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CONNECTED MAJORITY DOMINATION ON PRODUCT GRAPHS

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Abstract: This paper contributes a connected majority dominating set of a product graph G. A connected majority domination number $\gamma_{CM}(G)$ is determined for some product graphs such as Grid, Cylinder and Torus. Next we study connected majority dominating sets for generalized Petersen graphs and bounds of $\gamma_{CM}(G)$ are also established.

Keywords: Dominating set, Majority dominating set, Connected majority dominating set

I. INTRODUCTION

Definition 1.1 [2] Let G be a finite and simple graph with a vertex set V(G) and an edge set E(G). Then |V(G)| = p and |E(G)| = q. A subset S of V(G) is said to be a ***dominating set*** of G if every vertex in (V-S) is adjacent to at least one vertex in S. A dominating set is called ***minimal dominating set*** if no proper subset of S is a dominating set. The minimum cardinality of the minimal dominating set of G is called the ***domination number*** of G, denoted by $\gamma(G)$.

Definition 1.2 [1] A dominating set S is said to be a ***connected dominating set*** if the subgraph $\langle S \rangle$ induced by S is connected in G. A connected dominating set S is minimal if no proper subset of S is a connected dominating set. The minimum cardinality of the minimal connected dominating set of G is called the ***connected domination number***, denoted by $\gamma_c(G)$.

Definition 1.3 [5] A subset S of V(G) is said to be a ***Majority Dominating Set*** **(MDS)** if at least half of the vertices of V(G) are either in S or adjacent to elements of S. A majority dominating set S is minimal if no proper subset of S is a majority dominating set. The minimum cardinality of a minimal majority dominating set is called ***majority domination number***, denoted by $\gamma_M(G)$.

Definition 1.4 [6] A vertex v is called ***majority dominating vertex*** if the degree of v, $d(v) \geq \lceil \frac{p}{2} \rceil - 1$. For $S \subseteq V(G)$, a vertex $v \in S$ is called an ***enclave*** of S if $N[v] \subseteq S$ and $v \in S$ is an ***isolate*** of S if $N(v) \subseteq V - S$.

Definition 1.5 [4] A subset $S \subseteq V(G)$ is a ***Connected Majority Dominating set*** **(CMDS)** if

(i) S is a majority dominating set and

(ii) the subgraph $\langle S \rangle$ induced by S is connected in G.

The connected majority dominating set S is minimal if no proper subset of S is a connected majority dominating set. The minimum cardinality of a minimal connected majority dominating set is called the ***connected majority domination number*** and is denoted by $\gamma_{CM}(G)$. The maximum cardinality of a ***minimal connected majority dominating set*** of G is called *upper connected majority domination number* of G, denoted by $\Gamma_{CM}(G)$.

Theorem 1.6 [4] Let S be a Connected Majority Dominating (CMD) set of a graph G. Then S is minimal if and only if for every vertex $v \in S$, either the condition (i) or the condition (ii) holds.

(i) $|N[S]| = \lceil \frac{p}{2} \rceil$, then either v is an enclave of S or pn[v,S] $\cap (V - S) \neq \varphi$ and $\langle S \rangle$ is connected.

(ii) $|N[S]| > \lceil \frac{p}{2} \rceil$, then $|pn[v,S]| > |N[S]| - \lceil \frac{p}{2} \rceil$ and $\langle S \rangle$ is connected.

II. CMD NUMBER FOR GRID GRAPHS

Definition 2.1 [7] Let G and H be any two connected graphs with the vertex set (u_1,u_2,… u_n) and (v_1,v_2,… v_n) respectively. The (*Cartesian*) **product graph** K = G X H has V(K) = V(G) X V(H) and vertices (u_1,v_1) and (u_2,v_2) in V(K) are adjacent if and only if either $u_1 = u_2$ and v_1v_2 in E(H) or $v_1 = v_2$ and u_1u_2 in E(G). The ***grid*** is P_i X P_j; the ***cylinder*** is C_i X P_j, for $i \geq 3$ and $j \geq 3$; and the ***torus*** C_i X C_j, for $i \geq 3$ and $j \geq 3$.

