I. INTRODUCTION

The study to search for decision laws on the decision table by assessing the measures of decision laws as well as incremental approaches, determining decision laws … has been studied by many groups of authors, such as in [7], [8], … On the other hand, when the decision table is expanded into a decision block, then the study, proposing a model and algorithm to detect decision laws on the decision block has been studied by the authors as in [4], [5], [6], [3]. However, the proposed models and algorithms when smoothing and roughening the values of index attributes on the decision block have not been studied until now. The purpose of this paper is to study the some properties about smoothing, roughening the values of the condition index attribute or decision index attribute on the decision block and on the slice of the decision block. From the results found of the smoothing, roughening the condition equivalence class or decision equivalence class partial pullulate or pullulate smoothing, roughing the values of the condition index attribute or decision index attribute.

II. THE BASIC CONCEPT

II.1 The block, slice of the block

Definition II.1 [1]: Let block scheme \( \mathbf{R} = (\mathbf{id}; A_1, A_2, \ldots, A_n) \) is a finite set of elements, where \( \mathbf{id} \) is an empty finite set, \( A_i \) (\( i=1 \ldots n \)) is an attribute. Each attribute \( A_i (i=1 \ldots n) \) there is a corresponding value domain \( \text{dom}(A_i) \). A block \( \mathbf{r} \) on \( \mathbf{R} \), denoted \( \mathbf{r}(\mathbf{R}) \) consists of a finite number of elements that each element is a family of mappings from the index set to the value domain of the attributes \( A_i (i=1 \ldots n) \).

From the definition above, we denote \( \mathbf{r}(\mathbf{R}) = \{ t \mid t: \mathbf{id} \rightarrow \text{dom}(A_i) \} \), \( \mathbf{i}=1 \ldots n \). The block is denoted by \( \mathbf{r}(\mathbf{R}) \) or \( \mathbf{r}(\mathbf{id}; A_1, A_2, \ldots, A_n) \), sometimes without fear of confusion we simply denoted \( \mathbf{r} \).

Definition II.2 [2],[3]: Let \( \mathbf{R} = (\mathbf{id}; A_1, A_2, \ldots, A_n) \), \( \mathbf{r}(\mathbf{R}) \) is a block over \( \mathbf{R} \). For each \( x \in \mathbf{id} \) we denoted \( \mathbf{r}(\mathbf{R}_x) \) is a block with \( \mathbf{R}_x = (\{ x \}; A_1, A_2, \ldots, A_n) \) such that:

\[
\forall t \in \mathbf{r}(\mathbf{R}) \ni t = \{ t' : \mathbf{id} \rightarrow \text{dom}(A_i) \} \ni t' \ni \mathbf{i}=1 \ldots n.
\]

Then \( \mathbf{r}(\mathbf{R}_x) \) is called a slice of the block \( \mathbf{r}(\mathbf{R}) \) at point \( x \). Sometimes we denote \( \mathbf{r}_x \).

II.2 Information block

Definition II.3 [4]: Let block scheme \( \mathbf{R} = (\mathbf{id}; A_1, A_2, \ldots, A_n) \), \( \mathbf{r}(\mathbf{R}) \) is a block over \( \mathbf{R} \). Then, the information block is a tuple of four elements \( \mathbf{IB} = (\mathbf{U}, A, V, f) \) with \( \mathbf{U} \) is a set of objects of \( r \) called space objects, \( A = \bigcup_{i}^{n} \text{id}(i) \) is the set of index attributes of the object, \( V = \bigcup_{x}^{m} V(x) \), \( V(x) \) is the set of values of the objects corresponding to the index attribute \( x \), \( f \) is an information function \( \mathbf{UA} \rightarrow V(x) \) satisfy:

\[
\forall u \in \mathbf{U}, \forall x \in \mathbf{A} \text{ we have } f(u, x) \in V(x).
\]

We call \( f(u, x) \) is the value of the object \( u \) at the index attribute \( x \).

If \( V \) contains missing values in at least one index attribute \( x \), then we call \( \mathbf{IB} \) is an incomplete information block. In contrast \( \mathbf{IB} \) is a complete information block, or simply \( \mathbf{IB} \) is an information block.

