which often does not to be empty) having degrees of membership between 0 and 1. By membership function. :$\mu_A(x):A\to[0, 1]$ is the membership function and $\mu_A(x)$ is the grading value called as real numbers between 0 and 1 (cover with 0 and 1).

© 2020-2022, IJARCS All Rights Reserved

B. *Normal fuzzy set*

A fuzzy set A is called normal if there exist an element $x \in X$ whose membership value is one i.e., $\mu_A(x)=1$.

C. *Convex fuzzy set*

A fuzzy set A is convex if and only if for any $x \in X$ then the membership function of A is satisfies the condition $\mu_A\{\lambda x_1 + (1-\lambda)x_2\} \geq \min\{\mu_A(x_1),\mu_A(x_2)\}$, $0\leq \lambda \leq 1$.

D. *Triangular fuzzy number*

For a Triangular fuzzy number $\widehat{A_{3-tuples}}$ is represented by $\bar{A}$ (a, b, c) where a, b, c are real numbers and its membership function $\mu_{\bar{A}}$ is given by

$$\mu_{\bar{A}}(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x = b \\ \frac{c-x}{b-c}, & b \leq x \leq c \\ 0, & \text{other wise} \end{cases}$$

The representation of L-R can be recorded as $\bar{A}$ (a, b, c) = $\langle b, b-a, c-b\rangle_{LR}$ for L(x) = max (0,1-x)

III. SOLUTION PROCEDURE

A. *L-R triangular fuzzy number*

Fuzzy number is the L-R fuzzy number only if it has three positive numbers, m, a> 0, b> 0 and two positive, continuous and decreasing functions from the actual number [0,1].

(0) = R(0)=1

(1) = 0, $L(x) > 0$, $\lim_{x\to ?} L(x) = 0$

(1)= $R(x) > 0$, $\lim_{x\to ?} R(x) = 0$

$$\mu_{\bar{M}}(x) = \begin{cases} L\frac{m-x}{a}, & \text{if } x \in [m-a,b] \\ R\frac{c-x}{b-c}, & \text{if } x \in [m,m+b] \\ 0, & \text{other wise} \end{cases}$$

Fuzzy number $\bar{M}$ is the L-R fuzzy number then $\bar{M}$ = $\langle m,a,b\rangle LR$ m is called the average value or modal value of $\bar{M}$, a and b are called the left spread and right spread of $\bar{M}$ conventionally, $\bar{M}$ = $\langle m,0,0\rangle LR$. is the ordinary real number m, called fuzzy singleton.

Supp($\bar{M}$) = [m-a,m]∪[m,m+b]=[m-a,m+b]

B. L-R Fuzzy Arithmetical operators

if $\bar{M} = \langle m,a,b\rangle LR$ and $\bar{N} = \langle n,c,d\rangle LR$. are same fuzzy numbers then their sum is the same L-R fuzzy numbers and their difference is also the same L-R fuzzy numbers given by

$$\bar{M} + \bar{N} = \langle m+n, a+c, b+d\rangle LR$$
$$\bar{M} - \bar{N} = \langle m-n, a+d, b+c\rangle LR$$

The product of L-R fuzzy numbers $\bar{M} = \langle m,a,b\rangle LR$ and $\bar{N} = \langle n,c,d\rangle LR$ is given by

$$\bar{M}\,\bar{N} = \langle m.n, mc+na-ac, md+nb+bd\rangle LR$$

The quotient secant approximation of L-R fuzzy numbers $\bar{M} = \langle m,a,b\rangle LR$ and $\bar{N} = \langle n,c,d\rangle$ LR is given by

$$\frac{\bar{M}}{\bar{N}} = \frac{\langle m,a,b\rangle_{LR}}{\langle n,c,d\rangle_{LR}} = \left\langle \frac{m}{n}, \frac{md}{n(n+d)} + \frac{a}{n} - \frac{ad}{n(n+d)}, \frac{mc}{n(n-c)} + \frac{b}{n} + \frac{bc}{n(n-c)} \right\rangle_{LR}$$

C. L-R- type trapezoidal fuzzy numbers

A fuzzy number $\bar{M}$ = (m,n.α, β)LR is said to be LR-type trapezoidal fuzzy numbers if given its membership function

$$\mu_{\bar{M}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & \text{if } x \in m\,;\ \alpha > 0 \\ R\left(\frac{x-n}{\beta}\right), & \text{if } x \in n\,;\ \beta > 0 \\ 1, & \text{other wise} \end{cases}$$