Theorem 2.2

For a grid $G = P_2 \, X \, P_j$, $j \geq 3$,

$$\gamma_{CM}(G) = \begin{cases} \lceil \frac{p}{4} \rceil , & \text{if } j \text{ is odd} \\ \lceil \frac{p-1}{4} \rceil , & \text{if } j \text{ is even} \end{cases}$$

Proof: Consider the grid graph $G = P_2 \, X \, P_j$ (ladder graph) with vertex sets $\{v_{11}, v_{12}, \ldots v_{1j}\}$ in the first row and $\{v_{21}, v_{22}, \ldots v_{2j}\}$ in the second row respectively.

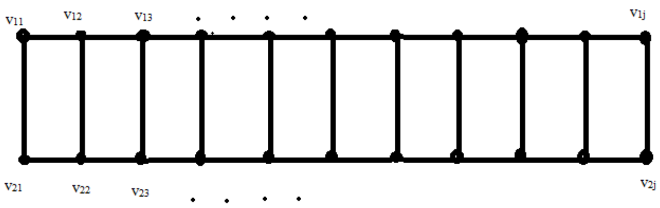

Figure 1. $G = P_2 \, X \, P_j$

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Case (i): Let j be odd. Consider the subset S of V(G) as follows $S = \{v_{12}, v_{13}, \dots v_{1t}\}$ with $|S| = \left\lceil \frac{p}{4} \right\rceil$ = t. $|N[S]| = \sum_{i=1}^{\lceil \frac{p}{4} \rceil} d(v_{1i}) - (|S| - 2) = \frac{p}{2} + 2 > \left\lceil \frac{p}{2} \right\rceil$. $\Longrightarrow$ S is a MD set of G. Since every vertex of S is of distance one, the induced subgraph ⟨S⟩ of S is connected. Hence, S is a CMD set of G.

$$\therefore \gamma_{CM}(G) \le |S| = \left\lceil \frac{p}{4} \right\rceil \quad (1)$$

Let $S' = S - \{v\}$ with $|S'| = \left\lceil \frac{p}{4} \right\rceil - 1$. Then $|N[S']| = \sum_{i=1}^{\lceil \frac{p}{4} \rceil - 1} d(v_{1i}) - (|S'| - 1) = \frac{p}{2} - 1 < \left\lceil \frac{p}{2} \right\rceil \Longrightarrow S'$ would not be a MD set of G. $\therefore \gamma_{CM}(G) > |S'| = \left\lceil \frac{p}{4} \right\rceil - 1$. Then

$$\gamma_{CM}(G) \ge \left\lceil \frac{p}{4} \right\rceil \quad (2)$$

From (1) and (2), we get $\gamma_{CM}(G) = \left\lceil \frac{p}{4} \right\rceil$,if j is odd

***Case (ii)*:** Let j be even. Choose first row vertices from V(G) and form the subset as $S = \{v_{12}, v_{13}, \dots v_{1t}\}$ with the size $|S| = \left\lceil \frac{p-1}{4} \right\rceil$.

Now, $|N[S]| = \sum_{i=1}^{\lceil \frac{p-1}{4} \rceil} d(v_{1i}) - (|S| - 2) = 2\left\lceil \frac{p-1}{4} \right\rceil + 2 > \left\lceil \frac{p}{2} \right\rceil$.

$\Longrightarrow$ S is a MD set of G. Since every vertex of S is of distance one, the induced subgraph ⟨S⟩ of S is connected. Hence, S is a CMD set of G.

$$\therefore \gamma_{CM}(G) \le |S| = \left\lceil \frac{p-1}{4} \right\rceil \quad (3)$$

Applying the same argument as in case (i) we get,

$$\gamma_{CM}(G) \ge \left\lceil \frac{p-1}{4} \right\rceil \quad (4)$$

From (3) and (4), we get $\gamma_{CM}(G) = \left\lceil \frac{p-1}{4} \right\rceil$ if j is even.