Definition II.4 [4]: Let block scheme \( \mathbf{R} = (\mathbf{id}; A_1, A_2, \ldots, A_n) \), \( \mathbf{r} \) is a block over \( \mathbf{R} \). \( x \) is the slice of the block \( \mathbf{r} \) at the point \( x \in \mathbf{id} \). Then the slice of the information block at \( x \) is a tuple of four elements \( \mathbf{IB}_x = (\mathbf{U}, A, V, f_x) \) with \( \mathbf{U} \) is a set of objects of \( r \) called space objects, \( A = \bigcup_{i}^{n} x(i) \) is the set of the index
attributes of the object on the slice at x, \( V_x = \bigcup_{x_i \in A} V_{x_i} \), is the set of values of the objects corresponding to the index attribute \( x^0 \), \( f \) is an information function \( U \times A_1 \rightarrow V_x \), satisfies: \( \forall u \in U, \forall x^0 \in A, \) we have \( f(u, x^0) \in V_{x^0} \).

**II.3 Relationships are indistinguishable**

**Definition II.5[5]**

Let information block \( IB = (U, A, V, f) \). Then for each index attribute set \( P \subseteq A \) we define an equivalence relation, \( \sim \), defined as follows:

\[
\forall u, v \in U | (u, v) \in \sim \text{ if and only if } f(u, x^0) = f(v, x^0),
\]

and called non-discriminatory relations:

\[
\forall u, v \in U | (u, v) \in \sim \text{ if and only if } x^0 \in x \sim \text{ and } \forall x \in x, x^0 \in x \sim \text{ and } \forall x \in x, x^0 \in x \sim
\]

From the definition we have:

\[
\forall u, v \in U | (u, v) \in \sim \text{ if and only if } x^0 \in x \sim \text{ and } \forall x \in x, x^0 \in x \sim \text{ and } \forall x \in x, x^0 \in x \sim
\]

Relation \( \sim \) divides \( U \) into equivalence classes, constitutes a subdivision of \( U \), sign \( U/\sim \) or simply \( U/\sim \).

**Proposition II.1 [5]**

Let decision block \( DB = (U, C, D, V, f) \), with \( U \) the space of objects, \( A = \bigcup_{x \in \text{index attributes}} A_D \) is divided into two sets \( C \) and \( D \) such that:

\[
C = \bigcup_{i=1}^k x(i) \quad D = \bigcup_{i=1}^n x(i)
\]

then information block \( IB = (U, A, \sim) \) with \( U \) the space of objects, \( A = \bigcup_{x \in \text{index attributes}} A_D \) is divided into two sets \( C \) and \( D \) such that:

\[
C = \bigcup_{i=1}^k x(i) \quad D = \bigcup_{i=1}^n x(i)
\]

\[
DB = (U, C \cup D, V, f)
\]

From the definition of the decision block, we see: \( C \cap D = A \), \( C \cap E = \emptyset \),

We can denote the decision block simply by: \( DB = (U, C \cup D) \).

**Definition II.8[5]**: Let decision block \( DB = (U, C \cup D, V, f) \), with \( C \) is the set of conditional index attributes and \( D \) is the set of decision index attributes.

The slice of the block decides at \( x (x \in D) \) is a tuples of four elements \( DB_x = (U, C \cup D, V, f, x) \) with \( U \) the set of objects of \( r \), called the space of objects

\[
C = \bigcup_{i=1}^k x(i) \quad D = \bigcup_{i=1}^n x(i)
\]

\[
V_x = \bigcup_{i=1}^k x(i) \quad f \text{ is an information function}\n\]

\[
U \times A \rightarrow V_x
\]

\[
\forall u \in U, \forall x \in A, f(u, x) \in V_x
\]

**II.4 Decision block**

**Definition II.7[5]**

Let information block \( IB = (U, A, V, f) \) with \( U \) the space of objects, \( A = \bigcup_{x \in \text{index attributes}} A_D \) is divided into two sets \( C \) and \( D \) such that:

\[
C = \bigcup_{i=1}^k x(i) \quad D = \bigcup_{i=1}^n x(i)
\]

Then information block \( IB \) is called the decision block and denoted by \( DB = (U, C \cup D, V, f) \), with \( C \) the set of conditional index attributes and \( D \) the set of decision index attributes.