D. Operations in LR-type trapezoidal fuzzy numbers

Using (J.Vahidi, 2013) , if M = (m,a,b,c,)& N= (n,d,e,f) then

© 2020-2022, IJARCS All Rights Reserved

M+N = (m,a,b,c,) + (n,d,e,f) = (m+n, b+f, a+d, c+f)

M-N = (m,a,b,c,) (n,d,e,f) = (m-n, a-d, b+f, c+e)

MN = (m,a,b,c) . (n,d,e,f) = (mn, ad, me+nb, af+cd)

$$\frac{M}{N} = \frac{(m,a,b,c)}{(n,d,e,f)} = \left(\frac{m}{d}, \frac{a}{n}, \frac{b}{f}, \frac{c}{e}\right)$$

IV. N-POLICY FM/FM/1 QUEUE MODEL

In L-R fuzzy literature studied some researchers like Mukeba Kanyinda Jean Pierre , Rostin Mekengo Matendo Mabela and B. Ulungu Ekunda Lukata [4] studied Computing Fuzzy Queueing Performance Measures by L-R method, N.Subashini and N.Anusheela [8] studied variance Performance Measures in terms of Crisp values for FM/FM/1by using L-R method and recently S.Vijaya, N.Srinivasan and M.V.Suresh [9] proposed L-R method to the minimal total expected cost of a crisp queue in infinite capacity model using mathematical non-linear parametric procedure by L-R type trapezoidal fuzzy numbers.

We consider the queuing system that customers reach according to the Poisson process and the service rate follows exponential distribution. The fuzzy arrival rate , service rate and vacation rate are all triangular and trapezoidal fuzzy numbers. Arrival rate, service rate and vacation are approximate and may be indicated by convex fuzzy sets. The objective is to decide the work N-approach to reduce the cost performance by considering the cost structure.

Let

Ch = system present holding cost per unit time for each customer ,

Cb = operation cost per unit time for keeping the server is on,

C_m = cost per unit time per cycle in startup,

Ct = timeout cost per unit time per cycle,

Cs = setup cost per cycle,

Cv = reward per unit time for being on server vacation and working secondarily.

The system performance measures are taken from the model ref [5].

A. *Optimal threshold*

$$N^* = \sqrt{\left(\frac{2\lambda\mu\gamma+2\lambda\mu c+c\lambda\gamma}{2c\mu\gamma}\right)^2 - \frac{2\lambda^2\mu-2\lambda\gamma\mu+2\lambda^2\gamma-\lambda\mu c+c^2}{c\mu\gamma} + \frac{2(\mu-\lambda)\left\{c_b\frac{\lambda}{\mu}+c_v\left(\frac{\lambda}{c}+\frac{\lambda}{\gamma}\right)+c_m\frac{\lambda}{\gamma}+c_t\frac{\lambda}{c}\cdot\right.}{\mu c_h}} - \frac{2\lambda\mu\gamma+2\lambda\mu c+c\lambda\gamma}{2c\mu\gamma}$$

B. *Eexpected systemlength*

$$L = \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu} + \frac{c\lambda(\gamma N^2+N\gamma+2\lambda)}{2\mu\gamma(\gamma Nc+\lambda c+\gamma\lambda)}$$

C. *Minimum Expected Cost*

$$T(N) = \frac{(\mu-\lambda)\gamma c}{(\gamma Nc+\lambda c+\gamma\lambda)}(C_h+1) + 2\frac{(N+\frac{\lambda}{\gamma})\lambda c}{(\gamma Nc+\lambda c+\gamma\lambda)} + \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{(\mu-\lambda)}{\mu} + C_m\frac{\lambda}{\gamma}p_0^0 + C_t\frac{\lambda}{\gamma}p_0^0 + Cs\,\lambda p_0^0 - C_v N p_0^0 + \frac{\lambda}{\gamma}C_b$$

Where $p_0^0 = \dfrac{(1-\frac{\lambda}{\mu})}{[N+\frac{\lambda}{v}+\frac{\lambda}{C}]}$