Theorem 2.3

For a grid $G = P_3 \, X \, P_j$, $j \ge 3$, $\gamma_{CM}(G) = \left\lceil \frac{p}{6} \right\rceil$

Proof: Consider the grid graph $G = P_3 \, X \, P_j$, $j \ge 3$. Let $V(G) = \{v_{11}, v_{12}, \dots, v_{1j}, v_{21}, \dots, v_{2j}, v_{31}, \dots v_{3j}\}$ and it forms I row, II row and III row respectively for G. Form a subset of V(G) as $S = \{v_{22}, v_{23}, \dots, v_{2t}\}$ with $|S| = \left\lceil \frac{p}{6} \right\rceil$. Since each vertex of S covers 3 vertices vertically, but the first and last vertices of S cover 4 vertices.

$\therefore |N[S]| = 3|S| + 2 \ge \left\lceil \frac{p}{2} \right\rceil$. Hence, S is a MD set. Since every vertex of S is of distance one, the induced subgraph ⟨S⟩ of S is connected. $\Longrightarrow$ S is a CMD set of G.

Now,

$$\gamma_{CM}(G) \le |S| = \left\lceil \frac{p}{6} \right\rceil \quad (1)$$

Let $S' = S - \{v\}$ with $|S'| = \left\lceil \frac{p}{6} \right\rceil - 1$.

Now $|N[S']| = 3|S'| - 3 + 2 = \frac{p}{2} - 1 < \left\lceil \frac{p}{2} \right\rceil \Rightarrow S'$ would not be a MD set of G.

$$\therefore \gamma_{CM}(G) > |S'| = \left\lceil \frac{p}{6} \right\rceil - 1$$

$$\Rightarrow \gamma_{CM}(G) \ge \left\lceil \frac{p}{6} \right\rceil \quad (2)$$

From (1) and (2), we obtain $\gamma_{CM}(G) = \left\lceil \frac{p}{6} \right\rceil$.

Corollary 2.4

For a grid $G = P_4 \, X \, P_j$,$j \ge 4$, $\gamma_{CM}(G) = \left\lceil \frac{p}{6} \right\rceil$.

Theorem 2.5

For a grid $G = P_5 \, X \, P_j$, $j \ge 5$,

$$\gamma_{CM}(G) = \begin{cases} \frac{p}{6} , & if \; j \equiv 0 \,(mod\; 6) \\ \left\lceil \frac{p}{6} \right\rceil , & if \; j \equiv 1 \,(mod\; 6) \\ \left\lceil \frac{p}{6} \right\rceil , & if \; j \equiv 2,3,4,5 \,(mod\; 6) \end{cases}$$

Proof: Consider the grid graph $G = P_5 \, X \, P_j$, $j \ge 5$, with $V(G) = \{v_{11}, \dots v_{1j}, v_{21}, \dots v_{2j}, v_{31}, \dots v_{3j}, v_{41}, \dots v_{4j}, v_{51}, \dots v_{5j}\}$ and it contains 5 rows.

Case (i): Let $j \equiv 0 (mod 6)$. Consider the set $S = \{v_{21}, v_{22}, \dots\}$ with $|S| = \frac{p}{6}$.

Now, $|N[S]| = 3\,|S| + 2 = \frac{p}{2} + 2 \ge \left\lceil \frac{p}{2} \right\rceil$. $\therefore$ S is a majority dominating set of G. Since every vertex of S is of distance one, the induced subgraph ⟨S⟩ of S is connected. $\therefore$ S is a connected majority dominating set of G. Then

$$\gamma_{CM}(G) \le |S| = \frac{p}{6} \quad (1)$$

Suppose, let $S' = S - \{v\}$ with $|S'| = \frac{p}{6} - 1$.

Then $|N[S']| = 3|S'| - 3 = \frac{p}{2} - 6 < \left\lceil \frac{p}{2} \right\rceil \Longrightarrow S'$ would not be a MD set of G and $\therefore \gamma_{CM}(G) > |S'| = \frac{p}{6} - 1$. Hence

$$\gamma_{CM}(G) \ge \frac{p}{6} \quad (2)$$

From (1) and (2) we get, $\gamma_{CM}(G) = \frac{p}{6}$, $if \; j \equiv 0 (mod\; 6)$.