From the definition of the decision block, we see: \( C \cap D = A \), \( C \cap E = \emptyset \),

We can denote the decision block simply by: \( DB = (U, C \cup D) \).

**Definition II.9[5]**: Let decision block \( DB = (U, C \cup D, V, f) \), with \( C \) is the set of conditional index attributes and \( D \) is the set of decision index attributes.

Then information block \( IB = (U, C \cup D, V, f) \) is called the decision block and denoted by \( DB = (U, C \cup D, V, f) \), with \( C \) the set of conditional index attributes and \( D \) the set of decision index attributes.

Then, support, accuracy and coverage of decision law \( C \rightarrow D \) on the block are:

- **Support**: \( \text{Sup}(C \cup D) = |C \cap D| \)

- **Accuracy**: \( \frac{|C \cap D|}{|D|} \)

- **Coverage**: \( \frac{|C \cap D|}{|C|} \)

**Definition II.10[5]**

Let decision block \( DB = (U, C \cup D) \), \( C \subseteq U \cap D \), \( D \subseteq U \cap D \), \( C \rightarrow D \)

\[
U \cap C \subseteq U \cap D \quad D \subseteq U \cap D
\]

Then, support, accuracy and coverage of decision law \( C \rightarrow D \) on the block are:

- **Support**: \( \text{Sup}(C \cup D) = |C \cap D| \)

- **Accuracy**: \( \frac{|C \cap D|}{|D|} \)

- **Coverage**: \( \frac{|C \cap D|}{|C|} \)
\textbf{Proposition II.2} \[5\]

Let decision block $\text{DB}(U, C_i, D_j)$ is the conditional equivalence class and decision equivalence class generated by $C, D$ respectively. If $\text{Acc}(C, D) \geq \delta$ and $\text{Cov}(C, D) \geq \beta$ then we call $(C, D)$ is the decision lawmeaning.

\textbf{Definition II.12} \[5\]

Let decision block $\text{DB}(U, C_i, D_j)$, with $U$ is the space of objects, $a \in C \cap D$. $V$ is the set of existing values of the index attribute $a$. Suppose $Z = \{x_j \in U \mid f(x_j) = z\}$ is the set of objects whose $z$ value is the index attribute $a$. If $Z$ is partitioned into two sets $W$ and $Y$ such that: $Z = W \cup Y$, $W \cap Y = \emptyset$ with $W = \{x_j \in U \mid f(x_j) = w\}$, $Y = \{x_j \in U \mid f(x_j) = y\}$ then we say the $z$ value of the index attribute $a$ is smoothed to two new values $w$ and $y$.

\textbf{Theorem II.1} \[6\]

Let decision block $\text{DB}(U, C_i, D_j, V, f)$, with $U$ is the space of objects, $a \in C \cap D$. $V$ is the set of existing values of the index attribute $a$. If $E, E_p, E_q$ are respectively the values of $x_p, x_q$ on the index attribute $a$ ($p \neq q$). If at any one time we have: $f(x_p, a) = f(x_q, a) = z$, ($z \in V_a$) then we say the two values $w, y$ of $a$ are roughened to the new value $z$.

\textbf{Theorem II.2} \[6\]

Let decision block $\text{DB}(U, C_i, D_j, V, f)$, with $U$ is the space of objects, $a \in C \cap D$. $V$ is the set of existing values of the index attribute $a$. Then, equivalent class $E, E_p, E_q$ are respectively the values of $x_p, x_q$ on the index attribute $a$ ($p \neq q$) made rough into new equivalent class $E_{p, q}$ and only if $\forall a \neq a: f(E_{p, q}) = f(E_{p, q})$.