In classical queuing theory, parameters such as arrival rate, service rate and vacation time are always positive, as these numbers are the same in a fuzzy environment given in vague and vague information. By L-R method described in the previous subsection,

performance measures can be calculated after replacing λ,μ and γ by their suitable L-R fuzzy values. Positive real numbers $k_1 = \lambda_2-\lambda_1$, $k_2 = \lambda_3-\lambda_2$ being spreads of $\tilde{\lambda}$; $l_1 = \mu_2 - \mu_1$, $l_2 = \mu_3 - \mu_2$, being spreads of $\tilde{\mu}$; and $v_1 = \gamma_2-\gamma_1$, $v_2 = \gamma_3-\gamma_2$ being spreads $\tilde{\gamma}$ and $m_1 = N_2 - N_1$, $m_2 = N_3 - N_2$ are respectively.

$$\tilde{\lambda} = \langle \lambda_2, \lambda_2-\lambda_1, \lambda_3-\lambda_2 \rangle LR$$
$$\tilde{\mu} = \langle \mu_2, \mu_2-\mu_1, \mu_3-\mu_2 \rangle LR$$
$$\tilde{\gamma} = \langle \gamma_2, \gamma_2-\gamma_1, \gamma_3-\gamma_2 \rangle LR$$
$$\tilde{N} = \langle N_2, N_2-N1, N_3-N_2 \rangle LR$$

From the optimal threshold N* can be computed using arithmetic of L-R fuzzy numbers as follows

N*=

$$\left(\frac{2\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR\,\gamma\langle\mu_2,\mu_2-\mu_1,\mu_3-\mu_2\rangle LR + 2\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR\,\mu c + c\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR\,\gamma}{2c\langle\mu_2,\mu_2-\mu_1,\mu_3-\mu_2\rangle LR\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle LR}\right)^2 -$$
$$\frac{2\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR^2\mu - 2\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR\,\gamma\mu + 2\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR^2\gamma - c\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR\,\mu + \langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR\,c^2}{c\langle\mu_2,\mu_2-\mu_1,\mu_3-\mu_2\rangle LR\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle LR} +$$
$$2(\langle\mu_2,\mu_2-\mu_1,\mu_3-\mu_2\rangle LR - \langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR)\left\{c_b\frac{\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR}{\mu} + c_v\left(\frac{\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR}{c} + \frac{\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR}{\gamma}\right) + c_m\frac{\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR}{\gamma} + c_t\frac{\lambda}{c} + c_s\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR\right\}$$

$$- \frac{2\langle\lambda_2,\lambda_2-\lambda_1\lambda_3-\lambda_2\rangle LR\gamma\langle\mu_2,\mu_2-\mu_1,\mu_3-\mu_2\rangle LR + 2\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR\mu c + c\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle LR}{2c\langle\mu_2,\mu_2-\mu_1,\mu_3-\mu_2\rangle LR\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle LR}$$

From expected system length L can be computed using arithmetic of L-R fuzzy numbers as follows

$$L = \frac{\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR^2}{\langle\mu_2,\mu_2-\mu_1,\mu_3-\mu_2\rangle LR(\langle\mu_2,\mu_2-\mu_1,\mu_3-\mu_2\rangle LR - \langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR)} +$$

© 2020-2022, IJARCS All Rights Reserved

$$\frac{\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR}{\langle\mu_2,\mu_2-\mu_1,\mu_3-\mu_2\rangle LR}+$$

$$\frac{c\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR\,(\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle LR\langle N_2,N_2-N,N_3-N_2\rangle LR^2+\langle N_2,N_2-N,N_3-N_2\rangle LR\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle LR+2\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR)}{2\langle\mu_2,\mu_2-\mu_1,\mu_3-\mu_2\rangle LR\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle LR(\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle LR\,\langle N_2,N_2-N,N_3-N_2\rangle LRc+\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR\,c+\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle LR\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR)}$$

$$T(N)=\frac{\langle\mu_2,\mu_2-\mu_1,\mu_3-\mu_2\rangle LR-\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle\,\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle LR\,c}{\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle LR\langle N_2,N_2-N,N_3-N_2\rangle LR\,\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LRc+\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle\,\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR}$$

$$(C_h+1)\quad+2\frac{(\langle N_2,N_2-N,N_3-N_2\rangle LR+\frac{\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR}{\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle LR}\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR\,c}{\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle LR\,\langle N_2,N_2-N,N_3-N_2\rangle LRc+\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR\,c+\gamma\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR}+$$