Case (ii): Let $j \equiv 1 (mod\; 6)$. Consider the set $S = \{v_{22}, v_{23}, \dots\}$ with $|S| = \left\lceil \frac{p}{6} \right\rceil$. Now, $|N[S]| = 3\,|S| + 2 = 3\left\lceil \frac{p}{6} \right\rceil + 2 = \frac{p}{2} + 2 \ge \left\lceil \frac{p}{2} \right\rceil$. $\Longrightarrow$ S is a MD set of G. Since every vertex of S is of distance one, S is a CMD set of G. Then

$$\gamma_{CM}(G) < |S| = \left\lceil \frac{p}{6} \right\rceil \quad (3)$$

Suppose $S' = S - \{v\}$ with $|S'| = \left\lceil \frac{p}{6} \right\rceil - 1$. Applying the same argument as in case (i), we get S' would not be a MD set of G.

$$\therefore \gamma_{CM}(G) > |S'| = \left\lceil \frac{p}{6} \right\rceil - 1 \quad (4)$$

From (3) and (4) we get, $\gamma_{CM}(G) = \left\lceil \frac{p}{6} \right\rceil$, $if \; j \equiv 1 (mod\; 6)$.

Case(iii): Let $j \equiv 2,3,4,5 (mod\; 6)$. Consider the set $S = \{v_{22}, v_{23}, \dots\}$ with $|S| = \left\lceil \frac{p}{6} \right\rceil$ .Now $|N[S]| = 3\,|S| + 2 = \frac{p}{2} + 2 > \left\lceil \frac{p}{2} \right\rceil$. $\Longrightarrow$ S is a MD set of G. Since every vertex of S is of distance one, S is a CMD set of G.

$$\therefore \gamma_{CM}(G) \le |S| = \left\lceil \frac{p}{6} \right\rceil \quad (5)$$

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Applying the same argument as in case (i), we get, $\gamma_{CM}(G) \geq \left\lceil \frac{p}{6} \right\rceil$ (6)

From (5) and (6) we get, $\gamma_{CM}(G) = \left\lceil \frac{p}{6} \right\rceil$, if $j \equiv 2,3,4,5 (mod\ 6)$.

Corollary 2.6

For a grid $G = P_6\ X\ P_j$, $j \geq 6$,

$$\gamma_{CM}(G) = \begin{cases} \frac{p}{6}, & if\ j \equiv 0\ (mod\ 6) \\ \left\lceil \frac{p}{6} \right\rceil, & if\ j \equiv 1,5\ (mod\ 6) \\ \left\lfloor \frac{p}{6} \right\rfloor, & if\ j \equiv 2,3,4\ (mod\ 6) \end{cases}$$

III. CMD NUMBER FOR CYLINDER AND TORUS GRAPHS

Proposition 3.1

1. For a Cylinder $G = C_3\ X\ P_j$, $\gamma_{CM}(C_3\ X\ P_j) = \left\lceil \frac{p}{6} \right\rceil$ for $j \geq 2$.
2. For a Torus $G = C_3\ X\ C_j$, $\gamma_{CM}(C_3\ X\ C_j) = \left\lceil \frac{p}{6} \right\rceil$ for $j \geq 3$

Proof:

1. Let $G = C_3\ X\ P_j$, $j \geq 2$. Let $V(G) = \{v_{11}, v_{12}, \ldots, v_{1j}, v_{21}, \ldots, v_{2j}, v_{31}, \ldots v_{3j}\}$. Consider a subset S of V(G) such that $S = \{v_{12}, v_{13}, \ldots, v_{1t}\}$ with $|S| = \left\lceil \frac{p}{6} \right\rceil$. Then $|N[S]| = \sum_{j=1}^{t} d(v_{1j}) - |S| + 2 = 3\left\lceil \frac{p}{6} \right\rceil + 2 > \left\lceil \frac{p}{2} \right\rceil$. $\Longrightarrow$ S is a MD set of G. Since every vertex of S is of distance one, S is a CMD set of G. Then applying the same argument as in Theorem 2.3, we get $\gamma_{CM}(G) = \left\lceil \frac{p}{6} \right\rceil$.