\textbf{Theorem II.3} \[6\]

Let decision block $\text{DB}(U, C_i, D_j)$, with $U$ is the space of objects:

\begin{align*}
\text{Cov}(C, D) &= \sum_{i=1}^{m} \sum_{j=1}^{n} \text{Sup}(C_i, D_j) \\
\end{align*}

\textbf{Definition II.13} \[5\]

Let decision block $\text{DB}(U, C_i, D_j)$, with $U$ is the space of objects, $a \in C \cap D$. $V$ is the set of existing values of the index attribute $a$. Suppose $Z = \{x_j \in U \mid f(x_j) = z\}$ is the set of objects whose $z$ value is the index attribute $a$. If $Z$ is partitioned into two sets $W$ and $Y$ such that: $Z = W \cup Y$, $W \cap Y = \emptyset$ with $W = \{x_j \in U \mid f(x_j) = w\}$, $Y = \{x_j \in U \mid f(x_j) = y\}$ then we say the $z$ value of the index attribute $a$ is smoothed to two new values $w$ and $y$.

\textbf{Theorem II.1} \[6\]

Let decision block $\text{DB}(U, C_i, D_j, V, f)$, with $U$ is the space of objects, $a \in C \cap D$. $V$ is the set of existing values of the index attribute $a$. If $E, E_p, E_q$ are respectively the values of $x_p, x_q$ on the index attribute $a$ ($p \neq q$). If at any one time we have: $f(x_p, a) = f(x_q, a) = z$, ($z \in V_a$) then we say the two values $w, y$ of $a$ are roughened to the new value $z$.

\textbf{Theorem II.2} \[6\]

Let decision block $\text{DB}(U, C_i, D_j, V, f)$, with $U$ is the space of objects, $a \in C \cap D$. $V$ is the set of existing values of the index attribute $a$. Then, equivalent class $E, E_p, E_q$ are respectively the values of $x_p, x_q$ on the index attribute $a$ ($p \neq q$) made rough into new equivalent class $E_{p, q}$ and only if $\forall a \neq a: f(E_{p, q}) = f(E_{p, q})$.

\textbf{Theorem II.3} \[6\]

Let decision block $\text{DB}(U, C_i, D_j)$, with $U$ is the space of objects:

\begin{align*}
\text{Cov}(C, D) &= \sum_{i=1}^{m} \sum_{j=1}^{n} \text{Sup}(C_i, D_j) \\
\end{align*}

\textbf{Definition II.12} \[5\]

Let decision block $\text{DB}(U, C_i, D_j)$, with $U$ is the space of objects, $a \in C \cap D$. $V$ is the set of existing values of the index attribute $a$. Suppose $Z = \{x_j \in U \mid f(x_j) = z\}$ is the set of objects whose $z$ value is the index attribute $a$. If $Z$ is partitioned into two sets $W$ and $Y$ such that: $Z = W \cup Y$, $W \cap Y = \emptyset$ with $W = \{x_j \in U \mid f(x_j) = w\}$, $Y = \{x_j \in U \mid f(x_j) = y\}$ then we say the $z$ value of the index attribute $a$ is smoothed to two new values $w$ and $y$.
III.1 Smoothing, roughening the conditional equivalent classes on the decision block and on the slice.

Proposition III.1
Let decision block $\text{blockDB}=\{U, C \cup D, V, f\}$, $a=x(0) \in C$, $V$ is the set of existing values of the conditional index attribute $a$. The $z$ value of $a$ is smoothed to two new values $w$ and $y$.

$C^m, D^m=\bigcup_{i=1}^{k} x(i)$

$C^s, D^s=\bigcup_{i=k}^{n} x(i)$

$U/C=\{C_1, C_2, \ldots, C_m\}$, $U/C^s=\{C_{s1}, C_{s2}, \ldots, C_{si}\}$.

$U/D=\{D_1, D_2, \ldots, D_j\}$, $U/D^s=\{D_{s1}, D_{s2}, \ldots, D_{sk}\}$.

Suppose that if the conditional equivalence class $C_i \in U/C$, $(f(C_i, a) = z)$ smoothed into two new conditional equivalence classes $C_p \cap C_q$ if $(f(C_i, a) = w)$, $f(C_q, a) = y$, with $w, y \notin V_a$. Then the slice $V_a$ exists equivalence class $C_{s1}$, satisfy: $C_{s1} \subseteq C_i$, also smoothed into two new conditional equivalence classes $C_{s1}$ and $C_{s1}'$.