$$\frac{\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR^2}{\langle\mu_2,\mu_2-\mu_1,\mu_3-\mu_2\rangle LR\,\langle\mu_2,\mu_2-\mu_1,\ldots-\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR}+$$

$$\frac{(\langle\mu_2,\mu_2-\mu_1,\mu_3-\mu_2\rangle LR-\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR)}{\langle\mu_2,\mu_2-\mu_1,\mu_3-\mu_2\rangle LR}$$

$$+C_m\frac{\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR}{\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle LR}p_0^0+Ct\frac{\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR}{\langle\gamma_2,\gamma_2-\gamma_1,\gamma_3-\gamma_2\rangle LR}p_0^0$$

$$+\quad Cs\ \langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle\ LR\ \ p_0^0\quad-\ C_vN\ \ p_0^0\ +$$

$$\frac{\langle\lambda_2,\lambda_2-\lambda_1,\lambda_3-\lambda_2\rangle LR}{\langle\mu_2,\mu_2-\mu_1,\mu_3-\mu_2\rangle LR}\ C_b$$

V. Numerical Example

A. Triangular fuzzy numbers

The arrival rate $\tilde{\lambda}$, service rate $\tilde{\mu}$ and vacation rate $\tilde{\gamma}$ are all triangular fuzzy numbers are represented by $\tilde{\lambda}$ =[1,2,3] , $\tilde{\mu}$ = [4,5,6] and $\tilde{\gamma}$ =[2,3,4] respectively. In their L-R decomposition, the arrival rate, the service rate and vacation are given by $\tilde{\lambda}$ =[2,1,1] , $\tilde{\mu}$ = [5,1,1] and $\tilde{\gamma}$ =[3,1,1], Cb = 300, Cm = 200, Ct = 30, Cs=500, Cv = 15 and Ch = 5 are monitoring operators c= 1 then optimum threshold calculated as

N*=

$$\sqrt{\left(\frac{2\langle 2,1,1\rangle LR\langle 5,1,1\rangle LR\langle 3,1,1\rangle LR+2\langle 2,1,1\rangle LR\langle 5,1,1\rangle LRc+1\langle 2,1,1\rangle LR\langle 3,1,1\rangle LR}{2\times 1\langle 5,1,1\rangle LR\langle 3,1,1\rangle LR}\right)^2-\frac{2\langle 2,1,1\rangle LR^2\langle 5,1,1\rangle LR-2\langle 2,1,1\rangle LR\langle 3,1,1\rangle LR\langle 5,1,1\rangle LR+2\langle 2,1,1\rangle LR^2\langle 3,1,1\rangle LR-\langle 2,1,1\rangle LR\langle 5,1,1\rangle LR1+1^2\langle 2,1,1\rangle LR}{1\langle 5,1,1\rangle LR\langle 3,1,1\rangle LR}+\frac{2(\langle 5,1,1\rangle LR-\langle 2,1,1\rangle LR)\left\{\begin{matrix}300\frac{\langle 2,1,1\rangle LR}{\langle 5,1,1\rangle LR}+15(\frac{\langle 2,1,1\rangle LR}{1}+\frac{\langle 2,1,1\rangle LR}{\langle 3,1,1\rangle LR})+\\200\frac{\langle 2,1,1\rangle LR}{\langle 3,1,1\rangle LR}+30\frac{\langle 2,1,1\rangle LR}{1}+500\langle 2,1,1\rangle LR\end{matrix}\right\}}{\langle 5,1,1\rangle LR\ 5}}$$

$$-\frac{2\langle 2,1,1\rangle LR\langle 5,1,1\rangle LR\langle 3,1,1\rangle LR+2\langle 2,1,1\rangle LR\langle 5,1,1\rangle LRc+c\langle 2,1,1\rangle LR\langle 3,1,1\rangle LR}{2c\langle 5,1,1\rangle LR\langle 3,1,1\rangle LR}$$