2. Let $G = C_3\ X\ C_j$, $j \geq 3$ be the 4-regular graph. Let $V(G) = \{v_{11}, v_{12}, \ldots, v_{1j}, v_{21}, \ldots, v_{2j}, v_{31}, \ldots v_{3j}\}$. Consider the set $S \subseteq V(G)$ as $S = \{v_{12}, v_{13}, \ldots, v_{1t}\}$ with $|S| = \left\lceil \frac{p}{6} \right\rceil$. Then $|N[S]| = \sum_{j=1}^{t} d(v_{1j}) - |S| + 2 = 3\left\lceil \frac{p}{6} \right\rceil + 2 > \left\lceil \frac{p}{2} \right\rceil$. Applying the same argument as in Theorem 2.3, we get $\gamma_{CM}(G) = \left\lceil \frac{p}{6} \right\rceil$.

Example 3.2 Consider the graph G = C_3 X P_5. Here |V(G)| = p = 15, S = {v_{12},v_{13}}is a CMD set of G. Hence, $\therefore\ \gamma_{CM}(G) = 2 = \left\lceil \frac{15}{6} \right\rceil$

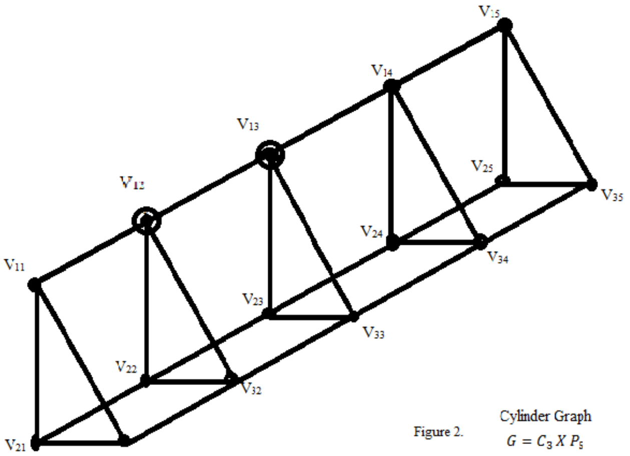

Figure 2. Cylinder Graph $G = C_3\ X\ P_5$

Example 3.3 Consider the graph G = C_3 X C_5. Here |V(G)| = p = 15, S = {v_{11},v_{12}}is a CMD set of G. Hence, $\therefore\ \gamma_{CM}(G) = 2 = \left\lceil \frac{15}{6} \right\rceil$

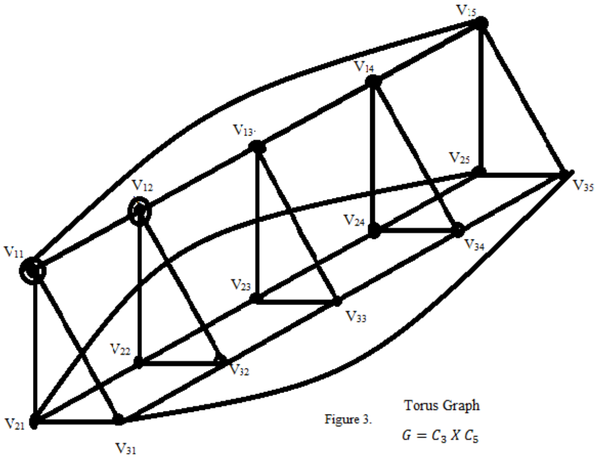

Figure 3. Torus Graph $G = C_3\ X\ C_5$

Corollary 3.4

1. For a cylinder $G = C_4\ X\ P_j$, $\gamma_{CM}(C_4\ X\ P_j) = \left\lceil \frac{p}{6} \right\rceil$ for $j \geq 2$.
2. For a Torus $G = C_4\ X\ C_j$, $\gamma_{CM}(C_4\ X\ C_j) = \left\lceil \frac{p}{6} \right\rceil$ for $j \geq 3$.