We say on the slice $C_{s1}$ then $C_{s1}$ is made friendly into two new conditional equivalence classes $C_{s1}$ and $C_{s1}'$ by the smoothing of $C_i$ into two new conditional equivalence classes $C_p \cap C_q$.

Proof
Assuming we have: $C_i \in U/C$, $(f(C_i, a) = z)$ smoothed into two new conditional equivalence classes $C_p \cap C_q$, $(f(C_i, a) = w)$, $f(C_q, a) = y$, with $w, y \notin V_a$. Therefore, to calculate the elements of these two new rows in the support matrix with slice $C_{s1}$, then the first calculate the values $\text{Sup}(C_{s1}, D_p)$ with $j=1,2,\ldots, h_0$. From there, we infer the values $\text{Sup}(C_{s1}', D_p)$ is the subtraction between $\text{Sup}(C_{s1}, D_p)$ and $\text{Sup}(C_{s1}', D_p)$ with $j=1,2,\ldots, h_0$.

Proposition III.2
Let decision block $\text{blockDB}=\{U, C \cup D, V, f\}$, $a=x(0) \in C$, $V$ is the set of existing values of the conditional index attribute $a$. The $z$ value of $a$ is smoothed to two new values $w$ and $y$.

$C^m=\bigcup_{i=1}^{k} x(i)$

$D^m=\bigcup_{i=k+1}^{n} x(i)$

$U/C=\{C_1, C_2, \ldots, C_m\}$, $U/C^s=\{C_{s1}, C_{s2}, \ldots, C_{si}\}$.

$U/D=\{D_1, D_2, \ldots, D_j\}$, $U/D^s=\{D_{s1}, D_{s2}, \ldots, D_{sk}\}$.

Suppose, if two conditional equivalence classes $C_p \in U/C$, $(f(C_p, a) = w)$, $f(C_q, a) = y$ is made friendly into two new conditional equivalence classes $C_p \cap C_q$. $C_{s1}$ satisfy: $C_{s1} \subseteq C_p \cap C_q$. $C_{s1}$ also is made friendly into two new conditional equivalence classes $C_{s1}$ and $C_{s1}'$.

We say on the slice $C_{s1}$ then $C_{s1}$ is made rough into $C_{s1}$ by the smoothing of $C_i$ into two new conditional equivalence classes $C_p \cap C_q$.

Proof
Assuming we have: $C_p \in U/C$, $(f(C_q, a) = w)$, $f(C_q, a) = y$ applying the results of proposition 1.1 we infer on the slice exists two conditional equivalence classes $C_{s1}$ $C_{s1}'$ satisfy: $C_{s1} \subseteq C_p \cap C_q$. From there we have: $f(u, a) = w$ with $u \in C_{s1} \cap C_q$. In the same way we also have: $f(u, a) = y$ with $u \in C_{s1}' \cap C_q$. On the other hand, assuming we have: two conditional equivalence classes $C_p \in U/C$ made friendly into two new conditional equivalence class $C_Q \in U/C$, according to the results of theorem 1.1 then we have:
∀a≠a, a∈C: f(Ca,a) = f(Cb,a) ⇒ ∀a≠a, a∈C' :
f(Ca,a) = f(Cb,a)(1)

In slices r, then we have:
C∈C, r∈U/C ⇒ ∀a≠a, a∈C': f(Ca,a) = f(Cb,a)(2)

Same, we also have:
C⊆C, r∈U/C ⇒ ∀a≠a, a∈C': f(Ca,a) = f(Cb,a)(3)

From (1), (2) and (3) we infer:
∀a≠a, a∈C' : f(Ca,a) = f(Cb,a).

Therefore, apply the necessary and sufficient conditions in the statement of the theorem 1.1, we have two conditional equivalents classes C, C′ is made rough sympathetic into C by the roughening off two conditional equivalents classes C, C′ to C.