N* = $\langle 16.56,24.85,30.64\rangle LR$

In their L-R decomposition, N = $[24.85,8.29,5.79]$

The expected system length calculated as

$$L=\frac{\langle 2,1,1\rangle LR^2}{\langle 5,1,1\rangle LR[\langle 5,1,1\rangle LR-\langle 2,1,1\rangle LR]}+\frac{\langle 2,1,1\rangle LR}{\langle 5,1,1\rangle LR}$$

$$+\frac{1\langle 2,1,1\rangle LR(\langle 3,1,1\rangle LR\langle 24.85,8.29,5.79\rangle LR^2+\langle 24.85,8.29,5.79\rangle LR\langle 3,1,1\rangle LR+2\langle 2,1,1\rangle LR)}{2\langle 5,1,1\rangle LR\langle 3,1,1\rangle LR(\langle 3,1,1\rangle LR\langle 24.85,8.29,5.79\rangle LR1+\langle 2,1,1\rangle LR1+\langle 3,1,1\rangle LR\langle 2,1,1\rangle LR)}$$

L = $\langle 1.235,0.88,5.2421\rangle LR$

$$T(N)=\frac{(\langle 5,1,1\rangle LR-\langle 2,1,1\rangle LR)\langle 3,1,1\rangle LRc}{(\langle 3,1,1\rangle LR\langle 24.85,8.29,5.79\rangle LRc+\langle 2,1,1\rangle LRc+\langle 3,1,1\rangle LR\langle 2,1,1\rangle LR)}$$

$$(5+1)+2\frac{(\langle 24.85,8.29,5.79\rangle LR+\frac{\langle 2,1,1\rangle LR}{\langle 3,1,1\rangle LR})\langle 2,1,1\rangle LRc}{(\langle 3,1,1\rangle LR\langle 24.85,8.29,5.79\rangle LRc+\langle 2,1,1\rangle LRc+\langle 3,1,1\rangle LR\langle 2,1,1\rangle LR)}$$

$$+\frac{\langle 2,1,1\rangle LR^2}{\langle 5,1,1\rangle LR(\langle 5,1,1\rangle LR-\langle 2,1,1\rangle LR)}+\frac{(\langle 5,1,1\rangle LR-\langle 2,1,1\rangle LR)}{\langle 5,1,1\rangle LR}+$$

$$200\frac{\langle 2,1,1\rangle LR\langle 2,1,1\rangle LR}{\langle 3,1,1\rangle LR}\langle 0.0220,0.0276,0.1311\rangle LR$$

$$+30\frac{\langle 2,1,1\rangle LR}{\langle 3,1,1\rangle LR}\langle 0.0220,0.0276,0.1311\rangle LR$$

$$+500\langle 2,1,1\rangle LR\langle 0.0220,0.0276,0.1311\rangle LR$$

$$-15\langle 24.85,8.29,5.79\rangle LR\langle 0.0220,0.0276,0.1311\rangle LR$$

$$+\frac{\langle 2,1,1\rangle LR}{\langle 5,1,1\rangle LR}300$$

T(N) = $\langle 152.68,163.71,391.704\rangle LR$

According to the performance measures the model values of optimum threshold N*, the expected system length L and the minimum expected cost T(N) are

© 2020-2022, IJARCS All Rights Reserved

respectively, m_{N^*} = 24.8, m_L= 1.2 , T(N) = 152.7 and their supports in the following intervals are given.
[24.85-8.29,24.85+5.79] = [16.6,30.6],
[1.2-0.8,1.2+5.2] = [0.4, 6.4],
[152.7-163.7, 152.7+391.7] = [-11,544.4].

Result

The model value of m_{N^*} = 24.8 is the most possible value and support m_{N^*} = [16.56,30.64] indicate that the optimal threshold N* is approximately between 16.6 and 30.6.
Expected system length L is approximately between 0 and 6 like 0 and 6 customers and The model value of m_L = 1 is the most possible value.
The model value of $m_{T(N)}$ = 152.7 is the most possible value and support m_{N^*} = [-11,544.4] indicate that the total expected cost is approximately between -11 and 544.4

B. Trapezoidal fuzzy numbers

The arrival rate $\tilde{\lambda}$, service rate $\tilde{\mu}$ and vacation time $\tilde{\gamma}$ are all trapezoidal fuzzy numbers are represented by $\tilde{\lambda}$ =[1,2,3,4] , $\tilde{\mu}$ = [4,5,6,7] and $\tilde{\gamma}$ =[2,3,4,5] respectively.C_b = 300, C_m = 200, C_t = 30, C_s=500, C_v = 15 and C_h = 5 are monitoring operators c= 1 then optimal threshold