Proposition 3.5

1. For a cylinder $G = C_5\ X\ P_j$, $j \geq 5$,

$$\gamma_{CM}(G) = \begin{cases} \frac{p}{6} & , if\ j \equiv 0 (mod\ 6) \\ \left\lceil \frac{p}{6} \right\rceil & , if\ j \equiv 1 (mod\ 6) \\ \left\lfloor \frac{p}{6} \right\rfloor & , if\ j \equiv 2,3,4,5 (mod\ 6) \end{cases}$$

2. For a Torus $G = C_5\ X\ C_j$, $j \geq 5$,

$$\gamma_{CM}(G) = \begin{cases} \frac{p}{6} & , if\ j \equiv 0 (mod\ 6) \\ \left\lceil \frac{p}{6} \right\rceil & , if\ j \equiv 1 (mod\ 6) \\ \left\lfloor \frac{p}{6} \right\rfloor & , if\ j \equiv 2,3,4,5 (mod\ 6) \end{cases}$$

IV. CMD NUMBER FOR GENERALIZED PETERSEN GRAPH

Result 4.1 For a Petersen graph with p = 10, q = 15, $\therefore\ \gamma_{CM}(G) = 2 = \left\lceil \frac{p-1}{4} \right\rceil$.

Definition 4.2 For each $n \geq 3$ and $0 < k < n$, $P(n,k)$ denotes the Generalized Petersen graph with vertex set $V(G) = \{u_1, u_2 \ldots u_n, v_1, v_2, \ldots v_n\}$ and the edge set $E(G) = \{u_i u_{i+1(mod\ n)}, u_i v_i, v_{i+k(mod\ n)}\}$, $1 \leq i \leq n$.

Theorem 4.3 For a generalized Petersen graph $G = P(n,1)$, $\gamma_{CM}(G) = \left\lceil \frac{n}{2} \right\rceil - 1$.

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Proof: Let us consider the vertices of the generalized Petersen graph $G = P(n,1)$ can be partitioned into two sets V_1 and V_2 such that $V = V_1 \cup V_2$ where the inner polygon has the vertex set as $V_1 = \{v_1, v_2, \dots, v_n\}$ and the outer polygon has the vertex set as $V_2 = \{u_1, u_2, \dots, u_n\}$.

Let |V(G)| = p = 2n. Consider the set $S \subseteq V$ such that $S = \{v_1, v_2, \dots, v_{\lceil \frac{n}{2} \rceil - 1}\}$ with $|S| = \lceil \frac{n}{2} \rceil - 1$ or $|S| = \lfloor \frac{p-1}{4} \rfloor$.

Now, let $|N[S]| = \sum_{i=1}^{\lceil \frac{n}{2} \rceil - 1} d(v_i) - (|S| - 2)$ $= 2\left(\lceil \frac{n}{2} \rceil - 1\right) + 2 = \lceil n \rceil \geq \lceil \frac{p}{2} \rceil$

$\therefore$ S is a MD set of G.

Since every vertex of S are of distance one, S is a CMD set of G.

$$\gamma_{CM}(G) \leq |S| = \left\lceil \frac{n}{2} \right\rceil - 1 \qquad (1)$$

Now, let $S' = S - \{v\}$ with $|S'| = \left(\lceil \frac{n}{2} \rceil - 1\right) - 1$. Then,

$$|N[S']| = \sum_{i=1}^{\lceil \frac{n}{2} \rceil - 2} d(v_i) - (|S| - 2) = 2\left(\left\lceil \frac{n}{2} \right\rceil - 2\right) + 2 = n - 2 < \left\lceil \frac{p}{2} \right\rceil.$$

$\therefore$ S' would not be a MD set of G.

$$\Rightarrow \gamma_{CM}(G) > |S'| = \left\lceil \frac{n}{2} \right\rceil - 2 \qquad (2)$$

From (1) and (2) we get,

$\gamma_{CM}(G) = \lceil \frac{n}{2} \rceil - 1$ or $\gamma_{CM}(G) = \lfloor \frac{p-1}{4} \rfloor$.

Corollary 4.4 For a generalized Petersen graph $G = P(n,2)$, $\gamma_{CM}(G) = \lfloor \frac{p-1}{4} \rfloor$.

Corollary 4.5 For a generalized Petersen graph $G = P(n,3)$, $\gamma_{CM}(G) = \lfloor \frac{p-1}{4} \rfloor$.