From the nature of the rough work two conditional equivalents classes C, C′ to C, we have:
C = (Ca ∪ C′) ⊇ (Cp ∪ Cq) = Cp.

From that:
C⊆C.

Proposition III.4
Let decision block DB=(U, C, D), a=x∈C, V is the set of existing values of the conditional index attribute a, the w and y values of a are roughened to the new value z

C = \{C1, C2, ..., Cm\}

D = \{D1, D2, ..., Dh\}

\[ D' = \bigcup_{i=1}^{n} x^{(i)}, x \in C \]

\[ U/C = \{C_{1,2,...,cm}\}, \quad U/C' = \{C1, C_{2,2,...,cm}\} \]

\[ U/D = \{D1, D_{2,2,...,dh}\}, \quad U/D' = \{D1, D_{2,2,...,dh}\} \]

Suppose, if C, C′ is made rough into new conditional equivalent class C, C′ and on the slice r, two conditional equivalents classes C, C′ are made rough sympathetic into C, then:

i) C ∪ C′ = C′

ii) ∀D∈U/D': Sup(C, D') = Sup(C, D)

Prove

i) Suppose we have: \( x \in C \cup C' \Rightarrow x \in C \) or \( x \in C' \). If \( x \in C \) then from the two conditional equivalents classes C, C′ is made rough into conditional equivalent class C, so we have:
\[ f(x,a) = f(C,a) \]

On the other hand, applying the results of theorem 2.1 we have:
∀a≠a: f(Ca,a) = f(Cb,a) ⇒ ∀a≠a: f(Ca,a) = f(Cb,a).

Therefore, we also prove that \( x \in C \).

So inference: (C∪C′)⊆C.

(5)

On the contrary, suppose \( x \in C \), because C, C′ made rough into C, applying the results of theorem 2.1 we have:
∀a≠a: f(Ca,a) = f(Cb,a) ⇒ f(x,a) = f(C,a).

On the other hand, because C, C′ made rough into conditional class C, so we have:
\[ f(x,a) = f(C,a) \]

Therefore, from \( x \in C \), we have:
\( x \in C \cup C' \).

So: C⊆(C′∪C′) (6)

Combined (5) and (6) we have: C′∪C′ = C.

ii) Because C, C′ are the conditional equivalent classes, so we have: C = C′ = ∅.

On the other hand: ∀D∈U/D: Sup(C, D') = Sup(C, D).

So inference: (D)∈U/D: Sup(C, D') = Sup(C, D).

Thus, we see two rows of matrix of support on the slice r, corresponding to the two conditional equivalents classes C, C′ is combined into a new row corresponding to the conditional equivalent class C.

Thus, we see two rows of matrix of support on the slice r, corresponding to the two conditional equivalents classes C, C′ is combined into a new row corresponding to the conditional equivalent class C.

III.2 Smoothing, rougheningthedecisionequivencelalssontheblock and onestice.

Proposition III.5
Let decision block DB=(U, C, D, f), a=x∈D, V is the set of existing values of the decision index attribute a, the z value of a is smoothed to two new values w and y.

C = \{C1, C2, ..., Cm\}

D = \{D1, D_{2,2,...,dh}\}

Suppose that if decision equivalent class D, ∈ U/D (f(D, a)=z) is smoothed into two decision equivalents classes D, D, (f(D, a)=w, f(D, a)=y), then on the slice r, we have:
\[ C_i \in D \]

We say on the slice r, decision equivalent class D, is smoothed sympathetic partially into two new decision equivalents classes D, and D, by the smoothing of D into two new decision equivalents classes D, and D,.

Proving this clause is similar to the proof of the proposition II.1.

Proposition III.6
Let decision block DB=(U, C, D), a=x∈D, V is the set of existing values of the decision index attribute a, the z value of a is smoothed to two new values w and y.