N* =

$$\sqrt{\left(\frac{2\langle 1,2,3,4\rangle LR\langle 4,5,6,7\rangle LR\langle 2,3,4,5\rangle LR+2\langle 1,2,3,4\rangle LR\langle 4,5,6,7\rangle LR1+1\langle 1,2,3,4\rangle LR\langle 2,3,4,5\rangle LR}{2\times 1\langle 4,5,6,7\rangle LR\langle 2,3,4,5\rangle LR}\right)^2 - \frac{2\langle 1,2,3,4\rangle LR^2\langle 4,5,6,7\rangle LR-2\langle 1,2,3,4\rangle LR\langle 2,3,4,5\rangle LR\langle 4,5,6,7\rangle LR+2\langle 1,2,3,4\rangle LR^2\langle 2,3,4,5\rangle LR-\langle 1,2,3,4\rangle LR\langle 4,5,6,7\rangle LR1+1^2\langle 1,2,3,4\rangle LR}{1\langle 4,5,6,7\rangle LR\langle 2,3,4,5\rangle LR} + \frac{2(\langle 4,5,6,7\rangle LR-\langle 1,2,3,4\rangle LR)\left\{300\frac{\langle 1,2,3,4\rangle LR}{\langle 4,5,6,7\rangle LR}+15\left(\frac{\langle 1,2,3,4\rangle LR}{1}+\frac{\langle 1,2,3,4\rangle LR}{\langle 2,3,4,5\rangle LR}\right)+200\frac{\langle 1,2,3,4\rangle LR}{\langle 2,3,4,5\rangle LR}+30\frac{\langle 1,2,3,4\rangle LR}{1}+500\langle 1,2,3,4\rangle LR\right\}}{\langle 4,5,6,7\rangle LR5}}$$

$$-\frac{2\langle 1,2,3,4\rangle LR\langle 4,5,6,7\rangle LR\langle 2,3,4,5\rangle LR+2\langle 1,2,3,4\rangle LR\langle 4,5,6,7\rangle LR1+1\langle 1,2,3,4\rangle LR\langle 2,3,4,5\rangle LR}{2\times 1\langle 4,5,6,7\rangle LR\langle 2,3,4,5\rangle LR}$$

$$N^* = \langle 5.1953,23.4738,31.9885,43.0744\rangle LR$$

The expected system length calculated as

$$L=\frac{\langle 1,2,3,4\rangle^2}{\langle 4,5,6,7\rangle LR[\langle 4,5,6,7\rangle LR-\langle 1,2,3,4\rangle LR]}+\frac{\langle 1,2,3,4\rangle LR}{\langle 4,5,6,7\rangle LR}$$

$$+\frac{1\langle 1,2,3,4\rangle(\langle 2,3,4,5\rangle LR\langle 5.1953,23.4738,31.9885,43.0744\rangle LR^2+\langle 5.1953,23.4738,31.9885,43.0744\rangle LR\langle 2,3,4,5\rangle LR+2\langle 1,2,3,4\rangle)}{2\mu\langle 2,3,4,5\rangle LR(\langle 2,3,4,5\rangle LR\langle 5.1953,23.4738,31.9885,43.0744\rangle LR1+\langle 1,2,3,4\rangle 1+\langle 2,3,4,5\rangle LR\langle 1,2,3,4\rangle}$$

$$L = \langle 0.23,17.13,0.7,11.23\rangle LR$$

$$T(N) = \frac{(4,5,6,7-\langle 1,2,3,4\rangle LR)\langle 2,3,4,5\rangle LR1}{\langle 2,3,4,5\rangle LR\langle 5.1953,23.4738,31.9885,43.0744\rangle LRc+\langle 1,2,3,4\rangle LR1+\langle 2,3,4,5\rangle LR\langle 1,2,3,4\rangle LR}$$

$$(5+1)+2\frac{(\langle 5.1953,23.4738,31.9885,43.0744\rangle LR+\frac{\langle 1,2,3,4\rangle LR}{\langle 2,3,4,5\rangle LR})\langle 1,2,3,4\rangle LR1}{(\langle 2,3,4,5\rangle LR\langle 5.1953,23.4738,31.9885,43.0744\rangle LR1+\langle 1,2,3,4\rangle LRc+\langle 2,3,4,5\rangle LR\langle 1,2,3,4\rangle LR)}+$$