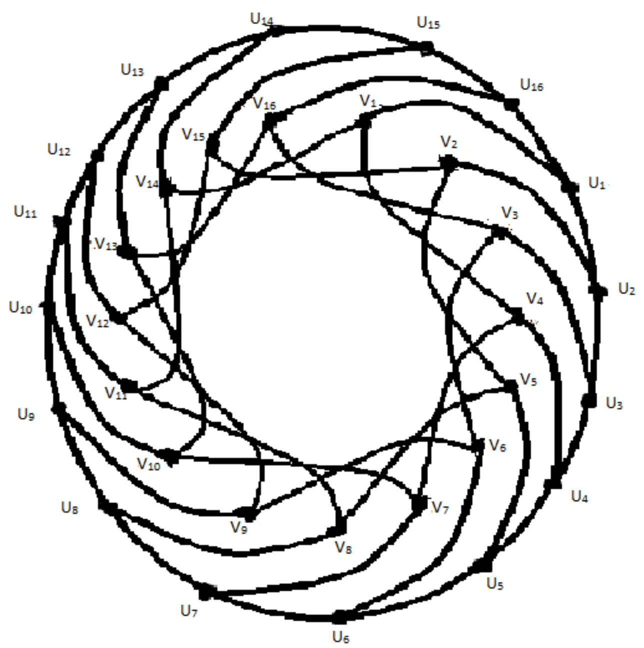

Figure 4. Generalised Petersen Graph G = P(16,3)

Corollary 4.6 For cubic graphs Grinberge and Tutte graph with p = 46, the CMD number for these graphs are $\gamma_{CM}(G) = 11 = \lfloor \frac{p-1}{4} \rfloor$.

Corollary 4.7 For all cubic graphs the CMD number $\gamma_{CM}(G) = \lfloor \frac{p-1}{4} \rfloor$.

V. BOUNDS FOR $\gamma_{CM}(G)$

Proposition 5.1 Let H be a connected spanning subgraph of a graph G. Then $\gamma_{CM}(G) \leq \gamma_{CM}(H)$.

Proposition 5.2 For any tree G with $p \geq 3$ vertices, $\lceil \frac{p}{2} \rceil - e \leq \gamma_{CM}(G) \leq p - e$.

Proof Let G be a tree of order $p \geq 3$ with $'e'$ end vertices.

Case(i): If the graph G has the end vertices $e < \lceil \frac{p}{2} \rceil$, then $\gamma_{CM}(G) = \lceil \frac{p}{2} \rceil - e$.

Case(ii): If the graph G has the end vertices $e \geq \lceil \frac{p}{2} \rceil$, then $\gamma_{CM}(G) \leq p - e$.

The bounds are sharp. For example, let $G = S(K_{1,10})$. Then $\gamma_{CM}(G) = 1 = \lceil \frac{p}{2} \rceil - e$.

And let $= K_{1,10}$. Then $\gamma_{CM}(G) = 1 = p - e$.

Corollary 5.3 For a product graph, the lower and upper bounds are $\lceil \frac{p}{6} \rceil \leq \gamma_{CM}(G) \leq \lfloor \frac{p}{4} \rfloor$. These bounds are sharp.

For example, 1. Let G = P_2 X P_{13}. Then $\gamma_{CM}(G) = 6 = \lfloor \frac{p}{4} \rfloor$.

2. Let G = C_4 X P_7. Then $\gamma_{CM}(G) = 4 = \lceil \frac{p}{6} \rceil$.

Proposition 5.4 For any graph G, $\gamma_M(G) \leq \gamma_{CM}(G) \leq \gamma_c(G)$.

For example, 1. let $G = K_p$, a Complete graph of $p \geq 2$. Then $\gamma_M(G) = \gamma_{CM}(G) = \gamma_c(G)$.

2. For a Caterpillar G, with $p = 22$, $\gamma_M(G) = 3, \gamma_{CM}(G) = 5$ and $\gamma_c(G) = 11$. Hence we get the inequality as $\gamma_M(G) < \gamma_{CM}(G) < \gamma_c(G)$.

VI. CONCLUSION

In this article, researcher thus introduced and discussed a new type of Domination parameter of a graph G. Connected Majority Domination number $\gamma_{CM}(G)$ is defined and it is determined for some classes of product graphs. Then bounds of $\gamma_{CM}(G)$ and Connected Majority Domination number for generalized Petersen graph are also established.

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