C = \{C1, C2, ..., Cm\}

D = \{D1, D_{2,2,...,dh}\}

\[ U/C = \{C_{1,2,...,cm}\}, \quad U/C' = \{C_{1,2,...,cm}\} \]

\[ U/D = \{D_{1,2,...,dh}\}, \quad U/D' = \{D_{1,2,...,dh}\} \]

Thus, suppose \( D \in U/D \), \( D, \in U/D' \), \( D, \in U/C' \), \( D_i \in U/C \), \( i=1,...,h_0 \), \( j=1,...,h_0 \), \( i=1,...,h_0 \), \( j=1,...,h_0 \),

we have:
\[ f(D, a)=w, f(D, a)=y \]

and on the slice r, D, is smoothed sympathetic partially into two new values w and y.
decision equivalences classes $D_x$ and $D_y$ then:

i. $D_x = D_x^b \cup D_x^c$,

ii. $\forall C_j \in U/C^b: \text{Sup}(C_j \cap D_x) = \text{Sup}(C_j \cap D_x^b) + \text{Sup}(C_j \cap D_x^c), \text{with } j = 1, 2, ..., t_c$.

Prove

i. From the smoothing of the decision equivalence class $D_x$, we see that: $D_x = D_x^b \cup D_x^c$.

ii. Assuming we have: $D_x$ is smoothed sympathetic partially into two new decision equivalence classes $D_x^b$ and $D_x^c$.

$\Rightarrow D_x = D_x^b \cup D_x^c$ and $D_x \cap D_x^b = \emptyset$.

Other way: $\forall C_j \in U/C^b: \text{Sup}(C_j \cap D_x) = |\text{Sup}(C_j \cap D_x^b) \cup \text{Sup}(C_j \cap D_x^c)|$.

We have: $D_x \cap D_x^b = \emptyset \Rightarrow (C_j \cap D_x) \cap (C_j \cap D_x^b) = \emptyset$.

Come on: $\text{Sup}(C_j \cap D_x) = |(C_j \cap D_x) \cup (C_j \cap D_x^b)| = |(C_j \cap D_x^b) + \text{Sup}(C_j \cap D_x^b, D_x^b)|$.

So we infer: $\forall C_j \in U/C^b: \text{Sup}(C_j \cap D_x) = \text{Sup}(C_j \cap D_x^b) + \text{Sup}(C_j \cap D_x^b, D_x^b)$, with $j = 1, 2, ..., t_c$.

From this result we see: columncorresponding to the decision equivalence class $D_x$ in the support matrix for slice $r_1$ will be split into two new columns corresponding to two new decision equivalence classes $D_x^b$ and $D_x^c$.

Therefore, to calculate the value of the elements of these two new columns in the support matrix with slice $r_1$, we first calculate the values $\text{Sup}(C_j \cap D_x^b)$ with $j = 1, 2, ..., t_c$. From there, we infer the values $\text{Sup}(C_j \cap D_x^c)$.

Proposition III.7

Let decision block $DB = (U, C \cup D, V, f)$, $a=x^b \in D$, $V_i$ is the set of existing values of the decision index attribute $a$, the $w$ and $y$ values of a are rounded to the new value $z$.

$$C = \bigcup_{i=1}^{t_c} x_i^{(i)}, D = \bigcup_{i=1}^{t_c} x_i^{(i)}, \text{and } C^b = \bigcup_{i=1}^{t_c} x_i^{(i)}.$$

$$D^b = \bigcup_{i=1}^{t_c} x_i^{(i)}, x \in \text{eid.}$$

$$U/C^b = \{C_1, C_2, ..., C_{t_c}\}, \text{ and } U/C^c = \{C_1, C_2, ..., C_{t_c}\}.$$  

Suppose, if two decision equivalence classes $D_x \cup D_y$ is made rough into new decision equivalence class $D_x \cup D_y$ ($f(D_x) = w$) and on the slice $r_1$ two decision equivalence classes $D_x$ and $D_y$ is made rough sympathetic into $D_x$ and $D_y$:

i. $D_x \cup D_y = D_x$, 

ii. $\forall C_j \in U/C^c: \text{Sup}(C_j \cap D_x) + \text{Sup}(C_j \cap D_y) = \text{Sup}(C_j \cap D_x, D_y)$, with $h = 1, 2, ..., t_c$.