$$\frac{\langle 1,2,3,4\rangle LR^2}{\langle 4,5,6,7\rangle LR(\langle 4,5,6,7\rangle LR-\langle 1,2,3,4\rangle LR)}+\frac{(\langle 4,5,6,7\rangle LR-\langle 1,2,3,4\rangle LR)}{\langle 4,5,6,7\rangle LR}+$$

$$200\frac{\langle 1,2,3,4\rangle LR}{\langle 2,3,4,5\rangle LR}\langle 0.0102,0.2240,0.0146,0.1060\rangle LR$$

$$+30\frac{\langle 1,2,3,4\rangle LR}{\langle 2,3,4,5\rangle LR}\langle 0.0102,0.2240,0.0146,0.1060\rangle LR$$

$$+500\langle 1,2,3,4\rangle LR\langle 0.0102,0.2240,0.0146,0.1060\rangle LR$$

$$-15\langle 5.1953,23.4738,31.9885,43.0744\rangle LR$$

$$\langle 0.0102,0.2240,0.0146,0.1060\rangle LR+\frac{\langle 1,2,3,4\rangle LR}{\langle 4,5,6,7\rangle LR}300$$

$$T(N) = \langle -11.9087,441.538,339.137,989.6315\rangle LR$$

According to the performance measures the model values of optimum threshold N*, the expected system length L and the minimum expected cost T(N) are respectively, m_{N^*} = 5.2, m_L= 0.23 , T(N) = -11.90.

Results

The optimum threshold m_{N^*} is 5.2. the left and right spreads are calculated as 5.2 and 43.1, expected system length $m_{\bar{L}}$ is 0.23. The left and right spreads are calculated as 0.23 and 11.23 and minimum expected cost $m_{T(N)}$ is -11.90 , The left and right spreads are calculated as -11.90 and 989.6 which give the lower and upper boundary of the measurement of the trapezoidal fuzzy number.

VI. CONCLUSION

The L-R method is used to analyze a N-policy FM/FM/1 model. The triangular and trapezoidal fuzzy numbers makes manipulation much easier. Numerical examples for triangular and trapezoidal fuzzy numbers are described. The model used to find the performance measures of fuzzy queues. The results are given in the L-R representation.

VII. REFERENCE

[1] L. A. Zadeh, "Fuzzy sets. Information and Control'', 1965, vol. 8 , pp. 338–353.

[2] R.J.Li and E.S.Lee, "Analysis of fuzzy queues, computers and Mathematics with Applications'', 1989 , 17(7),1143-1147.

[3] R.Srinivasan, "Fuzzy queueing model using DSW algorithm'', international Journal of Advanced Research in Mathematics and Computer Applications, 2013, 1(1), 1-6.

[4] Mukeba Kanyinda Jean Pierre and Rostin Mekengo Matendo Mabela, 2016, "Computing Fuzzy Queueing Performance Measures by L-R Method", In Journal of Fuzzy Set Valued Analysis April2015,

http://www.researchgate.net/publication/284217560 , DOI:10.5899/2015/jfsva-00226.

[5] K Satish Kumar, K. Chandan and A. Ankama Rao, 2017, “Optimal strategy analysis of N-policy M/M/1 vacation queueing system with server start up and time out”,

International Journal of Engineering Science Invention. ISSN (Online):2319-6734, Vol.6, issue11, 24-28.

[6] Aarthi, M. Shanmugasundari , Saranya.V, “A Study On Fuzzy Queue With N Policy By Triangular Fuzzy Numbers”,

2020, European Journal of Molecular & Clinical Medicine, ISSN 2515-8260,Volume 07, Issue 03,4544 4544.

[7] G. Kannadasan and D. Devi, “Fuzzy variant Vacations on Server Breakdowns using Pentagonal Numbers Advances and Applications in Mathematical Sciences Volume”, April 2021, 20, Issue 6, Pages 1003-1019 © 2021 Mili Publications.

[8] N.Subashini and N.Anusheela , (2021), “Variance Performance Measures in terms of Crisp Values for FM/FM/1by using L-R method”, Journal of Xi`an University of Architecture & Technology, Valume XIII, Issue 3 , ISSN No: 1006-7930,pp-211-215.

[9] S.Vijaya, N.Srinivasan and M.V.Suresh, (2021), “A study on Fuzzy Decision Problem of a Queueing System with Finite Capacity by LR Method” ,Turkish Journal of Computer and Mathematics Education, Vol.12 No.9, 2411-2419.

© 2020-2022, IJARCS All Rights Reserved