Prove

i. $\forall C_j \in U/C^c: \text{Sup}(C_j \cap D_x) + \text{Sup}(C_j \cap D_y) = \text{Sup}(C_j \cap D_x, D_y)$, with $h = 1, 2, ..., t_c$.

On the other hand, apply the results of the theorem 2.1 we have $\forall u, a: f(D_x \cap a) = f(D_y \cap a) = \{f(u, a) = \{f(D_x \cap a) = f(D_y \cap a) \Rightarrow u \in D_x \cup D_y$. Completely similar, if $u \in D_y$ then we also proved $u \in D_x \cup D_y$.

So inference: $(D_x \cup D_y) \subseteq D_x \cup D_y$.

On the contrary, suppose $u \in D_x \cup D_y$, because $D_x$ and $D_y$ is made rough into $D_x \cup D_y$ should apply the results of the theorem 2.1 we have: $\forall u, a: f(D_x \cap a) = f(D_y \cap a) = \{f(u, a) = \{f(D_x \cap a) = f(D_y \cap a)$.

- If $f(u, a) = w \Rightarrow f(u, a) = f(D_x \cap a) = u \in D_x \cup D_y$.

- If $f(u, a) = y \Rightarrow f(u, a) = f(D_y \cap a) = u \in D_y$.

So inference: $D_x \cup D_y \subseteq u \in D_x \cup D_y$.

Therefore, from $u \in D_x \cup D_y \Rightarrow u \in D_x \cup D_y$.

So: $(D_x \cup D_y) \subseteq (D_x \cup D_y)$.

(8)

Combined (7) and (8) we have: $(D_x \cup D_y) = D_x \cup D_y$.

ii. Because $D_x$, $D_y$ are decision equivalence classes, so we have: $D_x \cup D_y = \emptyset$.

On the other hand: $\forall C_j \in U/C^c: \text{Sup}(C_j \cap D_x) = |C_j \cap D_x| = |C_j \cap D_x| = |C_j \cap D_y|$.

We have: $D_x \cap D_y = \emptyset \Rightarrow (D_x \cap C_j) \cap (D_y \cap C_j) = \emptyset$.

Inferred: $\text{Sup}(C_j \cap D_x) = |C_j \cap D_x| = |C_j \cap D_y| + |C_j \cap D_y| = \text{Sup}(C_j \cap D_x) + \text{Sup}(C_j \cap D_y)$.

So inference: $\forall C_j \in U/C^c: \text{Sup}(C_j \cap D_x) + \text{Sup}(C_j \cap D_y) = \text{Sup}(C_j \cap D_x)$, with $h = 1, 2, ..., t_c$.

Thus, we see two columns of the support matrix on the slice corresponds to two decision equivalence classes $D_x$, $D_y$ made rough sympathetic into a new column corresponding to the decision equivalence class $D_x$. The value of each element of the new column corresponds to $D_x$ as the total value of two elements of two columns corresponding to two decision equivalence classes $D_x$ and $D_y$.

IV. CONCLUSIONS

From the initial results on the decision block, the paper proposes and demonstrates some of the results of the relationship between roughing, smoothing the values of...
conditional attributes or decisions for conditionalequivalents classe or decision equivalence classes on the decision blocks and on the slices. The smoothing of conditionalequivalents classes or decision equivalence classes on the decision block have a sympathetic partially the smoothing of conditionalequivalents classes or decision equivalence classes respectively on the slice. The roughening of conditionalequivalents classes or decision equivalence classes on the decision block have a sympathetic the roughening of conditionalequivalents classes or decision equivalence classes on the slice. From these results, calculation of support matrix on the slices same is define as the calculation of the support matrix on the block when the smoothing, roughening of conditionalequivalents classes or decision equivalence classes.

In special cases, the index set \( id = \{x\} \), the information blocks degenerate into information systemsthenthese results coincide with the results reported by many authors for the information system. On the basis of these results we can study the reverse relationship between slices of information block with that block itself, in case the objects of the information block are changed, some other results may be considered in individual cases of information blocks, it adds the theoretical results of the exploitation of decision rules on information blocks.

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VI. REFERENCES